Filters and Attenuators

CHAPTER 17

Wave filters were first invented by G.A Campbell and O.I. Lobel of the Bell Telephone Laboratories. A filter is a reactive network that freely passes the desired bands of frequencies while almost totally suppressing all other bands. A filter is constructed from purely reactive elements, for otherwise the attenuation would never becomes zero in the pass band of the filter network. Filters differ from simple resonant circuits in providing a substantially constant transmission over the band which they accept; this band may lie between any limits depending on the design. Ideally, filters should produce no attenuation in the desired band, called the *transmission* band or *pass* band, and should provide total or infinite attenuation at all other frequencies, called *attenuation* band or *stop* band. The frequency which separates the transmission band and the attenuation band is defined as the cut-off frequency of the wave filters, and is designated by f_c.

Filter networks are widely used in communication systems to separate various voice channels in carrier frequency telephone circuits. Filters also find applications in instrumentation, telemetering equipment, etc. where it is necessary to transmit or attenuate a limited range of frequencies.

A filter may, in principle, have any number of pass bands separated by attenuation bands. However, they are classified into four common types, viz. low pass, high pass, band pass and band elimination.

Decibel and Neper

The attenuation of a wave filter can be expressed in decibels or nepers. Neper is defined as the natural logarithm of the ratio of input voltage (or current) to the output voltage (or current), provided that the network is properly terminated in its characteristic impedance Z_0.
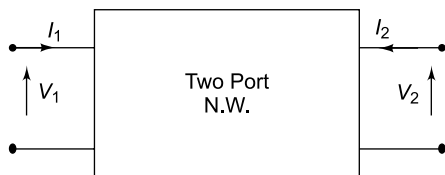

Fig. 17.1 (a)

From Fig. 17.1 (a) the number of nepers, $N = \log_e \left| \dfrac{V_1}{V_2} \right|$ or $\log_e \left| \dfrac{I_1}{I_2} \right|$.

A neper can also be expressed in terms of input power, P_1 and the output power P_2 as $N = 1/2 \log_e P_1/P_2$.

A decibel is defined as ten times the common logarithms of the ratio of the input power to the output power.

$$\therefore \quad \text{Decibel } D = 10 \log_{10} \frac{P_1}{P_2}$$

The decibel can be expressed in terms of the ratio of input voltage (or current) and the output voltage (or current.)

$$D = 20 \log_{10} \left| \frac{V_1}{V_2} \right| = 20 \log_{10} \left| \frac{I_1}{I_2} \right|$$

$\therefore$ One decibel is equal to 0.115 N.

Low Pass Filter

By definition, a low pass (LP) filter is one which passes without attenuation all frequencies up to the cut-off frequency f_c, and attenuates all other frequencies greater than f_c. The attenuation characteristic of an ideal LP filter is shown in Fig. 17.1 (b). This transmits currents of all frequencies from zero up to the cut-off frequency. The band is called pass band or transmission band. Thus, the pass band for the LP filter is the frequency range 0 to f_c. The frequency range over which transmission does not take place is called the stop band or attenuation band. The stop band for a LP filter is the frequency range above f_c.
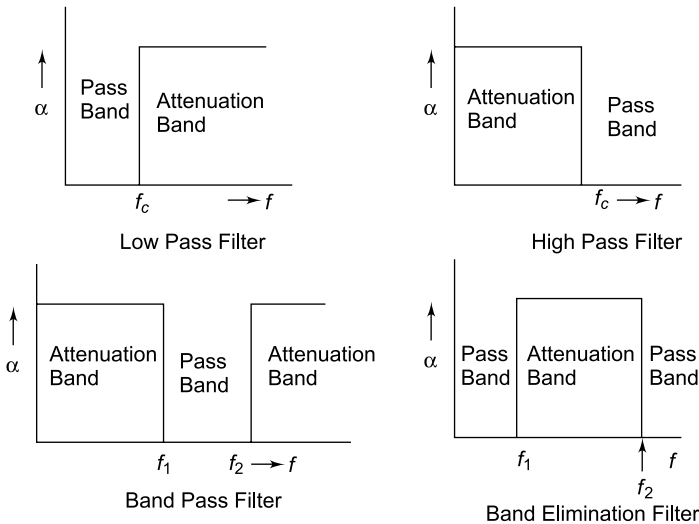

Fig. 17.1 (b)

High Pass Filter

A high pass (HP) filter attenuates all frequencies below a designated cut-off frequency, f_c, and passes all frequencies above f_c. Thus the pass band of this filter is the frequency range above f_c, and the stop band is the frequency range below f_c. The attenuation characteristic of a HP filter is shown in Fig. 17.1 (b).

Band Pass Filter

A band pass filter passes frequencies between two designated cut-off frequencies and attenuates all other frequencies. It is abbreviated as *BP filter*. As shown in Fig.17.1 (b), a BP filter has two cut-off frequencies and will have the pass band $f_2 - f_1$; f_1 is called the lower cut-off frequency, while f_2 is called the upper cut-off frequency.

Band Elimination Filter

A band elimination filter passes all frequencies lying outside a certain range, while it attenuates all frequencies between the two designated frequencies. It is also referred as band stop filter. The characteristic of an ideal band elimination filter is shown in Fig. 17.1 (b).

All frequencies between f_1 and f_2 will be attenuated while frequencies below f_1 and above f_2 will be passed.

17.2   FILTER NETWORKS

Ideally a filter should have zero attenuation in the pass band. This condition can only be satisfied if the elements of the filter are dissipationless, which cannot be realized in practice. Filters are designed with an assumption that the elements of the filters are purely reactive. Filters are made of symmetrical T, or π sections. T and π sections can be considered as combinations of unsymmetrical L sections as shown in Fig. 17.2.
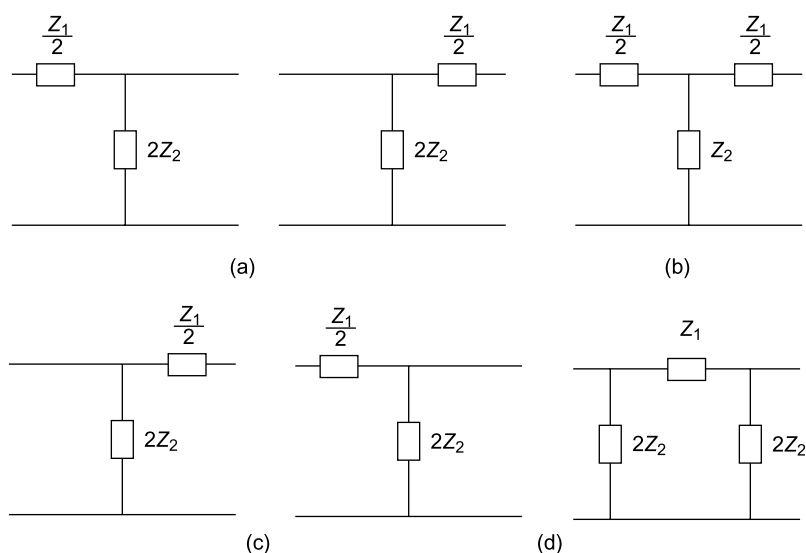

Fig. 17.2

The ladder structure is one of the commonest forms of filter network. A cascade connection of several T and π sections constitutes a ladder network. A common form of the ladder network is shown in Fig. 17.3.

Figure 17.3 (a) represents a T section ladder network, whereas Fig. 17.3 (b) represents the π section ladder network. It can be observed that both networks are identical except at the ends.
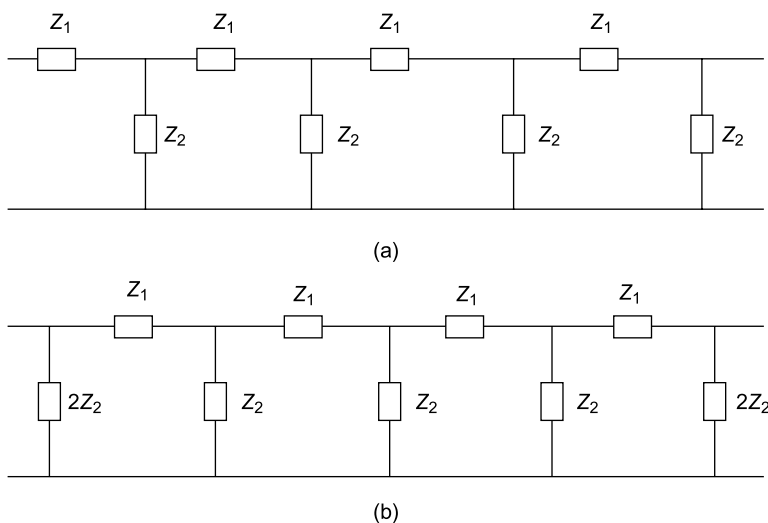
(a)

(b)

Fig. 17.3

17.3 EQUATIONS OF FILTER NETWORKS

The study of the behaviour of any filter requires the calculation of its propagation constant γ, attenuation α, phase shift β and its characteristic impedance Z_0.

T-Network

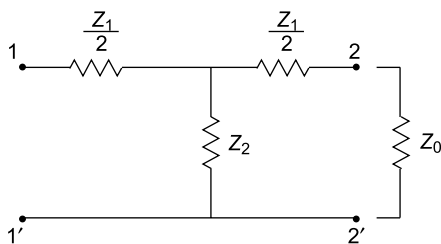Consider a symmetrical *T*-network as shown in Fig. 17.4.

Fig. 17.4

As has already been mentioned in Section 16.13, if the image impedances at port 1-1$'$ and port 2-2$'$ are equal to each other, the image impedance is then called the characteristic, or the iterative impedance, Z_0. Thus, if the network in Fig. 17.4 is terminated in Z_0, its input impedance will also be Z_0. The value of input impedance for the *T*-network when it is terminated in Z_0 is given by

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\dfrac{Z_1}{2} + Z_0\right)}{\dfrac{Z_1}{2} + Z_2 + Z_0}$$

also $\quad Z_{in} = Z_0$

$\therefore \quad$
$$Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\dfrac{Z_1}{2} + Z_0\right)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1 Z_2 + 2Z_2 Z_0)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

The characteristic impedance of a symmetrical *T*-section is

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2} \tag{17.1}$$

Z_{0T} can also be expressed in terms of open circuit impedance Z_{0c} and short circuit impedance Z_{sc} of the *T*-network. From Fig. 17.4, the open circuit impedance $Z_{0c} = \frac{Z_1}{2} + Z_2$ and

$$Z_{sc} = \frac{Z_1}{2} + \frac{\dfrac{Z_1}{2} \times Z_2}{\dfrac{Z_1}{2} + Z_2}$$

$$Z_{sc} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$$

$$Z_{0c} \times Z_{sc} = Z_1Z_2 + \frac{Z_1^2}{4}$$

$$= Z_{0T}^2 \quad \text{or} \quad Z_{0T} = \sqrt{Z_{0c}Z_{sc}} \tag{17.2}$$

Propagation Constant of T-Network

By definition the propagation constant γ of the network in Fig. 17.5 is given by
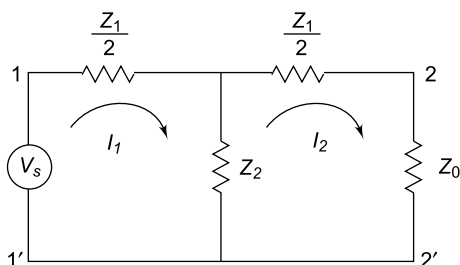$\gamma = \log_e I_1/I_2$

Writing the mesh equation for the 2nd mesh, we get

Fig. 17.5

$$I_1Z_2 = I_2\left(\frac{Z_1}{2} + Z_2 + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{\dfrac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^\gamma$$

$$\therefore \quad \frac{Z_1}{2} + Z_2 + Z_0 = Z_2e^\gamma$$

$$Z_0 = Z_2(e^\gamma - 1) - \frac{Z_1}{2} \tag{17.3}$$

The characteristic impedance of a *T*-network is given by

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \qquad (17.4)$$

Squaring Eqs 17.3 and 17.4 and subtracting Eq. 17.4 from Eq. 17.3, we get

$$Z_2^2 (e^\gamma - 1)^2 + \frac{Z_1^2}{4} - Z_1 Z_2 (e^\gamma - 1) - \frac{Z_1^2}{4} - Z_1 Z_2 = 0$$

$$Z_2^2 (e^\gamma - 1)^2 - Z_1 Z_2 (1 + e^\gamma - 1) = 0$$

$$Z_2^2 (e^\gamma - 1)^2 - Z_1 Z_2 e^\gamma = 0$$

$$Z_2 (e^\gamma - 1)^2 - Z_1 e^\gamma = 0$$

$$(e^\gamma - 1)^2 = \frac{Z_1 e^\gamma}{Z_2}$$

$$e^{2\gamma} + 1 - 2e^\gamma = \frac{Z_1}{Z_2 e^{-\gamma}}$$

Rearranging the above equation, we have

$$e^{-\gamma} (e^{2\gamma} + 1 - 2e^\gamma) = \frac{Z_1}{Z_2}$$

$$(e^\gamma + e^{-\gamma} - 2) = \frac{Z_1}{Z_2}$$

Dividing both sides by 2, we have

$$\frac{e^\gamma + e^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2} \qquad (17.5)$$

Still another expression may be obtained for the complex propagation constant in terms of the hyperbolic tangent rather than hyperbolic cosine.

$$\sinh \gamma = \sqrt{\cos h^2 \gamma - 1}$$

$$= \sqrt{\left(1 + \frac{Z_1}{2Z_2}\right)^2 - 1} = \sqrt{\frac{Z_1}{Z_1} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

$$\sinh \gamma = \frac{1}{Z_2} \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \frac{Z_{0T}}{Z_2} \qquad (17.6)$$

Dividing Eq. 17.6 by Eq. 17.5, we get

$$\tanh \gamma = \frac{Z_{0T}}{Z_2 + \dfrac{Z_1}{2}}$$

But $\qquad Z_2 + \dfrac{Z_1}{2} = Z_{0c}$

Also from Eq. 17.2, $Z_{0T} = \sqrt{Z_{0c} Z_{sc}}$

$$\tanh \gamma = \sqrt{\dfrac{Z_{sc}}{Z_{0c}}}$$

Also $\qquad \sinh \dfrac{\gamma}{2} = \sqrt{\dfrac{1}{2}(\cosh \gamma - 1)}$

Where $\qquad \cosh \gamma = 1 + (Z_1 / 2Z_2)$

$$= \sqrt{\dfrac{Z_1}{4Z_2}} \tag{17.7}$$

π-Network

Consider asymmetrical π-section shown in Fig. 17.6. When the network is terminated in Z_0 at port 2-2′, its input impedance is given by
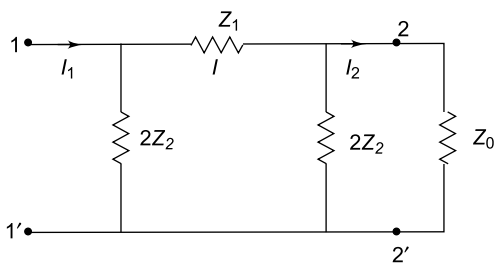

Fig. 17.6

$$Z_{in} = \dfrac{2Z_2 \left[Z_1 + \dfrac{2Z_2 \, Z_0}{2Z_2 + Z_0} \right]}{Z_1 + \dfrac{2Z_2 \, Z_0}{2Z_2 + Z_0} + 2Z_2}$$

By definition of characteristic impedance, $Z_{in} = Z_0$

$$Z_0 = \dfrac{2Z_2 \left[Z_1 + \dfrac{2Z_2 \, Z_0}{2Z_2 + Z_0} \right]}{Z_1 + \dfrac{2Z_2 \, Z_0}{2Z_2 + Z_0} + 2Z_2}$$

$$Z_0 Z_1 + \dfrac{2Z_2 Z_0^2}{2Z_2 + Z_0} + 2Z_0 Z_2 = \dfrac{2Z_2 (2Z_1 Z_2 + Z_0 Z_1 + 2Z_0 Z_2)}{(2Z_2 + Z_0)}$$

$$2Z_0 Z_1 Z_2 + Z_1 Z_0^2 + 2Z_0^2 Z_2 + 4Z_2^2 Z_0 + 2Z_2 Z_0^2$$
$$= 4Z_1 Z_2^2 + 2Z_0 Z_1 Z_2 + 4Z_0 Z_2^2$$

$$Z_1 Z_0^2 + 4Z_2 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2 (Z_1 + 4Z_2) = 4Z_1 Z_2^2$$

$$Z_0^2 = \dfrac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Rearranging the above equation leads to

$$Z_0 = \sqrt{\dfrac{Z_1 Z_2}{1 + Z_1 / 4Z_2}} \tag{17.8}$$

which is the characteristic impedance of a symmetrical π-network,

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + Z_1^2 / 4}}$$

From Eq. 17.1 $\quad Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$

$$\therefore \quad Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} \qquad (17.9)$$

$Z_{0\pi}$ can be expressed in terms of the open circuit impedance Z_{0c} and short circuit impedance Z_{sc} of the π network shown in Fig. 17.6 exclusive of the load Z_0.

From Fig. 17.6, the input impedance at port 1-1′when port 2-2′ is open is given by $Z_{0C} = \dfrac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2}$

Similarly, the input impedance at port 1-1′ when port 2-2′ is short circuited is given by $Z_{sc} = \dfrac{2Z_1 Z_2}{2Z_2 + Z_1}$

Hence $\quad Z_{0c} \times Z_{sc} = \dfrac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = \dfrac{Z_1 Z_2}{1 + Z_1 / 4Z_2}$

Thus from Eq. 17.8

$$Z_{0\pi} = \sqrt{Z_{0c} \, Z_{sc}} \qquad (17.10)$$

Propagation Constant of π-Network

The propagation constant of a symmetrical π-section is the same as that for a symmetrical *T*-section.

i.e. $\qquad \cosh \gamma = 1 + \dfrac{Z_1}{2Z_2}$

17.4 CLASSIFICATION OF PASS BAND AND STOP BAND

It is possible to verify the characteristics of filters from the propagation constant of the network. The propagation constant γ, being a function of frequency, the pass band, stop band and the cut-off point, i.e. the point of separation between the two bands, can be identified. For symmetrical *T* or π-section, the expression for propagation constant γ in terms of the hyperbolic functions is given by Eqs 17.5 and 17.7 in Section 17.3. From Eq. 17.7, $\sinh \dfrac{\gamma}{2} = \sqrt{\dfrac{Z_1}{4Z_2}}$.

If Z_1 and Z_2 are both pure imaginary values, their ratio, and hence $Z_1/4Z_2$, will be a pure real number. Since Z_1 and Z_2 may be anywhere in the range from $-j_\alpha$ t$_0$ $+ j_\alpha$, $Z_1/4Z_2$

may also have any real value between the infinite limits. Then $\sinh \dfrac{\gamma}{2} = \sqrt{Z_1}/\sqrt{4Z_2}$ will also have infinite limits, but may be either real or imaginary depending upon whether $Z_1/4Z_2$ is positive or negative.

We know that the propagation constant is a complex function $\gamma = \alpha + j\beta$, the real part of the complex propagation constant α, is a measure of the change in magnitude of the current or voltage in the network, known as the attenuation constant. β is a measure of the difference in phase between the input and output currents or voltages, known as phase shift constant. Therefore α and β take on different values depending upon the range of $Z_1/4Z_2$. From Eq. 17.7, we have

$$\sinh \frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2} + \frac{j\beta}{2}\right) = \sinh \frac{\alpha}{2} \cos \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2}$$

$$= \sqrt{\frac{Z_1}{4Z_2}} \qquad (17.11)$$

Case A

If Z_1 and Z_2 are the same type of reactances, then $\left|\dfrac{Z_1}{4Z_2}\right|$ is real and equal to say $\alpha + x$.

The imaginary part of the Eq. 17.11 must be zero.

$$\therefore \qquad \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = 0 \qquad (17.12)$$

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = x \qquad (17.13)$$

α and β must satisfy both the above equations.

Equation 17.12 can be satisfied if $\beta/2 = 0$ or $n\pi$, where $n = 0, 1, 2, ...,$ then

$\cos \beta/2 = 1$ and $\sinh \alpha/2 = x = \sqrt{\dfrac{Z_1}{4Z_2}}$

That x should be always positive implies that

$$\left|\frac{Z_1}{4Z_2}\right| > 0 \text{ and } \alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \qquad (17.14)$$

Since $\alpha \ne 0$, it indicates that the attenuation exists.

Case B

Consider the case of Z_1 and Z_2 being opposite type of reactances, i.e. $Z_1/4Z_2$ is negative, making $\sqrt{Z_1/4Z_2}$ imaginary and equal to say Jx

$\therefore$ The real part of the Eq. 17.11 must be zero.

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = 0 \qquad (17.15)$$

$$\cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = x \qquad (17.16)$$

Both the above equations must be satisfied simultaneously by α and β. Equation 17.15 may be satisfied when $\alpha = 0$, or when $\beta = \pi$. These conditions are considered separately hereunder.

(i) When $\alpha = 0$; from Eq. 17.15, $\sinh \alpha/2 = 0$. And from Eq. 17.16 $\sin \beta / 2 = x = \sqrt{Z_1 / 4Z_2}$. But the sine can have a maximum value of 1. Therefore, the above solution is valid only for negative $Z_1/4Z_2$, and having maximum value of unity. It indicates the condition of pass band with zero attenuation and follows the condition as

$$-1 \le \frac{Z_1}{4Z_2} \le 0$$

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}} \tag{17.17}$$

(ii) When $\beta = \pi$, from Eq. 17.15, $\cos \beta/2 = 0$. And from Eq. 17.16, $\sin \beta/2 = \pm 1$; $\cosh \alpha/2 = x = \sqrt{Z_1 / 4Z_2}$.

Since $\cosh \alpha/2 \ge 1$, this solution is valid for negative $Z_1/4Z_2$, and having magnitude greater than, or equal to unity. It indicates the condition of stop band since $\alpha \ne 0$.

$$-\alpha \le \frac{Z_1}{4Z_2} \le -1$$

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \tag{17.18}$$

It can be observed that there are three limits for case A and B. Knowing the values of Z_1 and Z_2, it is possible to determine the case to be applied to the filter. Z_1 and Z_2 are made of different types of reactances, or combinations of reactances, so that, as the frequency changes, a filter may pass from one case to another. Case A and (ii) in case B are attenuation bands, whereas (i) in case B is the transmission band.

The frequency which separates the attenuation band from pass band or vice versa is called cut-off frequency. The cut-off frequency is denoted by f_c, and is also termed as nominal frequency. Since Z_0 is real in the pass band and imaginary in an attenuation band, f_c is the frequency at which Z_0 changes from being real to being imaginary. These frequencies occur at

$$\frac{Z_1}{4Z_2} = 0 \text{ or } Z_1 = 0 \qquad \qquad 17.18(a)$$

$$\frac{Z_1}{4Z_2} = -1 \text{ or } Z_1 + 4Z_2 = 0 \qquad 17.18(b)$$

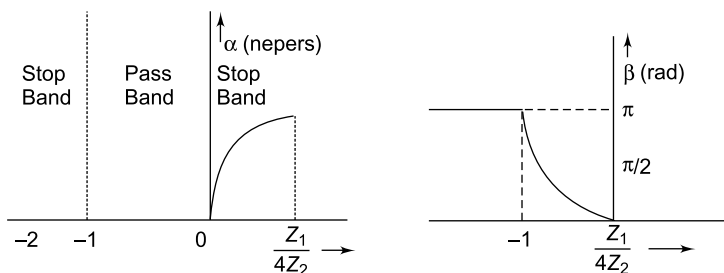The above conditions can be represented graphically, as in Fig. 17.7.

Fig. 17.7

17.5   **CHARACTERISTIC IMPEDANCE IN THE PASS AND STOP BANDS**

Referring to the characteristic impedance of a symmetrical T-network, from Eq. 17.1 we have

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

If Z_1 and Z_2 are purely reactive, let $Z_1 = jx_1$ and $Z_2 = jx_2$, then

$$Z_{0T} = \sqrt{-x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right)} \qquad (17.19)$$

A pass band exists when x_1 and x_2 are of opposite reactances and

$$-1 < \frac{x_1}{4x_2} < 0$$

Substituting these conditions in Eq. 17.19, we find that Z_{0T} is positive and real. Now consider the stop band. A stop band exists when x_1 and x_2 are of the same type of reactances; then $x_1/4x_2 > 0$. Substituting these conditions in Eq. 17.19, we find that Z_{0T} is purely imaginary in this attenuation region. Another stop band exists when x_1 and x_2 are of the same type of reactances, but with $x_1/4x_2 < -1$. Then from Eq. 17.19, Z_{0T} is again purely imaginary in the attenuation region.

Thus, in a pass band if a network is terminated in a pure resistance $R_0(Z_{0T} = R_0)$, the input impedance is R_0 and the network transmits the power received from the source to the R_0 without any attenuation. In a stop band Z_{0T} is reactive. Therefore, if the network is terminated in a pure reactance (Z_0 = pure reactance), the input impedance is reactive, and cannot receive or transmit power. However, the network transmits voltage and current with 90° phase difference and with attenuation. It has already been shown that the characteristic impedance of a symmetrical π-section can be expressed in terms of T. Thus, from Eq. 17.9, $Z_{0\pi} = Z_1 Z_2 / Z_{0T}$.

Since Z_1 and Z_2 are purely reactive, $Z_{0\pi}$ is real if Z_{0T} is real, and Z_{0x} is imaginary if Z_{0T} is imaginary. Thus the conditions developed for T-sections are valid for π sections.

17.6   **CONSTANT—K LOW PASS FILTER**

A network, either T or π, is said to be of the constant-k type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1 Z_2 = k^2 \qquad (17.20)$$

where Z_1 and Z_2 are impedances in the T and π sections as shown in Fig. 17.8. Equation 17.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. k is a real constant, that is the resistance. k is often termed as design impedance or nominal impedance of the constant k-filter.

The constant k, T or π type filter is also known as the *prototype* because other more complex networks can be derived from it. A prototype T and π-sections are shown in
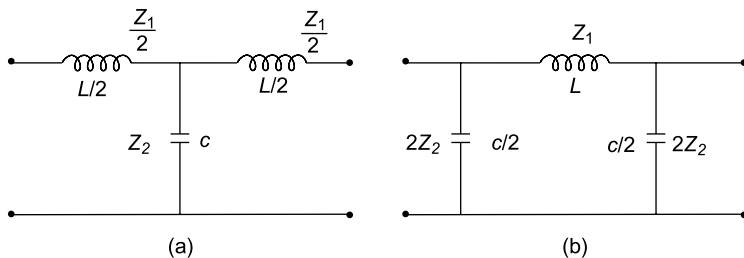
Fig. 17.8

Fig. 17.8 (a) and (b), where $Z_1 = j\omega_L$ and $Z_2 = 1/j\omega_C$. Hence $Z_1 Z_2 = \dfrac{L}{C} = k^2$ which is independent of frequency.

$$Z_1 Z_2 = k^2 = \frac{L}{C} \quad \text{or} \quad k = \sqrt{\frac{L}{C}} \tag{17.21}$$

Since the product Z_1 and Z_2 is constant, the filter is a constant-k type. From Eq. 17.18(a) the cut-off frequencies are $Z_1/4Z_2 = 0$,

i.e.
$$\frac{-\omega^2 LC}{4} = 0$$

i.e. $f = 0$ and $\dfrac{Z_1}{4Z_2} = -1$

$$\frac{-\omega^2 LC}{4} = -1$$

or
$$f_c = \frac{1}{\pi\sqrt{LC}} \tag{17.22}$$

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig. 17.9. The cut-off frequency at the intersection of the curves Z_1 and $- 4Z_2$ is indicated as f_c. On the X-axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1 = -4Z_2$.
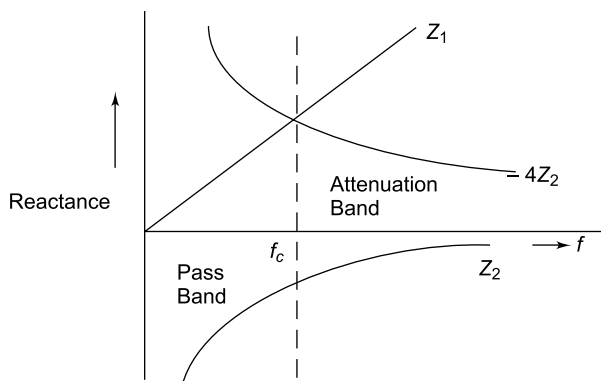

Fig. 17.9

All the frequencies above f_c lie in a stop or attenuation band. Thus, the network is called a low-pass filter. We also have from Eq. 17.7 that

$$\sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{J\omega\sqrt{LC}}{2}$$

From Eq. 17.22 $\sqrt{LC} = \dfrac{1}{f_c \pi}$

$$\therefore \quad \sinh \frac{\gamma}{2} = \frac{j2\pi f}{2\pi f_c} = j\frac{f}{f_c}$$

We also know that in the pass band

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$-1 < \frac{-\omega^2 LC}{4} < 0$$

$$-1 < -\left(\frac{f}{f_c}\right)^2 < 0$$

or

$$\frac{f}{f_c} < 1$$

and

$$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right); \alpha = 0$$

In the attenuation band,

$$\frac{Z_1}{4Z_2} < -1, \text{ i.e. } \frac{f}{f_c} < 1$$

$$\alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right] = 2\cosh^{-1}\left(\frac{f}{f_c}\right); \beta = \pi$$

The plots of α and β for pass and stop bands are shown in Fig. 17.10.

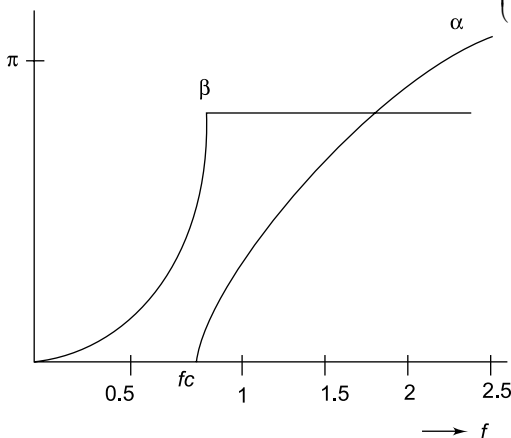Thus, from Fig. 17.10, $\alpha = 0, \beta = 2\sinh^{-1}\left(\frac{f}{f_c}\right)$ for $f < f_c$

Fig. 17.10

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right); \beta = \pi \text{ for } f > f_c$$

The characteristic impedance can be calculated as follows

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f}{f_c}\right)^2} \qquad (17.23)$$

From Eq. 17.23, Z_{0T} is real when $f < f_c$, i.e. in the pass band at $f = f_c$, $Z_{0T} = 0$; and for $f > f_c$, Z_{0T} is imaginary in the attenuation band, rising to infinite reactance at infinite frequency. The variation of Z_{0T} with frequency is shown in Fig. 17.11.
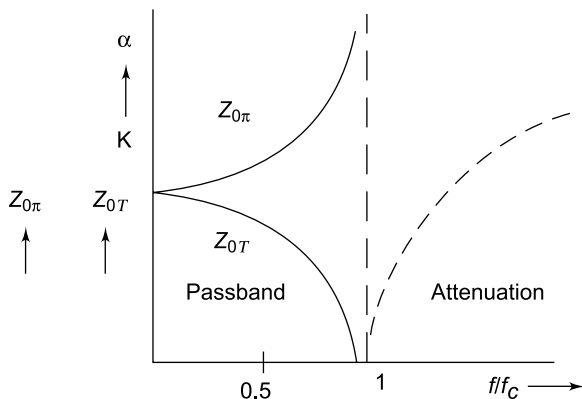

Fig. 17.11

Similarly, the characteristic impedance of a π-network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1 - \left(\dfrac{f}{f_c}\right)^2}} \qquad (17.24)$$

The variation of $Z_{0\pi}$ with frequency is shown in Fig. 17.11. For $f < f_c$, $Z_{0\pi}$ is real; at $f = f_c$, $Z_{0\pi}$ is infinite, and for $f > f_c$, $Z_{0\pi}$ is imaginary. A low pass filter can be designed from the specifications of cut-off frequency and load resistance.

At cut-off frequency, $Z_1 = -4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C}$$

$$\pi^2 f_c^2 LC = 1$$

Also we know that $k = \sqrt{L/C}$ is called the design impedance or the load resistance

$$\therefore \qquad k^2 = \frac{L}{C}$$

$$\pi^2 f_c^2 \, k^2 C^2 = 1$$

$C = \dfrac{1}{\pi f_c k}$ gives the value of the *shunt capacitance*

and $L = k^2 C = \dfrac{k}{\pi f_c}$ gives the value of the series inductance.

Example 17.1   *Design a low pass filter (both π and T-sections) having a cut-off frequency of 2 kHz to operate with a terminated load resistance of 500 Ω.*

Solution   It is given that $k = \sqrt{\dfrac{L}{C}} = 500\ \Omega$, and $f_c = 2000$ Hz

We know that $\quad L = \dfrac{k}{\pi f_c} = \dfrac{500}{3.14 \times 2000} = 79.6\ \text{mH}$

$$C = \dfrac{1}{\pi f_c k} = \dfrac{1}{3.14 \cdot 2000 \cdot 500} = 0.318\ \mu\text{F}$$

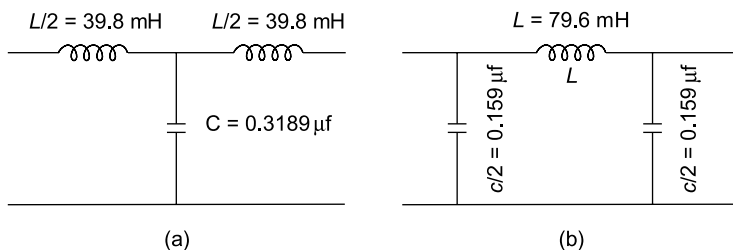The T and π-sections of this filter are shown in Fig. 17.12 (a) and (b) respectively.

Fig. 17.12

17.7   CONSTANT *K*-HIGH PASS FILTER

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig. 17.8. The prototype high pass filters are shown in Fig. 17.13, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.
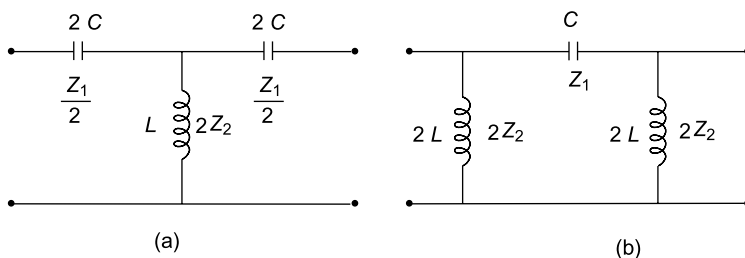
Fig. 17.13

Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type. Thus, $Z_1 Z_2$ are given by

$$Z_1 Z_2 = \dfrac{-j}{\omega C}\, j\omega L = \dfrac{L}{C} = k^2$$

$$k = \sqrt{\dfrac{L}{C}}$$

The cut-off frequencies are given by $Z_1 = 0$ and $Z_1 = -4Z_2$.

$$Z_1 = 0 \text{ indicates } \dfrac{j}{\omega C} = 0 \text{, or } \omega \to \infty$$

From $\quad Z_1 = -4Z_2$

$$\frac{-j}{\omega C} = -4j\omega L$$

$$\omega^2 LC = \frac{1}{4}$$

or $$f_c = \frac{1}{4\pi\sqrt{LC}}$$ (17.25)

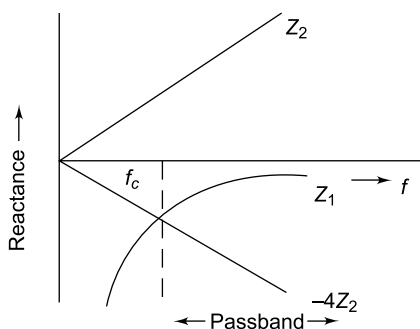The reactances of Z_1 and Z_2 are sketched as functions of frequency as shown in Fig. 17.14.

Fig. 17.14

As seen from Fig. 17.14, the filter transmits all frequencies between $f = f_c$ and $f = \infty$. The point f_c from the graph is a point at which $Z_1 = -4Z_2$.

From Eq. 17.7,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}}$$

From Eq. 17.25, $f_c = \frac{1}{4\pi\sqrt{LC}}$

$$\therefore \qquad \sqrt{LC} = \frac{1}{4\pi f_c}$$

$$\therefore \qquad \sinh\frac{\gamma}{2} = \sqrt{\frac{-(4\pi)^2(f_c)^2}{4\omega^2}} = j\frac{f_c}{f}$$

In the pass band, $-1 < \frac{Z_1}{4Z_2} < 0$, $\alpha = 0$ or the region in which $\frac{f_c}{f} < 1$ is a pass band

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)$$

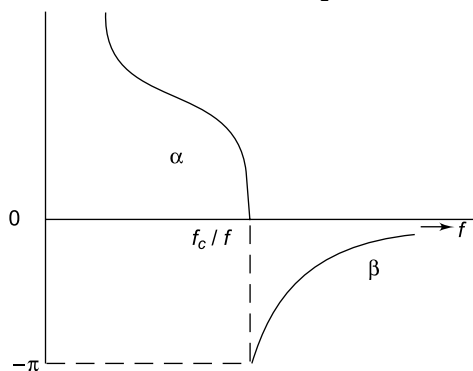In the attenuation band $\frac{Z_1}{4Z_2} < -1$, i.e. $\frac{f_c}{f} > 1$

Fig. 17.15

$$\alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right]$$

$$= 2\cos^{-1}\left(\frac{f_c}{f}\right); \quad \beta = -\pi$$

The plots of α and β for pass and stop bands of a high pass filter network are shown in Fig. 17.15.

A high pass filter may be designed similar to the low pass filter by choosing a resistive load r equal to the constant k, such that

$$R = k = \sqrt{L/C}$$

$$f_c = \frac{1}{4\pi\sqrt{L/C}}$$

$$f_c = \frac{k}{4\pi L} = \frac{1}{4\pi Ck}$$

Since $\sqrt{C} = \dfrac{L}{k}$,

$$L = \frac{k}{4\pi f_c} \text{ and } C = \frac{1}{4\pi f_c k}$$

The characteristic impedance can be calculated using the relation

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

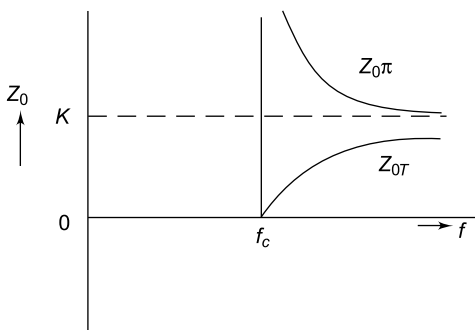Similarly, the characteristic impedance of a π-network is given by

Fig. 17.16

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k^2}{Z_{0T}}$$

$$= \frac{k}{\sqrt{1 - \left(\dfrac{f_c}{f}\right)^2}} \qquad (17.26)$$

The plot of characteristic impedances with respect to frequency is shown in Fig. 17.16.

Example 17.2 *Design a high pass filter having a cut-off frequency of 1 kHz with a load resistance of 600 Ω.*

Solution It is given that $R_L = K = 600\ \Omega$ and $f_c = 1000$ Hz

$$\therefore \qquad L = \frac{K}{4\pi f_c} = \frac{600}{4 \times \pi \times 1000} = 47.74 \text{ mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi \times 600 \times 1000} = 0.133\ \mu\text{F}$$

The T and π-sections of the filter are shown in Fig. 17.17.

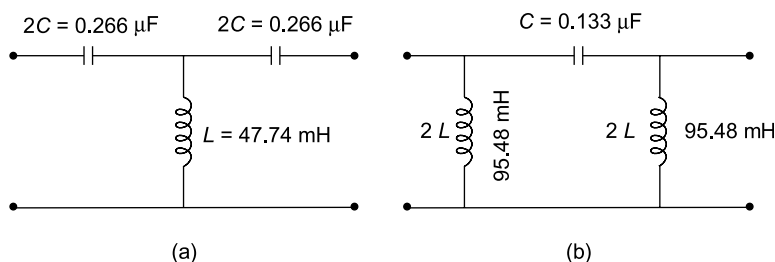(a)   (b)

Fig. 17.17

17.8 *m*-DERIVED T-SECTION

It is clear from Figs 17.10 and 17.15 that the attenuation is not sharp in the stop band for *k*-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedances be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'α' in the pass band. If the constant *k* section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called *m-derived filter*. Suppose a prototype T-network shown in Fig. 17.18 (a) has the series arm modified as shown in Fig. 17.18 (b), where *m* is a constant. Equating the characteristic impedance of the networks in Fig. 17.18, we have
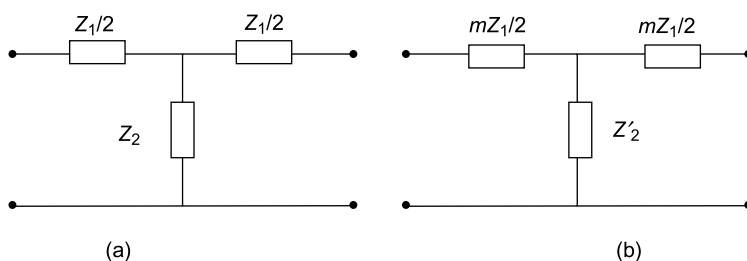

(a)   (b)

Fig. 17.18

$$Z_{0T} = Z_{0T'}$$

where $Z_{0T'}$ is the characteristic impedance of the modified (*m-derived*) T-network.

$$\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{m^2 Z_1^2}{4} + m Z_1 Z_2'}$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + m Z_1 Z_2'$$

$$m Z_1 Z_2' = \frac{Z_1^2}{4}(1 - m^2) + Z_1 Z_2$$

$$Z_2' = \frac{Z_1}{4m}(1 - m^2) + \frac{Z_2}{m} \tag{17.27}$$

It appears that the shunt arm Z_2' consists of two impedances in series as shown in Fig. 17.19.
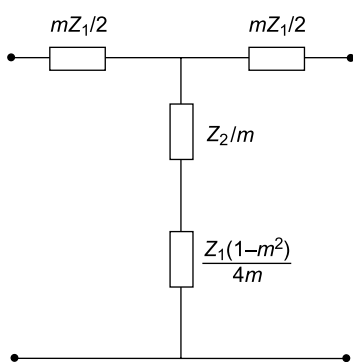

Fig. 17.19

From Eq. 17.27, $\dfrac{1 - m^2}{4m}$ should be positive to realize the impedance Z_2' physically, i.e. $0 < m < 1$. Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to π section network. Suppose a prototype π-network shown in Fig. 17.20 (a) has the shunt arm modified as shown in Fig. 17.20 (b).

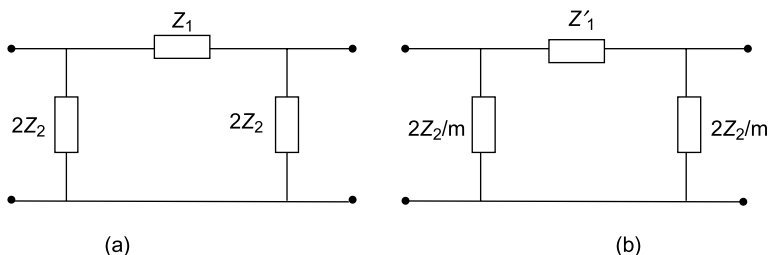The characteristic impedances of the prototype and its modified sections have to be equal for matching.

(a)   (b)

Fig. 17.20

$$Z_{0\pi} = Z_{0\pi}'$$

where $Z_{0\pi}'$ is the characteristic impedance of the modified (m-derived) π-network.

$$\therefore \qquad \sqrt{\frac{Z_1 Z_2}{1 + \dfrac{Z_1}{4 Z_2}}} = \sqrt{\frac{Z_1' \dfrac{Z_2}{m}}{1 + \dfrac{Z_1'}{4 \cdot Z_2 / m}}}$$

Squaring and cross multiplying the above equation results as under.

$$(4Z_1Z_2 + mZ_1'Z_1) = \frac{4Z_1'Z_2 + Z_1Z_1'}{m}$$

$$Z_1'\left(\frac{Z_1}{m} + \frac{4Z_2}{m} - mZ_1\right) = 4Z_1Z_2$$

or

$$Z_1' = \frac{Z_1Z_2}{\dfrac{Z_1}{4m} + \dfrac{Z_2}{m} - \dfrac{mZ_1}{4}}$$

$$= \frac{Z_1Z_2}{\dfrac{Z_2}{m} + \dfrac{Z_1}{4m}(1 - m^2)}$$

$$Z_1' = \frac{Z_1Z_2 \dfrac{4m^2}{(1 - m^2)}}{\dfrac{Z_2 4m^2}{m(1 - m^2)} + Z_1 m} = \frac{mZ_1 \dfrac{Z_2 4m}{(1 - m^2)}}{mZ_1 + \dfrac{Z_2 4m}{(1 - m^2)}} \tag{17.28}$$

It appears that the series arm of the m-derived π section is a parallel combination of mZ_1 and $4mZ_2/1 - m^2$. The derived m section is shown in Fig. 17.21.

m-Derived Low Pass Filter

In Fig. 17.22, both m-derived low pass T and π filter sections are shown. For the T-section shown in Fig. 17.22 (a), the shunt arm is to be chosen so that it is resonant at some

Fig. 17.21

frequency f_α above cut-off frequency f_c.

If the shunt arm is series resonant, its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency. Thus, at f_α

$$\frac{1}{m\omega_r C} = \frac{1 - m^2}{4m} \omega_r L, \text{ where } \omega_r \text{ is the}$$

resonant frequency

(a)   (b)

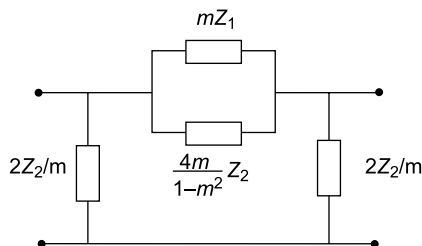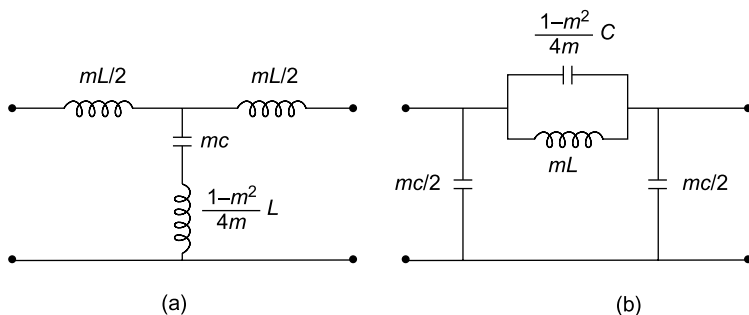**Fig. 17.22**

$$\omega_r^2 = \frac{4}{(1-m^2)LC}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Since the cut-off frequency for the low pass filter is $f_c = \dfrac{1}{\pi\sqrt{LC}}$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \qquad (17.29)$$

or $\qquad m = \sqrt{1-\left(\dfrac{f_c}{f_\infty}\right)^2} \qquad (17.30)$

If a sharp cut-off is desired, f_∞ should be near to f_c. From Eq. 17.29, it is clear that for the smaller the value of m, f_∞ comes close to f_c. Equation 17.30 shows that if f_c and f_∞ are specified, the necessary value of m may then be calculated. Similarly, for m-derived π section, the inductance and capacitance in the series arm constitute a resonant circuit. Thus, at f_∞ a frequency corresponds to infinite attenuation, i.e. at f_∞

$$m\omega_r L = \frac{1}{\left(\dfrac{1-m^2}{4m}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

Since, $\qquad f_c = \dfrac{1}{\pi\sqrt{LC}}$

$$f_r = \frac{f_c}{\sqrt{1-m^2}} = f_\infty \qquad (17.31)$$

Thus for both m-derived low pass networks for a positive value of m $(0 < m < 1)$, $f_\infty > f_c$. Equations 17.30 or 17.31 can be used to choose the value of m, knowing f_c and f_r. After the value of m is evaluated, the elements of the T or π-networks can be found from Fig. 17.22. The variation of attenuation for a low pass m-derived section can be verified from $\alpha = 2\cosh^{-1}\sqrt{Z_1/4Z_2}$ for $f_c < f < f_\infty$. For $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ for the prototype.

$\therefore \qquad \alpha = 2\cosh^{-1}\dfrac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_\infty}\right)^2}}$

and $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_c}\right)^2(1-m)^2}}$$

Figure 17.23 shows the variation of α, β and Z_0 with respect to frequency for an m-derived low pass filter.
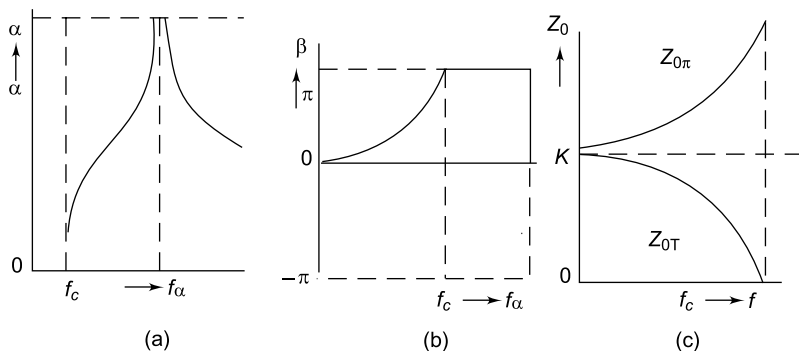

Fig. 17.23

Example 17.3   *Design a m-derived low pass filter having cut-off frequency of 1 kHz, design impedance of 400 Ω, and the resonant frequency 1100 Hz.*

Solution $k = 400\ \Omega, f_c = 1000\ \text{Hz}; f_\alpha = 1100\ \text{Hz}$

From Eq. 17.30

$$m = \sqrt{1-\left(\frac{f_c}{f_\alpha}\right)^2} = \sqrt{1-\left(\frac{1000}{1100}\right)^2} = 0.416$$

Let us design the values of L and C for a low pass, K-type filter (prototype filter). Thus,

$$L = \frac{k}{\pi f_c} = \frac{400}{\pi \times 1000} = 127.32\ \text{mH}$$

$$C = \frac{1}{\pi k f_c} = \frac{1}{\pi \times 400 \times 1000} = 0.795\ \mu\text{F}$$

The elements of m-derived low pass sections can be obtained with reference to Fig. 17.22.

Thus the T-section elements are

$$\frac{mL}{2} = \frac{0.416 \times 127.32 \times 10^{-3}}{2} = 26.48\ \text{mH}$$

$$mC = 0.416 \times 0.795 \times 10^{-6} = 0.33\ \mu\text{F}$$

$$\frac{1-m^2}{4m}L = \frac{1-(0.416)^2}{4 \cdot 0.416} \times 127.32 \times 10^{-3} = 63.27 \text{ mH}$$

The π-section elements are

$$\frac{mC}{2} = \frac{0.416 \times 0.795 \times 10^{-6}}{2} = 0.165 \text{ } \mu\text{F}$$

$$\frac{1-m^2}{4m} \times C = \frac{1-(0.416)^2}{4 \times 0.416} \times 0.795 \times 10^{-6} = 0.395 \text{ } \mu\text{F}$$

$$mL = 0.416 \times 127.32 \times 10^{-3} = 52.965 \text{ mH}$$

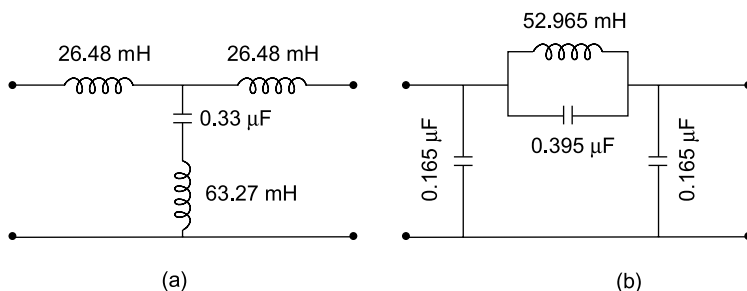The *m*-derived LP filter sections are shown in Fig. 17.24.

Fig. 17.24

m-derived High Pass Filter

In Fig. 17.25 both *m*-derived high pass T and π-sections are shown.

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency, or the frequency corresponds to infinite attenuation.

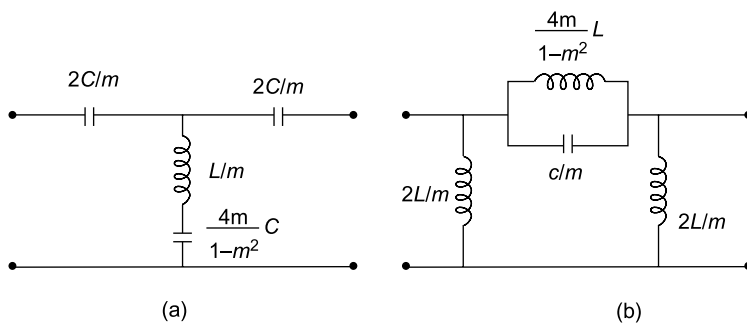$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

Fig. 17.25

$$\omega_r^2 = \omega_\infty^2 = \frac{1}{\dfrac{L}{m}\dfrac{4m}{1-m^2}C} = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

From Eq. 17.25, the cut-off frequency f_c of a high pass prototype filter is given by

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\infty = f_c\sqrt{1-m^2} \tag{17.32}$$

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \tag{17.33}$$

Similarly, for the m-derived π-section, the resonant circuit is constituted by the series arm inductance and capacitance. Thus, at f_∞

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\dfrac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\infty^2 = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$
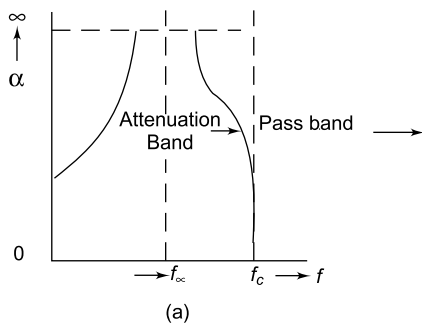

(a)

Fig. 17.26

Thus, the frequency corresponding to infinite attenuation is the same for both sections.

Equation 17.33 may be used to determine m for a given f_∞ and f_c. The elements of the m-derived high pass T or π-sections can be found from Fig. 17.25. The variation of α, β and Z_0 with frequency is shown in Fig. 17.26.
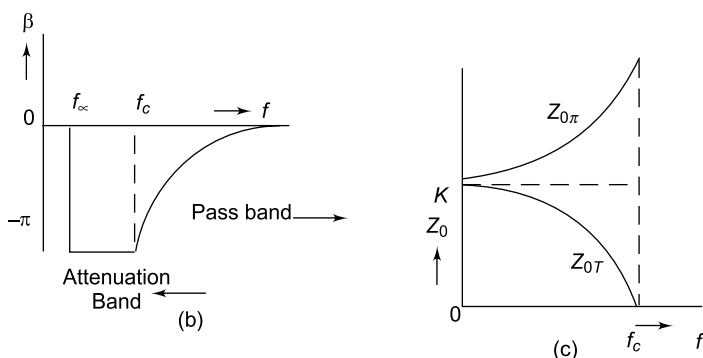
Fig. 17.26

Example 17.4   *Design a m-derived highpass filter with a cut-off frequency of 10 kHz; design impedance of 5 Ω and m = 0.4.*

Solution   For the prototype high pass filter,

$$L = \frac{k}{4\pi f_c} = \frac{500}{4\times\pi\times10000} = 3.978 \text{ mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi\times500\times10000} = 0.0159 \text{ μF}$$

The elements of *m*-derived high pass sections can be obtained with reference to Fig. 17.25. Thus, the *T*-section elements are

$$\frac{2C}{m} = \frac{2\times0.0159\times10^{-6}}{0.4} = 0.0795 \text{ μF}$$

$$\frac{L}{m} = \frac{3.978\times10^{-3}}{0.4} = 9.945 \text{ mH}$$

$$\frac{4m}{1-m^2}C = \frac{4\times0.4}{1-(0.4)^2}\times0.0159\times10^{-6} = 0.0302 \text{ μF}$$

The π-section elements are

$$\frac{2L}{m} = \frac{2\times0.0159\times10^{-3}}{0.4} = 19.89 \text{ mH}$$

$$\frac{4m}{1-m^2}\times L = \frac{4\times0.4}{1-(0.4)^2}\times3.978\times10^{-3} = 7.577 \text{ mH}$$

$$\frac{C}{m} = \frac{0.0159}{0.4}\times10^{-6} = 0.0397 \text{ μF}$$

T and π sections of the *m*-derived highpass filter are shown in Fig. 17.27.
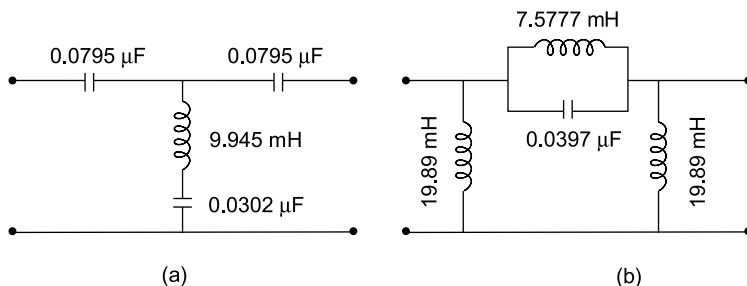

Fig. 17.27

17.9   **BAND PASS FILTER**

As already explained in Section 17.1, a band pass filter is one which attenuates all frequencies below a lower cut-off frequency f_1 and above an upper cut-off frequency f_2. Frequencies lying between f_1 and f_2 comprise the pass band, and are transmitted with zero attenuation. A band pass filter may be obtained by using a low pass filter followed by a high pass filter in which the cut-off frequency of the LP filter is above the cut-off frequency of the HP filter, the over lap thus allowing only a band of frequencies to pass. This is not economical in practice; it is more economical to combine the low and high pass functions into a single filter section.

Consider the circuit in Fig. 17.28, each arm has a resonant circuit with same resonant frequency, i.e. the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band pass characteristic.
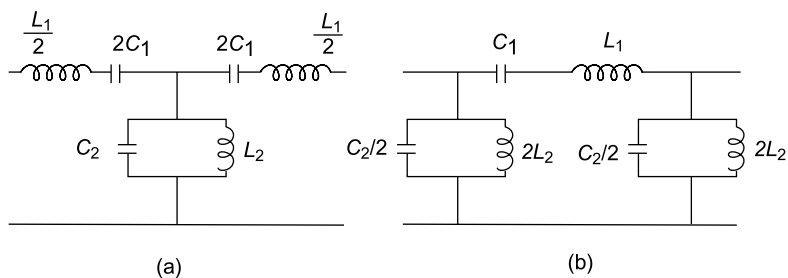

Fig. 17.28

For this condition of equal resonant frequencies.

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1} \text{ for the series arm}$$

from which, $\omega^2_0 L_1 C_1 = 1$ (17.34)

and $\dfrac{1}{\omega_0 C_2} = \omega_0 L_2$ for the shunt arm

from which, $\omega^2_0 L_2 C_2 = 1$ (17.35)

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \tag{17.36}$$

The impedance of the series arm, Z_1 is given by

$$Z_1 = \left(j\omega L_1 - \frac{j}{\omega C_1} \right) = j \left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1} \right)$$

The impedance of the shunt arm, Z_2 is given by

$$Z_2 = \frac{j\omega L_2 \dfrac{1}{j\omega C_2}}{j\omega L_2 + \dfrac{1}{j\omega C_2}} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = j \left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1} \right) \left(\frac{j\omega L_2}{1 - \omega^2 L_2 C_2} \right)$$

$$= \frac{-L_2}{C_1} \left(\frac{\omega^2 L_1 C_1 - 1}{1 - \omega^2 L_2 C_2} \right)$$

From Eq. 17.36, $L_1 C_1 = L_2 C_2$

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

where k is constant. Thus, the filter is a constant k-type. Therefore, for a constant k-type in the pass band.

$-1 < \dfrac{Z_1}{4Z_2} < 0$, and at cut-off frequency

$$Z_1 = -4Z_2$$

$$Z_1^2 = -4Z_1 Z_2 = -4k^2$$

$\therefore \qquad\qquad Z_1 = \pm j2k$

i.e. the value of Z_1 at lower cut-off frequency is equal to the negative of the value of Z_1 at the upper cut-off frequency.

$\therefore \qquad\qquad \left(\dfrac{1}{j\omega_1 C_1} + j\omega_1 L_1 \right) = - \left(\dfrac{1}{j\omega_2 C_1} + j\omega_2 L_1 \right)$

or $\qquad\qquad \left(\omega_1 L_1 - \dfrac{1}{\omega_1 C_1} \right) = \left(\dfrac{1}{\omega_2 C_1} - \omega_2 L_1 \right)$

$$(1 - \omega_1^2 L_1 C_1) = \frac{\omega_1}{\omega_2} (\omega_2^2 L_1 C_1 - 1) \tag{17.37}$$

From Eq. 17.34, $L_1 C_1 = \dfrac{1}{\omega_0^2}$

Hence Eq. 17.37 may be written as

$$\left(1 - \frac{\omega_1^2}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

$$(\omega_0^2 - \omega_1^2)\omega_2 = \omega_1(\omega_2^2 - \omega_0^2)$$

$$\omega_0^2 \omega_2 - \omega_1^2 \omega_2 = \omega_1 \omega_2^2 - \omega_1 \omega_0^2$$

$$\omega_0^2(\omega_1 + \omega_2) = \omega_1 \omega_2(\omega_2 + \omega_1)$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$f_0 = \sqrt{f_1 f_2} \tag{17.38}$$
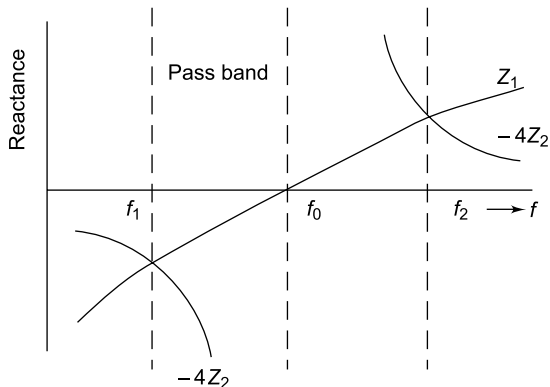
Thus, the resonant frequency is the geometric mean of the cut-off frequencies. The variation of the reactances with respect to frequency is shown in Fig. 17.29.

Design   If the filter is terminated in a load resistance $R = K$, then at the lower cut-off frequency

$$\left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -2 jk$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2k$$

$$1 - \omega^2{}_1 C_1 L_1 = 2k\omega_1 C_1$$

Fig. 17.29

Since

$$L_1 C_1 = \frac{1}{\omega_0^2}$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2k\omega_1 C_1$$

or

$$1 - \left(\frac{f_1}{f_0}\right)^2 = 4\pi k f_1 C_1$$

$$1 - \frac{f_1^2}{f_1 f_2} = 4\pi k f_1 C_1 \qquad (\because f_0 = \sqrt{f_1 f_2})$$

$$f_2 - f_1 = 4\pi k f_1 f_2 C_1$$

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} \tag{17.39}$$

Since $\qquad L_1 C_1 = \dfrac{1}{\omega_0^2}$

$$L_1 = \dfrac{1}{\omega_0^2 C_1} = \dfrac{4\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

$$L_1 = \dfrac{k}{\pi(f_2 - f_1)} \qquad\qquad (17.40)$$

To evaluate the values for the shunt arm, consider the equation

$$Z_1 Z_2 = \dfrac{L_2}{C_1} = \dfrac{L_1}{C_2} = k^2$$

∴ $\qquad\qquad L_2 = C_1 k^2 = \dfrac{(f_2 - f_1)k}{4\pi f_1 f_2} \qquad\qquad (17.41)$

and $\qquad\qquad C_2 = \dfrac{L_1}{k^2} = \dfrac{1}{\pi(f_2 - f_1)k} \qquad\qquad (17.42)$

Equations 17.39 through 17.42 are the design equations of a prototype band pass filter. The variation of α, β with respect to frequency is shown in Fig. 17.30.
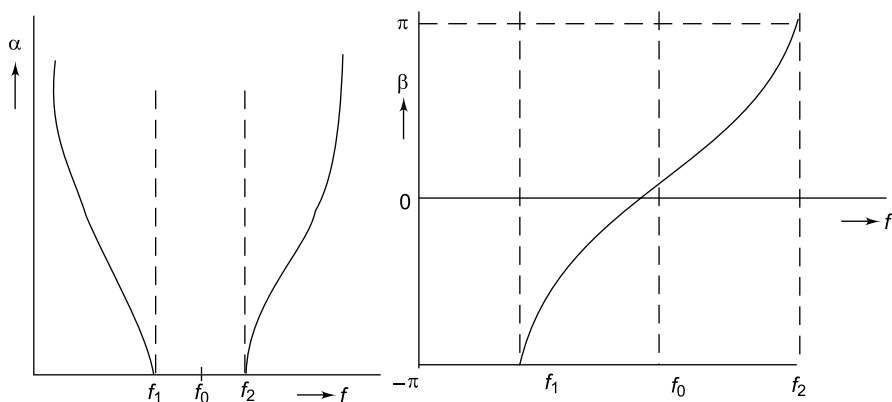

Fig. 17.30

Example 17.5   *Design k-type band pass filter having a design impedance of 500 Ω and cut-off frequencies 1 kHz and 10 kHz.*

Solution $k = 500\ \Omega; f_1 = 1000\ \text{Hz}; f_2 = 10000\ \text{Hz}$

From Eq. 17.40,

$$L_1 = \dfrac{k}{\pi(f_2 - f_1)} = \dfrac{500}{\pi\,9000} = \dfrac{55.55}{\pi}\,\text{mH} = 16.68\ \text{mH}$$

From Eq. 17.39,

$$C_1 = \dfrac{f_2 - f_1}{4\pi k f_1 f_2} = \dfrac{9000}{4\times\pi\times 500\times 1000\times 10000} = 0.143\ \mu\text{F}$$

From Eq. 17.41,

$$L_2 = C_1 k^2 = 3.57 \text{ mH}$$

From Eq. 17.42,

$$C_2 = \frac{L_1}{k^2} = 0.0707\,\mu\text{F}$$

Each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = \frac{17.68}{2} = 8.84 \text{ mH}$$

$$2C_1 = 2 \times 0.143 = 0.286 \ \mu\text{F}$$

And the shunt arm elements of the network are given by

$$C_2 = 0.0707 \ \mu\text{F and } L_2 = 3.57 \text{ mH}$$

For the constant-k, π section filter the elements of the series arm are

$$C_1 = 0.143 \ \mu\text{F and } L_1 = 16.68 \text{ mH}$$

The elements of the shunt arms are

$$\frac{C_2}{2} = \frac{0.0707}{2} = 0.035 \ \mu\text{F}$$

$$2L_2 = 2 \times 0.0358 = 0.0716 \text{ H}$$

17.10   **BAND ELIMINATION FILTER**

A band elimination filter is one which passes without attenuation all frequencies less than the lower cut-off frequency f_1, and greater than the upper cut-off frequency f_2. Frequencies lying between f_1 and f_2 are attenuated. It is also known as band stop filter. Therefore, a band stop filter can be realized by connecting a low pass filter in parallel with a highpass section, in which the cut-off frequency of low pass filter is below that of a high pass filter. The configurations of T and π constant k band stop sections are shown in Fig. 17.31. The band elimination filter is designed in the same manner as is the band pass filter.
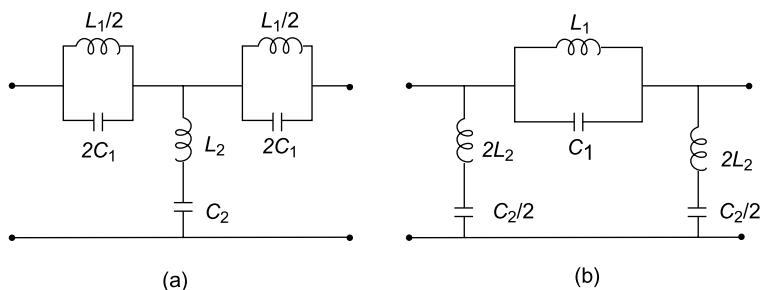

(a)   (b)

Fig. 17.31

As for the band pass filter, the series and shunt arms are chosen to resonate at the same frequency ω_0. Therefore, from Fig. 17.31 (a), for the condition of equal resonant frequencies

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm}$$

or

$$\omega_0^2 = \frac{1}{L_1 C_1} \qquad (17.43)$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \quad \text{for the shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2} \qquad (17.44)$$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = k$$

Thus

$$L_1 C_1 = L_2 C_2 \qquad (17.45)$$

It can be also verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2 \qquad (17.46)$$

and

$$f_0 = \sqrt{f_1 f_2} \qquad (17.47)$$

At cut-off frequencies, $Z_1 = -4Z_2$

Multiplying both sides with Z_2, we get

$$Z_1 Z_2 = -4Z_2^2 = k^2$$

$$Z_2 = \pm j\frac{k}{2} \qquad (17.48)$$

If the load is terminated in a load resistance, $R = k$, then at lower cut-off frequency

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{k}{2}$$

$$\frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{k}{2}$$

$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{k}{2}$$

From Eq. 17.44, $L_2 C_2 = \dfrac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{k}{2}\omega_1 C_2$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = k\pi f_1 C_2$$

$$C_2 = \frac{1}{k\pi f_1}\left[1 - \left(\frac{f_1}{f_0}\right)^2\right]$$

Since

$$f_0 = \sqrt{f_1 f_2}$$

$$C_2 = \frac{1}{k\pi}\left[\frac{1}{f_1} - \frac{1}{f_2}\right]$$

$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] \tag{17.49}$$

From Eq. 17.44, $\omega_0^2 = \dfrac{1}{L_2 C_2}$

$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

Since

$$f_0 = \sqrt{f_1 f_2}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} \tag{17.50}$$

Also from Eq. 17.46,

$$k^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$\therefore \qquad L_1 = k^2 C_2 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right) \tag{17.51}$$
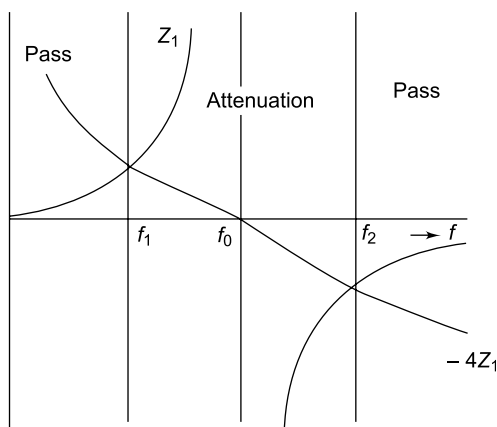

Fig. 17.32

and $C_1 = \dfrac{L_2}{k^2}$ (17.52)

$$= \frac{1}{4\pi k(f_2 - f_1)}$$

The variation of the reactances with respect to frequency is shown in Fig. 17.32.

Equation 17.49 through Eq. 17.52 are the design equations of a prototype band elimination filter. The variation of α, β with respect to frequency is shown in Fig. 17.33.
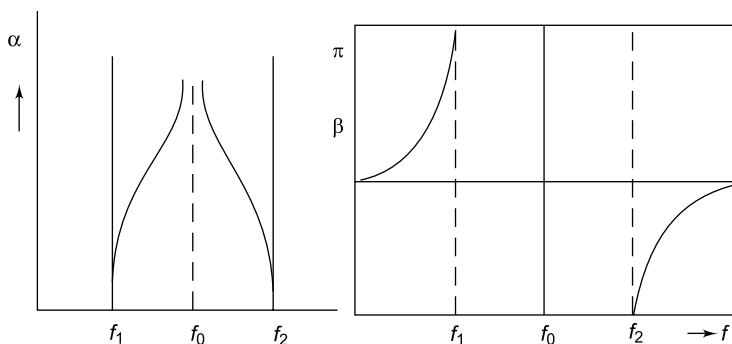
Fig. 17.33

Example 17.6   *Design a band elimination filter having a design impedance of 600 Ω and cut-off frequencies f_1 = 2 kHz and f_2 = 6 kHz.*

Solution $(f_2 - f_1) = 4$ kHz

Makinkg use of the Eqs 17.49 through 17.52 in Section 17.10, we have

$$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 2000 \times 6000} = 63 \text{ mH}$$

$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{1}{4 \times \pi \times 600(4000)} = 0.033 \text{ μF}$$

$$L_2 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{600}{4\pi(4000)} = 12 \text{ mH}$$

$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] = \frac{1}{600 \times \pi}\left[\frac{4000}{2000 \times 6000}\right] = 0.176 \text{ μF}$$

Each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = 31.5 \text{ mH}$$

$$2C_1 = 0.066 \text{ μF}$$

And the shunt arm elements of the network are

$$L_2 = 12 \text{ mH and } C_2 = 0.176 \text{ μF}$$

For the constant k, π-section filter the elements of the series arm are

$$L_1 = 63 \text{ mH, } C_1 = 0.033 \text{ μF}$$

and the elements of the shunt arms are

$$2L_2 = 24 \text{ mH and } \frac{C_2}{2} = 0.088 \text{ μF}$$

17.11 ATTENUATORS

An attenuator is a two-port resistive network and is used to reduce the signal level by a given amount. In a number of applications, it is necessary to introduce a specified loss between source and a matched load without altering the impedance relationship. Attenuators may be used for this purpose. Attenuators may be symmetrical or asymmetrical, and can be either fixed or variable. A fixed attenuator with constant attenuation is called a *pad*. Variable attenuators are used as volume controls in radio broadcasting sections. Attenuators are also used in laboratory to obtain small value of voltage or current for testing circuits.

The increase or decrease in power due to insertion or substitution of a new element in a network can be conveniently expressed in decibels (dB), or in nepers. In other words, attenuation is expressed either in decibels (dB) or in nepers. Accordingly, the attenuation offered by a network in decibels is

$$\text{Attenuation in dB} = 10 \ \log_{10} \left(\frac{P_1}{P_2} \right) \tag{17.53}$$

where P_1 is the input power and P_2 is the output power.

For a properly matched network, both terminal pairs are matched to the characteristic resistance, R_0 of the attenuator.

Hence,
$$\frac{P_1}{P_2} = \frac{I_1^2 R_0}{I_2^2 R_0} = \frac{I_1^2}{I_2^2} \tag{17.54}$$

where I_1 is the input current and I_2 is the output current leaving the port.

or
$$\frac{P_1}{P_2} = \frac{V_1^2}{V_2^2} \tag{17.55}$$

where V_1 is the voltage at port 1 and V_2 is the voltage at port 2

Hence, attenuation in dB $= 20 \ \log_{10} \left(\frac{V_1}{V_2} \right) \tag{17.56}$

$$= 20 \ \log_{10} \left(\frac{I_1}{I_2} \right) \tag{17.57}$$

If
$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = N \tag{17.58}$$

then
$$\frac{P_1}{P_2} = N^2$$

and
$$\text{dB} = 20 \log_{10} N \tag{17.59}$$

or
$$N = \text{antilog} \left(\frac{\text{dB}}{20} \right) \tag{17.60}$$

17.12 T-TYPE ATTENUATOR

Basically, there are four types of attenuators, T, π, lattice and bridged T-type. The basic design principles are discussed in the following Sections. Figure 17.34 shows

the symmetrical *T*-attenuator. An attenuator is to be designed for desired values of characteristic resistance, R_0 and attenuation.
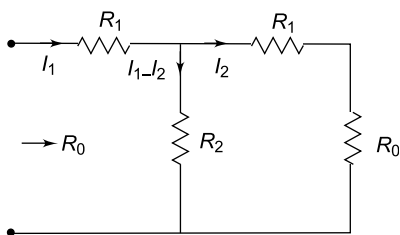

Fig. 17.34

The values of the arms of the network can be specified in terms of characteristic impedance, Z_0, and propagation constant, γ, of the network. The network in the figure is a symmetrical resistive circuit; hence $Z_0 = R_0$ and $\gamma = \alpha$. The design equations can be obtained by applying Kirchhoff's law to the network in Fig. 17.34.

$$R_2 (I_1 - I_2) = I_2 (R_1 + R_0)$$

$$I_2 (R_2 + R_1 + R_0) = I_1 R_2$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0 + R_2}{R_2} = N \qquad (17.61)$$

The characteristic impedance of the attenuator is R_0 when it is terminated in a load of R_0

Hence,
$$R_0 = R_1 + \frac{R_2(R_1 + R_0)}{R_1 + R_0 + R_2}$$

Substituting Eq. 17.61, we have

$$R_0 = R_1 + \frac{\left(R_1 + R_0\right)}{N}$$

$$N R_0 = N R_1 + R_1 + R_0$$

$$R_0(N - 1) = R_1 (N + 1)$$

$$R_1 = \frac{R_0 (N - 1)}{N + 1} \qquad (17.62)$$

From Eq. 17.61, we have

$$N R_2 = R_1 + R_0 + R_2$$

$$(N - 1)R_2 = (R_1 + R_0)$$

Substituting the value of R_1 from Eq. 17.62, we have

$$(N - 1)R_2 = R_0 \frac{(N - 1)}{N + 1} + R_0$$

$$(N - 1)R_2 = \frac{2 N R_0}{(N + 1)}$$

$$R_2 = \frac{2 N R_0}{N^2 - 1} \qquad (17.63)$$

Equations 17.62 and 17.63 are the design equations for the symmetrical *T*-attenuator.

Example 17.7   *Design a T-pad attenuator to give an attenuation of 60 dB and to work in a line of 500 Ω impedance.*

Solution $N = \dfrac{I_1}{I_2} = \text{antilog} \dfrac{D}{20}$

$\qquad\qquad = \text{antilog} \dfrac{60}{20} = 1000$

Each of the series arm is given by

$$R_1 = \frac{R_0(N-1)}{N+1} = 500\frac{(1000-1)}{(1000+1)} = 499 \ \Omega$$

The shunt arm resistor R_2 is given by

$$R_2 = \frac{2N}{N^2-1} = R_0 = \frac{2\times1000}{(1000)^2-1}\times500 = 1 \ \Omega$$

17.13   **π-TYPE ATTENUATOR**

Figure 17.35 shows symmetrical attenuator. The series and shunt arm of the attenuator can be specified in terms of Z_0 and propagation constant γ. In this case also, the network is formed by resistors and is symmetrical, hence $Z_0 = R_0$ and $\gamma = \alpha$. From the fundamental equations, we have
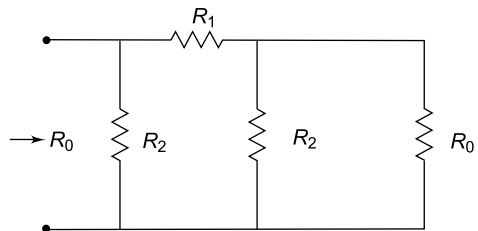

Fig. 17.35

$$R_1 = R_0 \sinh \alpha \qquad\qquad (17.64)$$

$$R_2 = R_0 \coth \alpha/2 \qquad\qquad (17.65)$$

$$\therefore \quad R_1 = R_0 \frac{e^\alpha - e^{-\alpha}}{2} \qquad (17.66)$$

By definition of propagation constant

$$e^\gamma = \frac{I_1}{I_2} = N$$

Here $\gamma = \alpha$ and $e^\alpha = N$

Therefore, Eq. 17.66 can be written as

$$R_1 = R_0 \frac{N - \dfrac{1}{N}}{2} = R_0 \frac{N^2-1}{2N} \qquad\qquad (17.67)$$

Similarly, from Eq. 17.65,

$$R_2 = R_0 \frac{\cosh \alpha/2}{\sinh \alpha/2} = R_0 \frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}}$$

$$R_2 = R_0 \frac{e^\alpha + 1}{e^\alpha - 1} = R_0 \frac{(N-1)}{(N+1)} \qquad\qquad (17.68)$$

Equations 17.67 and 17.68 are the design equations for the symmetrical π-attenuator.

Example 17.8 Design a π-type attenuator to give 20 dB attenuation and to have a characteristic impedance of 100 Ω.

Solution Given $R_0 = 100\ \Omega$, $D = 20$ dB.

$$N = \text{Antilog } \frac{D}{20} = 10$$

$$R_1 = R_0 \frac{(N^2 - 1)}{2N} = 100 \frac{(10^2 - 1)}{2 + 10} = 495\ \Omega$$

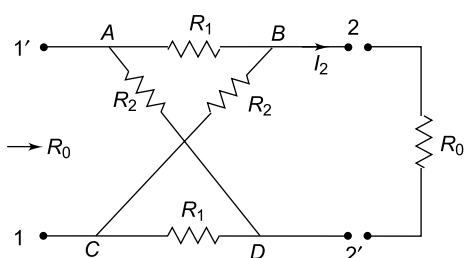$$R_2 = R_0 \frac{(N + 1)}{(N - 1)} = 100 \left(\frac{10 + 1}{10 - 1} \right) = 122.22\ \Omega$$

17.14 LATTICE ATTENUATOR

Fig. 17.36

A symmetrical resistance lattice is shown in Fig. 17.36. The series and the diagonal arm of the network can be specified in terms of the characteristic impedance Z_0 and propagation constant γ.

It has already been stated and proved that characteristic impedance of symmetrical network is the geometric mean of the open and short circuit impedance. The circuit in Fig. 17.36 is redrawn as in Fig. 17.37 to calculate the open and short circuit impedances.
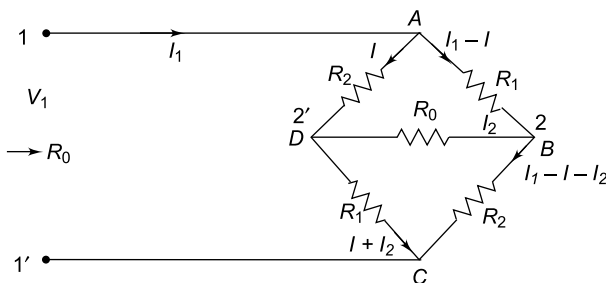

Fig. 17.37

Thus, $Z_{sc} = \dfrac{2R_1 R_2}{R_1 + R_2}$

$Z_{0c} = \dfrac{R_1 + R_2}{2}$

Hence, $Z_0 = R_0 = \sqrt{Z_{0c} Z_{sc}}$

$R_0 = \sqrt{R_1 R_2}$

In Fig. 17.37 the input impedance at 1-1' is R_0 when the network is terminated in R_0 at 2-2'. By applying Kirchhoff's voltage law, we get

$$V_1 = I_1R_0 = (I_1 - I)R_1 + I_2R_0 + (1 + I_2)R_1$$

$$I_1R_0 = R_1(I_1 + I_2) + I_2R_0$$

$$I_1(R_0 - R_1) = I_2(R_1 + R_0)$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0}{R_0 - R_1} = \frac{1 + \dfrac{R_1}{R_0}}{1 - \dfrac{R_1}{R_0}} \tag{17.69}$$

$$N = e^\alpha = \frac{I_1}{I_2} = \frac{1 + \dfrac{R_1}{R_0}}{1 - \dfrac{R_1}{R_0}} \tag{17.70}$$

$$e^\alpha = \frac{1 + \sqrt{R_1 / R_2}}{1 - \sqrt{R_1 / R_2}}$$

The propagation constant
$$\alpha = \log \left[\frac{1 + \sqrt{\dfrac{R_1}{R_2}}}{1 - \sqrt{\dfrac{R_1}{R_2}}} \right] \tag{17.71}$$

From Eq. 17.70

$$N\left(1 - \frac{R_1}{R_0}\right) = \left(1 + \frac{R_1}{R_0}\right)$$

$$R_1 = R_0 \frac{(N-1)}{(N+1)} \tag{17.72}$$

Similarly, we can express $R_2 = R_0 \dfrac{(N+1)}{(N-1)}$ $\tag{17.73}$

Equations 17.72 and 17.73 are the design equations for lattice attenuator.

Example 17.9   *Design a symmetrical lattice attenuator to have characteristic impedance of 800 Ω and attenuation of 20 dB.*

Solution   Given $R_0 = 800\ \Omega$ and $D = 20$ dB

$$N = \text{antilog}\ \frac{D}{20} = \text{antilog}\ \frac{20}{20} = 10$$
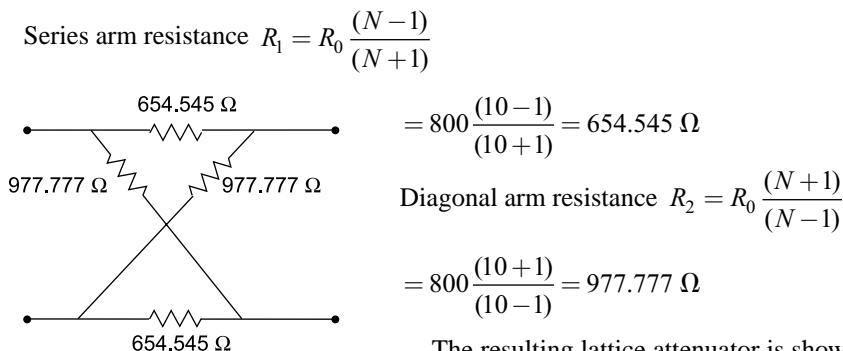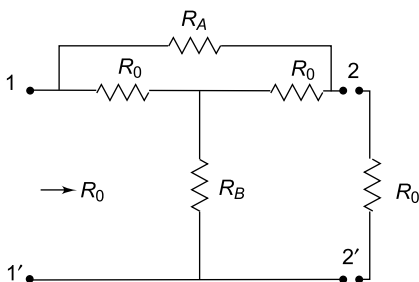
From the design equations of lattice attenuator

Series arm resistance $R_1 = R_0 \dfrac{(N-1)}{(N+1)}$

Fig. 17.38

$$= 800 \dfrac{(10-1)}{(10+1)} = 654.545 \ \Omega$$

Diagonal arm resistance $R_2 = R_0 \dfrac{(N+1)}{(N-1)}$

$$= 800 \dfrac{(10+1)}{(10-1)} = 977.777 \ \Omega$$

The resulting lattice attenuator is shown in Fig. 17.38.

17.15   BRIDGED-*T* ATTENUATOR

Fig. 17.39
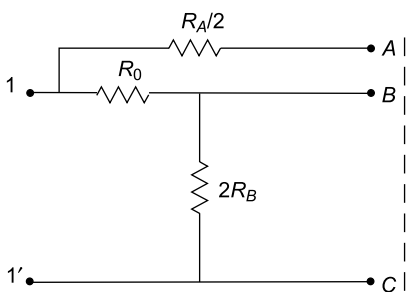
Fig. 17.40

A bridged-*T* attenuator is shown in Fig. 17.39. In this case also since the attenuator is formed by resistors only, $Z_0 = R_0$ and $\gamma = \alpha$. The bridged-*T* network may be designed to have any characteristic resistance R_0 and desired attenuation by making $R_A R_B = R^2{}_0$. Here R_A and R_B are variable resistances and all other resistances are equal to the characteristic resistance R_0 of the network.

A symmetrical resistance lattice network can be converted into an equivalent *T*, π or bridged-*T* resistance network using the bisection theorem. We can obtain the design equations of the bridged-*T* attenuator by bisection theorem. A bisected half sections is shown in Fig. 17.40. According to the bisection theorem, a network having mirror image symmetry can be reduced to an equivalent lattice structure. The series arm of the equivalent lattice is found by bisecting the given network into two parts, short circuiting all the cut wires and equating the series impedance of the lattice to the input impedance of the bisected network; the diagonal arm is equal to the input impedance of the bisected network when cut wires are open circuited.

From Fig. 17.40, when the cut wires *A*, *B*, *C* are shorted, the input resistance of the network is given by

$$R_{sc} = \frac{R_0 \times R_{A/2}}{R_0 + R_{A/2}} = \frac{R_0 R_A}{2 R_0 + R_A} \tag{17.74}$$

This resistance is equal to the series arm resistance of the lattice network shown in Fig. 17.36.

$$\therefore \qquad \frac{R_0 R_A}{2R_0 + R_A} = R_1 \qquad (17.75)$$

From Eq. 17.72, we have

$$R_1 = R_0 \frac{(N-1)}{(N+1)}$$

Hence, $\qquad \dfrac{R_0 R_A}{(2R_0 + R_A)} = R_0 \dfrac{(N-1)}{(N+1)}$

From which $R_A = R_0 (N-1)$ $\qquad (17.76)$

From Fig. 17.40, when the cut wires *A*, *B*, *C* are open, the input resistance of the network is given by

$$R_{0c} = (R_0 + 2R_B) \qquad (17.77)$$

This resistance is equal to the diagonal arm resistance of the lattice network shown in Fig. 17.36.

$$\therefore \qquad R_0 + 2R_B = R_2 \qquad (17.78)$$

From Eq. 17.73, we have

$$R_2 = R_0 \frac{(N+1)}{(N-1)}$$

Hence $\quad (R_0 + 2R_B) = R_0 \dfrac{(N+1)}{(N-1)}$

From which $\quad R_B = \dfrac{R_0}{N-1}$ $\qquad (17.79)$

Equations 17.76 and 17.79 are the design equations for bridged-*T* attenuator.

Example 17.10 *Design a symmetrical bridged T - attenuator with an attenuation of 20 dB and terminated into a load of 500 Ω.*

Solution $D = 20\ dB;\ R_0 = 500\,\Omega$

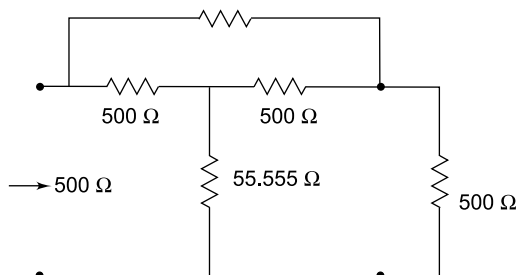**Fig. 17.41**

$N = \text{antilog}\,\dfrac{D}{20} = \text{antilog}\,\dfrac{20}{20} = 10$

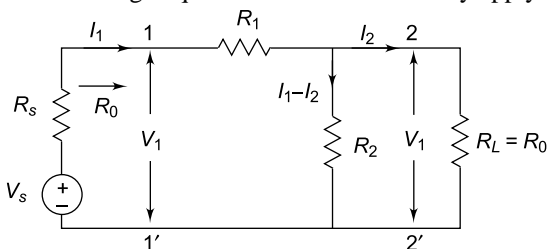$R_A = R_0(N-1) = 500\,(10-1) = 4500\,\Omega$

$R_B = \dfrac{R_0}{(N-1)} = \dfrac{500}{(10-1)} = 55.555\,\Omega$

The desired configuration of the attenuator is shown in Fig.17.41.

17.16   L-TYPE ATTENUATOR

An *L*-type asymmetrical attenuator is shown in Fig. 17.42. The attenuator is connected between a source with source resistance $R_s = R_0$ and load resistance $R_L = R_0$.

The design equations can be obtained by applying simple laws.

Fig. 17.42

$$V_2 = (I_1 - I_2)R_2 = I_2 R_L$$

$$\text{or} \quad I_1 R_2 = I_2 (R_2 + R_L)$$

$$\frac{I_1}{I_2} = \frac{R_2 + R_L}{R_2} = N \quad (17.80)$$

$$1 + \frac{R_L}{R_2} = N$$

$$R_2 = \frac{R_L}{N-1} \tag{17.81}$$

As $R_L = R_0$, Eq. 17.81 can be written as

$$R_2 = \frac{R_0}{N-1} \tag{17.82}$$

The resistance of the network as viewed from 1-1′ into the network is

$$R_0 = R_1 + \frac{R_2 R_0}{R_2 + R_0}$$

$$R_1 = \frac{R_0^2}{R_2 + R_0} \tag{17.83}$$

Substituting the value of R_2 from Eq. 17.82, we have

$$R_1 = \frac{R_0^2}{\dfrac{R_0}{N-1} + R_0} = \frac{R_0^2 (N-1)}{R_0 + R_0 (N-1)}$$

$$R_1 = R_0 \frac{(N-1)}{N} \tag{17.84}$$

Equations 17.82 and 17.84 are the design equations. Attenuation *N* of the network can be varied by varying the values of R_1 and R_2.

Example 17.11   *Design a L-type attenuator to operate into a load resistance of 600 Ω with an attenuation of 20 dB.*

Solution $N = \text{antilog}\ \dfrac{dB}{20} = \text{antilog}\ \dfrac{20}{20} = 10$

The series arm of the attenuator is given by

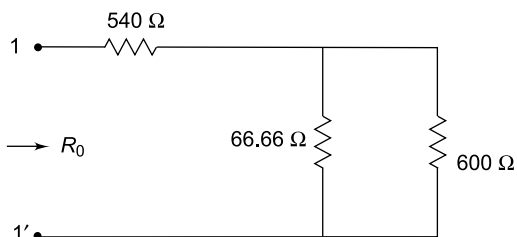$$R_1 = R_0 \left(\frac{N-1}{N} \right) = 600 \left(\frac{10-1}{10} \right) = 540\ \Omega$$

Fig. 17.43

The shunt arm of the attenuator is given by

$$R_2 = \frac{R_0}{N-1} - \frac{600}{9} = 66.66\,\Omega$$

The desired configuration of the network is shown in Fig. 17.43.

17.17 EQUALIZERS

Equalizers are networks designed to provide compensation against distortions that occur in a signal while passing through an electrical network. In general, any electrical network has attenuation distortion and phase distortion. Attenuation distortion occurs due to non-uniform attenuation against frequency characteristics. Phase distortion occurs due to phase delay against frequency characteristics. An attenuation equalizer is used to compensate attenuation distortion in any network. These equalizers are used in medium to high frequency carrier telephone systems, amplifiers, transmission lines and speech reproduction, etc. A phase equalizer is used to compensate phase distortion in any network. These equalizers are used in TV signal transmission lines and in facsimile.

17.18 INVERSE NETWORK

The geometrical mean of two impedances Z_1 and Z_2 is a real number and they are said to be inverse if
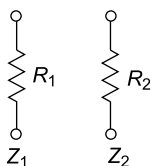

Fig. 17.44

$$Z_1 Z_2 = R_0^2$$

where R_0 is a resistance
Consider $Z_1 = R_1$ and $Z_2 = R_2$
The product $Z_1 Z_2$ is a real number
Therefore, the two impedances are said to be inverse if they satisfy the relation $Z_1 Z_2 = R_1 R_2 = R_0^2$.
In another case, consider $Z_1 = j\omega L$ and $Z_2 = \dfrac{1}{j\omega C}$

Fig. 17.45

$$Z_1 Z_2 = \frac{j\omega L}{j\omega C} = \frac{L}{C}$$
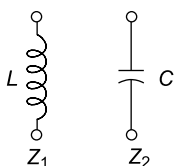
The product $Z_1 Z_2$ is a real number
Therefore, the two impedances are inverse.
Similarly,

Let $Z_1 = R_1 + j\omega L$ (17.85)

and $Z_2 = \dfrac{R_2 \dfrac{1}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} = \dfrac{-jR_2}{\omega C R_2 - j} \cdot \dfrac{\omega C R_2 + j}{\omega C R_2 + j}$ (17.86)

$$= \frac{R_2 - j\omega CR_2^2}{1+\omega^2 C^2 R_2^2}$$

$$Z_1 Z_2 = (R_1 + j\omega L) \left(\frac{R_2 - j\omega CR_2^2}{1+\omega^2 C^2 R_2^2} \right)$$

$$= \frac{R_1 R_2 + \omega^2 R_2^2 LC + j(\omega LR_2 - \omega CR_1 R_2^2)}{1+\omega^2 C^2 R_2^2} \qquad (17.87)$$

The imaginary part of the above equation must be zero.

Therefore, we get $\omega LR_2 = \omega CR_1 R_2^2$

$$\frac{L}{C} = R_1 R_2 = R_0^2 \qquad (17.88)$$

The two impedances Z_1 and Z_2 are inverse, when the above condition is satisfied. An inverse network may be obtained by

(i)   Converting each series branch into parallel branch and vice-versa.

(ii) Converting each resistance element R into a corresponding resistive element $\dfrac{R_0^2}{R}$.

(iii) Converting each inductance L into capacitance $C^1 = \dfrac{L}{R_0^2}$.

(iv) Converting each capacitance C into inductance $L^1 = CR_0^2$.

Example 17.12   *Obtain the inverse network of the network shown in Fig. 17.46.*
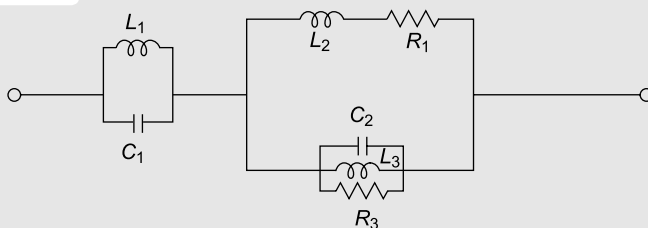

Fig. 17.46

Solution   The parallel branch is converted into a series branch and vice-versa. The capacitance is replaced by inductance and vice-versa. The resistance is replaced by another resistance as shown in Fig. 17.47.
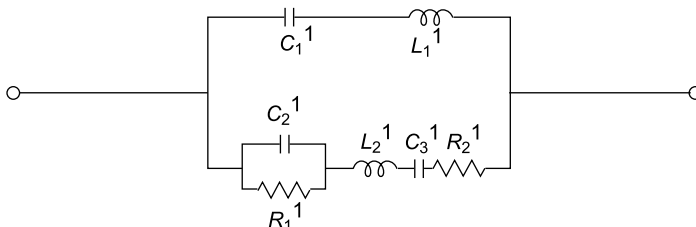

Fig. 17.47

Where $C_1^1 = \dfrac{L_1}{R_0^2}, iL_1^1 = C_1 R_0^2, R_1^1 = \dfrac{R_0^2}{R_1}$

$$C_2^1 = \dfrac{L_2}{R_0^2}, iL_2^1 = C_2 R_0^2, R_2^1 = \dfrac{R_0^2}{R_2}$$

$$C_3^1 = \dfrac{L_3}{R_0^2} \text{ and } R_0 = \text{design impedance.}$$
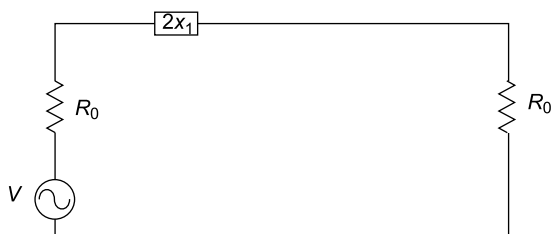
17.19 SERIES EQUALIZER

Fig. 17.48

The series equalizer is a two terminal network connected in series with a network to be corrected. (see Fig. 17.48)

Let $N =$ Input to output power ratio of the load

$D =$ Attenuation in decibels

$R_0 =$ Resistance of the load as well as source
$P_1 =$ Input power
$P_1 =$ Load power
$2X_1 =$ Reactance of the equalizer
$V_{max} =$ Voltage applied to the network

Attenuation $D = \log_{10} N$

or $N = \text{antilog} \left(\dfrac{D}{10} \right)$ (17.89)

$$N = \dfrac{\text{Maximum power delivered to the load when equlizer is not present}}{\text{Power delivered to the load when equalizer is present}}$$

$$N = \dfrac{P_i}{P_l}$$

$$P_i = \left(\dfrac{V_{max}}{2R_0} \right)^2 R_0 = \dfrac{V_{max}^2}{4R_0}$$

When the equalizer is connected,

$$l_1 = \dfrac{V_{max}}{\sqrt{(2R_0)^2 + (2X_1)^2}}$$

$$P_l = \left[\dfrac{V_{max}}{\sqrt{(2R_0)^2 + (2X_1)^2}} \right]^2 R_0$$

$$= \left| \frac{V_{\max}^2}{4(R_0^2 + X_1^2)} \right| R_0 \tag{17.90}$$

Therefore, $N = \dfrac{P_i}{P_l} = \dfrac{V_{\max}^2 / 4R_0}{\dfrac{V_{\max}^2 \, R_0}{4(R_0^2 + X_1^2)}} = 1 + \dfrac{X_1^2}{R_0^2}$ \hfill (17.91)

By knowing the values of R_0 and N, X_1 can be determined.

17.20   FULL SERIES EQUALIZER
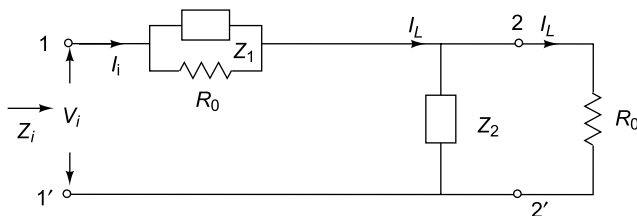
Figure 17.49 shows the configuration of full series equalizer.

Fig. 17.49

The circuit is a constant resistance equalizer satisfying the relation $Z_1 Z_2 = R_0^2$. The input impedance is given by

$$Z_i = \frac{R_0 Z_1}{R_0 + Z_1} + \frac{R_0 Z_2}{R_0 + Z_2}$$

$$= \frac{R_0[2Z_1 Z_2 + R_0(Z_1 + Z_2)]}{R_0^2 + R_0(Z_1 + Z_2) + Z_1 Z_2} \tag{17.92}$$

If we substitute $Z_1 Z_2 = R_0^2$ in the above equation

$$Z_i = R_0$$

$$|V_i| = I_i Z_i = I_i R_0$$

$$|V_l| = I_i Z_i = I_i \frac{R_0 Z_2}{R_0 + Z_2} \tag{17.93}$$

$$N = \left| \frac{V_i}{V_l} \right|^2 = \left| \frac{R_0 + Z_2}{Z_2} \right|^2 = 1 + \frac{R_0^2}{X_2^2} \tag{17.94}$$

$$= 1 + \frac{X_1^2}{R_0^2}$$

Since Z_1 and Z_2 are pure reactances and $X_1 X_2 = R_0^2$

(i) When $X_1 = \omega L$,

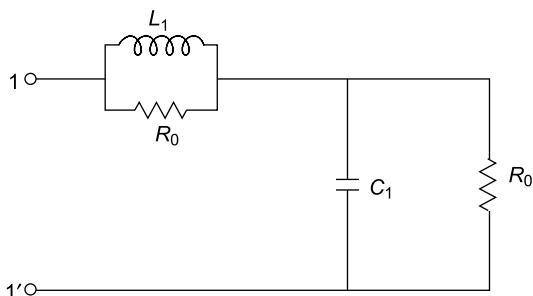$$X_2 = \frac{1}{\omega C_1} \text{ since both are inverse}$$

Fig. 17.50

The full series equalizer is shown in Fig. 17.50.

Where $\dfrac{L_1}{C_1} = R_0^2$

From the equation

$$N = 1 + \frac{X_1^2}{R_0^2}$$

$$= 1 + \frac{\omega^2 L_1^2}{R_0^2}$$

By knowing the values of N and R_0, the elemental values of L_1, C_1 may be obtained.
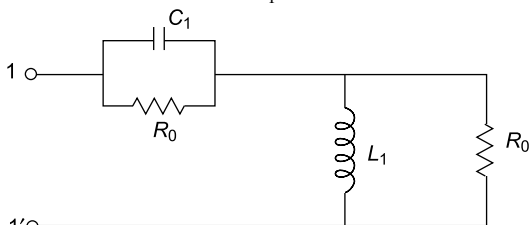
(ii) When $X_1 = \dfrac{1}{\omega C_1}$,

Fig. 17.51

$X_2 = \omega L_1$

The full series equalizer is shown in Fig. 17.51.

Here $\dfrac{L_1}{C_1} = R_0^2$

From the equation

$$N = 1 + \frac{R_0^2}{X_2^2} = 1 + \frac{R_0^2}{\omega^2 L_1^2}$$

By knowing the values of N and R_0, the values of L_1, C_1 may be obtained.

17.21 SHUNT EQUALIZER

The shunt equalizer is a two terminal network connected in shunt with a network to be corrected.

Let N = Input to output power ratio

D = Attenuation in decibels

R_0 = Source resistance/load resistance

I_s = Source current

I_l = Load current

P_i = Input power

P_l = Load power

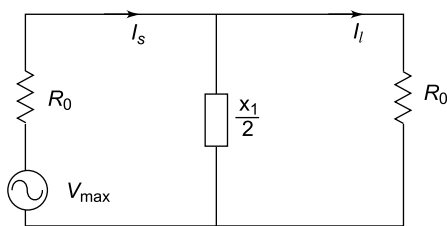$\dfrac{X_1}{2}$ = Reactance of shunt equalizer

Fig. 17.52

The shunt equalizer connected to the network is shown in Fig. 17.52.

Source current

$$I_s = \frac{V_{max}}{R_0 + \left(R_0 \,//\, \dfrac{jX_1}{2}\right)} \tag{17.95}$$

$$= \frac{V_{max}}{R_0 + \left[\dfrac{jX_1 R_0}{2R_0 + jX_1}\right]}$$

$$= \frac{V_{max}[2R_0 + jX_1]}{2R_0(R_0 + jX_1)}$$

Load current $I_l = I_s \dfrac{jX_1/2}{R_0 + \dfrac{jX_1}{2}} = I_s \dfrac{jX_1}{2R_0 + jX_1}$ $\qquad$ (17.96)

Substituting I_s in the above equation

$$I_l = \frac{V_{max}\,jX_1}{2R_0(R_0 + jX_1)} \tag{17.97}$$

Power delivered to the load

$$P_l = |I_l|^2\, R_0 = \frac{V_{max}^2 X_1^2}{4R_0(R_0^2 + X_1^2)} \tag{17.98}$$

and $\qquad P_i = V_{max}^2 / 4R_0$

Therefore, $\qquad N = \dfrac{P_i}{P_l} = \dfrac{\dfrac{V_{max}^2}{4R_0}}{\dfrac{V_{max}^2 X_1^2}{4R_0(R_0^2 + X_1^2)}}$

$$\therefore \qquad N = 1 + \left(\frac{R_0}{X_1}\right)^2 \tag{17.99}$$

By knowing the values of R_0 and N, X_1 can be determined.

17.22 FULL SHUNT EQUALIZER

Figure 17.53 shows the full shunt equalizer. It is also a constant resistance equalizer which satisfies the equation $Z_1 Z_2 = R_0^2$.
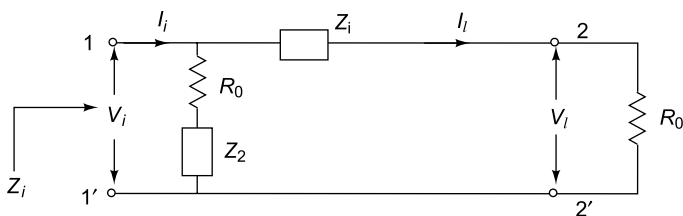
Fig. 17.53

The input impedance is given by

$$Z_i = \frac{(R_0 + Z_2)(R_0 + Z_1)}{2R_0 + Z_1 + Z_2} \quad (17.100)$$

$$= \frac{Z_1 Z_2 + R_0^2 + R_0(Z_1 + Z_2)}{2R_0 + Z_1 + Z_2}$$

$$Z_l = R_0$$

Since $Z_1 Z_2 = R_0^2$,

$$V_i = I_i Z_i = I_i R_0$$

$$V_l = I_l R_0$$

$$\frac{V_i}{V_l} = \frac{I_i}{I_l}$$

But $\quad I_l = I_i \dfrac{(R_0 + Z_2)}{2R_0 + Z_1 + Z_2} \quad (17.101)$

$$\frac{I_i}{I_l} = \frac{Z_1 + Z_2 + 2R_0}{R_0 + Z_2} \quad (17.102)$$

Multiplying both numerator and denominator by Z_1, we get

$$\frac{I_i}{I_l} = \frac{Z_1^2 + Z_1 Z_2 + 2R_0 Z_1}{Z_1 R_0 + Z_1 Z_2}$$

$$\frac{I_i}{I_l} = \frac{(Z_1 + R_0)^2}{R_0(Z_1 + R_0)} = \frac{Z_1 + R_0}{R_0}$$

Therefore, $\quad N = \left|\dfrac{V_i}{V_l}\right|^2 = \left|\dfrac{I_i}{I_l}\right|^2 = \left|\dfrac{R_0 + Z_1}{R_0}\right|^2$

$$N = 1 + \frac{X_1^2}{R_0^2} = 1 + \frac{R_0^2}{X_2^2} \quad (17.103)$$

since Z_1 and Z_2 are pure reactances and are equal to X_1 and X_2 respectively.

By knowing the values or R_0 and N_1 the elemental values X_1 and X_2 can be obtained.

(i) When $X_1 = \omega L_1$

   X_2 becomes $\dfrac{1}{\omega C_1}$

   The circuit is shown in Fig. 17.54.
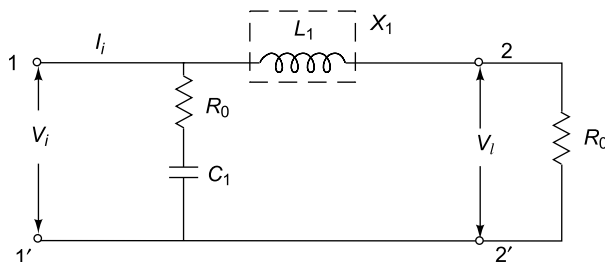

Fig. 17.54

(ii) When $X_1 = \dfrac{1}{\omega C_1}$

   X_2 becomes ωL_1
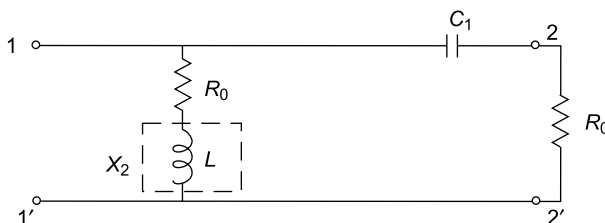
   The circuit is shown in Fig. 17.55.

Fig. 17.55

17.23   CONSTANT RESISTANCE EQUALIZER

The disadvantage of a reactance equalizer either in a shunt equalizer or a series equalizer is that, the variation of impedance with frequency causes impedance mismatch which results in reflection losses. A four terminal equalizer which offers a constant resistance at all frequencies avoids reflection loss when terminated in its design impedance. Constant resistance equalizer is a four terminal network which can be T, π, lattice and bridged-T network type. All these types have characteristic impedance satisfying the relation $Z_1 Z_2 = R_0{}^2$.

17.24   BRIDGE-*T* ATTENUATION EQUALIZER

The network shown in Fig. 17.56 is a bridged-T attenuation equalizer. Let Z_1 be a parallel combination of resistor R_1 and inductance L_1. To provide a constant

resistance the impedance Z_2 must be an inverse of Z_1 which is a series combination of R_2 and a capacitor C_1. Let R_0 be the design resistance.
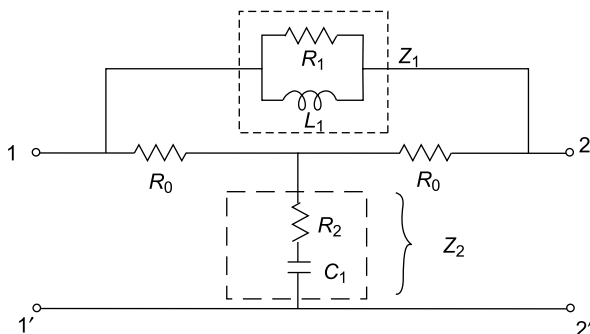
Then, $Z_1 Z_2 = R_0^2$

Fig. 17.56   *Bridged-T attenuation equalizer*

The propagation constant for a bridged-T network is given by

$$\gamma = \ln\left[1 + \frac{Z_1}{Z_0}\right] = \ln\left[1 + \frac{Z_0}{Z_2}\right] \tag{17.104}$$

But $Z_0 = R_0$

And $Z_1 = \dfrac{jR_1 \omega L_1}{R_1 + \omega L_1}$ \hfill (17.105)

Therefore, the propagation constant

$$\gamma = \ln\left[1 + \frac{jR_1 \omega L_1}{R_0(R_1 + j\omega L_1)}\right] \tag{17.106}$$

$$\alpha + j\beta = \ln\left[\frac{R_0 R_1 + j\omega L_1(R_0 + R_1)}{R_0 R_1 + j\omega L_1 R_0}\right] \tag{17.107}$$

Equating real parts on both sides

$$\alpha = \ln\left[\frac{(R_0 R_1)^2 + \omega^2 L_1^2 R_0^2 + \omega^2 L_1^2 R_1^2 + 2\omega^2 L_1^2 R_0 R_1}{R_0^2 R_1^2 + \omega^2 L_1^2 R_0^2}\right]^{1/2}$$

$$= \frac{1}{2}\ln\left[1 + \frac{\omega^2 L_1^2 R_1(2R_0 + R_1)}{R_0^2(R_1^2 + \omega^2 L_1^2)}\right] \tag{17.108}$$

and $R_1 R_2 = R_0^2 = \dfrac{L_1}{C_1}$ \hfill (17.109)

The elements may be calculated from the above design Eqs (17.108) and (17.109).

17.25 BRIDGED-*T* PHASE EQUALIZER

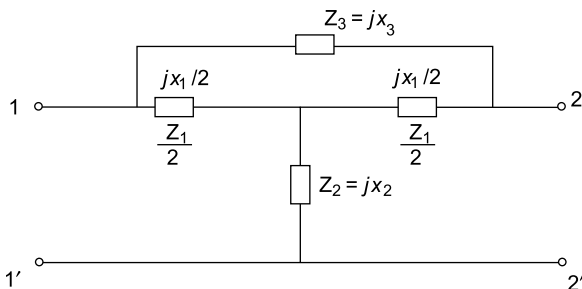A bridged-*T* phase equalizer is shown in Fig. 17.57.

Fig. 17.57

It consists of only pure reactances.
The characteristics impedance is given by

$$Z_0 = \left[\frac{Z_1 Z_3 (Z_1 + 4Z_2)}{4(Z_1 + Z_3)} \right]^{1/2} \tag{17.110}$$

From the Fig. 17.57, $Z_3 = jX_3$, $\dfrac{Z_1}{2} = jX_1$, $Z_2 = jX_2$ and $Z_0 = R_0$.

$$R_0^2 = \frac{2jX_1 \cdot jX_3 (2jX_1 + 4jX_2)}{4(2jX_1 + 4jX_3)} \tag{17.111}$$

$$= \frac{-X_1 X_3 (X_1 + 2X_2)}{2X_1 + X_3}$$

Let X_1 and X_3 be made inverse

$$jX_1 . jX_3 = R_0^2$$

$$-X_1 X_3 = R_0^2 \tag{17.112}$$

Substituting this in the above equation, we get

$$X_2 = \frac{X_1 + X_3}{2} \tag{17.113}$$

The propagation constant is given by

$$e^\gamma = \frac{Z_0(Z_1 + Z_3) + (Z_1 Z_3 / 2)}{Z_0(Z_1 + Z_3) - (Z_1 Z_3 / 2)} \tag{17.114}$$

$$e^\gamma - 1 = \frac{Z_1 Z_3}{Z_0(Z_1 + Z_3) - (Z_1 Z_3 / 2)}$$

and similarly,

$$e^\gamma + 1 = \frac{2Z_0(Z_1 + Z_3)}{Z_0(Z_1 + Z_3) - \left(\frac{Z_1 Z_3}{2}\right)}$$

From the above equations

$$\frac{e^\gamma - 1}{e^\gamma + 1} = \tanh\frac{\gamma}{2} = \frac{Z_1 Z_3}{2Z_0(Z_1 + Z_3)} = \frac{2jX_1 \cdot jX_3}{2R_0(2jX_1 + jX_3)}$$

$$= \frac{2R_0^2}{2R_0 j(2X_1 + X_3)} \tag{17.115}$$

$$\tanh\frac{\gamma}{2} = \frac{2R_0^2}{2R_0 j\left(2X_1 - \dfrac{R_0^2}{X_1}\right)}$$

$$\frac{\gamma}{2} = \tanh^{-1}\frac{jR_0 X_1}{R_0^2 - 2X_1^2}$$

$$\therefore \quad \alpha + j\beta = 2j\tan^{-1}\frac{R_0 X_1}{R_0^2 - 2X_1^2}$$

Equating the real and imaginary parts, we get

$$\alpha = 0$$

$$\beta = 2\tan^{-1}\left(\frac{R_0 X_1}{R_0^2 - 2X_1^2}\right) \tag{17.116}$$

Equations 17.112, 17.113 and 17.116 are the design equations of a bridged-T phase equalizer.

17.26   LATTICE ATTENUATION EQUALIZER

The constant resistance lattice attenuation equalizer is shown in Fig. 17.58. The element Z_1 represents series arm and Z_2 represents diagonal arm as shown in Fig. 17.58. The equalizer is a constant resistance equalizer such that Z_1 must be inverse of Z_2 to the design resistance R_0.
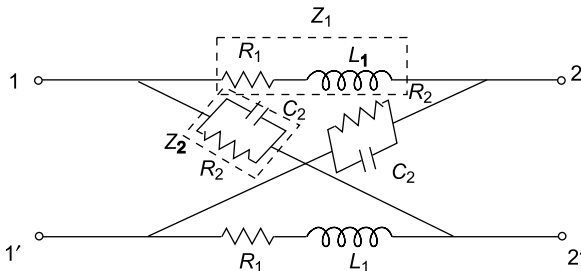

Fig. 17.58

$$Z_1 Z_2 = R_0{}^2$$

$$R_1 R_2 = \frac{L_1}{C_1} R_0^2 \tag{17.117}$$

The propagation constant of a lattice network is given by

$$\gamma = \ln\left| \frac{1 + \dfrac{Z_1}{R_0}}{1 - \dfrac{Z_1}{R_0}} \right| = \ln\left| \frac{1 + \dfrac{Z_2}{R_0}}{\dfrac{Z_2}{R_0} - 1} \right| \tag{17.118}$$

$$\alpha + j\beta = \ln\left| \frac{1 + \dfrac{R_1 + j\omega L_1}{R_0}}{1 - \dfrac{R_1 + j\omega L_1}{R_0}} \right| \tag{17.119}$$

$$\alpha + j\beta = \ln\left[\frac{(R_0 + R_1) + j\omega L_1}{(R_0 - R_1) - j\omega L_1} \right]$$

Equating real parts on both sides

$$\alpha = \ln\left[\frac{(R_0 + R_1)^2 + \omega^2 L_1^2}{(R_0 - R_1)^2 + \omega^2 L_1^2} \right]^{1/2}$$

$$N = e^{\alpha} = \left[\frac{(R_0 + R_1)^2 + \omega^2 L_1^2}{(R_0 - R_1)^2 + \omega^2 L_1^2} \right]^{1/2} \tag{17.120}$$

On the other hand if $X_1 = \dfrac{1}{\omega C_1}$

$$N = e^{\alpha} = \left[\frac{(R_0 + R_1)^2 + \dfrac{1}{\omega^2 C_1^2}}{(R_0 - R_1)^2 + \dfrac{1}{\omega^2 C_1^2}} \right]^{1/2} \tag{17.121}$$

Equations (17.117) and (17.121) are called design equations for the lattice attenuator network.

17.27   LATTICE PHASE EQUALIZER

The lattice phase equalizer is shown in Fig. 17.59. It consists of only reactive components. This is also a constant resistance equalizer which satisfies the equation $Z_1 Z_2 = R_0^2$.

Z_1 is the series arm impedance and Z_2 is the shunt arm impedance as shown in Fig. 17.59.
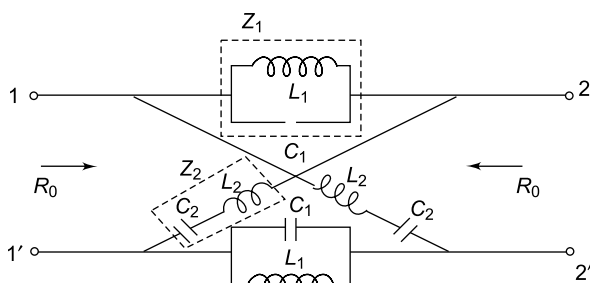
Fig. 17.59

The propagation constant is given by

$$\tanh\left(\frac{\gamma}{2}\right) = \left(\frac{Z_1}{R_0}\right) = \sqrt{\frac{Z_1}{Z_2}}$$

$$\therefore \qquad \tanh\left(\frac{\gamma}{2}\right) = \frac{j\omega L_1 / j\omega C_1}{R_0\left(j\omega L_1 + \dfrac{1}{j\omega C_1}\right)}$$

$$\tanh\left(\frac{\gamma}{2}\right) = \frac{j\omega L_1}{R_0(1 - \omega^2 L_1 C_1)}$$

$$\gamma = 2\tanh^{-1}\left[\frac{j\omega L_1}{R_0(1 - \omega^2 L_1 C_1)}\right]$$

$$= 2j\tan^{-1}\left[\frac{\omega L_1}{R_0(1 - \omega^2 L_1 C_1)}\right]$$

$$\alpha + j\beta = 2j\tan^{-1}\left[\frac{\omega L_1}{R_0(1 - \omega^2 L_1 C_1)}\right]$$

Here $\qquad \alpha = 0$

$$\beta = 2\tan^{-1}\left[\frac{\omega L_1}{R_0(1 - \omega^2 L_1 C_1)}\right]$$

The above expression gives the phase delay in a lattice phase equalizer.

Additional Solved Problems

Problem 17.1 Design a low pass π-section filter with a cut-off frequency of 2 kHz to operate with a load resistance of 400 Ω.

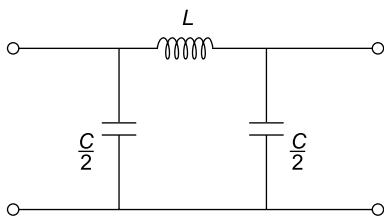**Solution** The π-section low pass filter is shown in Fig. 17.60.

Fig. 17.60

Cut-off frequency $f_c = 2$ kHz

Load resistance $K = 400\,\Omega = R_L$

Inductance

$$L = \frac{K}{\pi f_c} = \frac{400}{\pi \times 2 \times 10^3} = 63.66 \text{ mH}$$

Capacitance

$$C = \frac{1}{K \pi f_c} = \frac{1}{400 \times \pi \times 2 \times 10^3} = 0.3978 \ \mu\text{F}$$

Problem 17.2 Design an m-derived low pass filter having cut-off frequency of 1.5 kHz with a nominal impedance of 500Ω, and resonant frequency is 1600 Hz.

Solution we have $f_c = 1.5$ kHz ; $k = 500\ \Omega$, and $f_\alpha = 1600$ Hz

For an m - derived section, the value of $m = \sqrt{1 - \left(\dfrac{f_c}{f_\alpha}\right)^2}$

$$= \sqrt{1 - \left(\frac{1.5 \times 10^3}{1600}\right)^2} = 0.3479$$

For the prototype low pass section $L = \dfrac{k}{\pi f_c}$

$$= \frac{500}{\pi \times 1.5 \times 10^3} = 0.1061 \text{H} = z_1$$

$$C = \frac{1}{\pi k f_c} = \frac{1}{\pi \times 500 \times 1.5 \times 10^3} = 0.4244 \ \mu\text{F} \ = z$$

The *T*-section elements are

$$\frac{mz_1}{2} = \frac{mL}{2} = \frac{0.3479 \times 0.1061}{2} = 18.45 \text{ mH}$$

$$mz = mc = 0.3479 \times 0.4244 \times 10^{-6} = 0.147 \ \mu\text{F}$$

and $\left(\dfrac{1 - m^2}{4m}\right) z_1 = \left(\dfrac{1 - m^2}{4m}\right) L = 6 \text{ mH}$

The π - section elements are

$$\frac{mc}{2} = \frac{0.3479 \times 0.4244 \times 10^{-6}}{2} = 0.0738 \ \mu\text{F}$$

$$mL = 0.3479 \times 0.1061 = 36.91 \text{ mH}$$

and $\left(\dfrac{1 - m^2}{4m}\right) c = 0.268 \ \mu\text{F}$
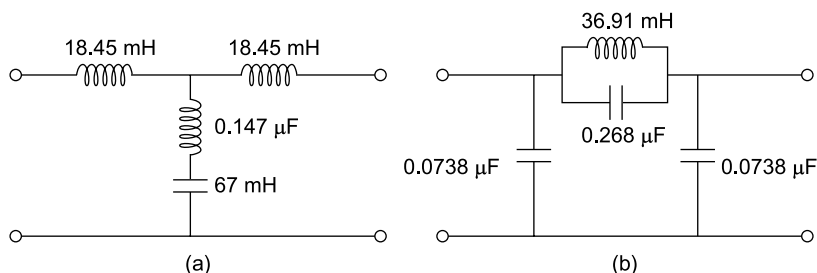
The filter sections are shown in Fig. 17.61

Fig. 17.61

Problem 17.3 *Design a band elimination filter having a design impedance of 500 Ω and cut-off frequencies* $f_1 = 1$ *kHz and* $f_2 = 5$ *kHz.*

Solution We have $f_1 = 1$ kHz ; $f_2 = 5$ kHz ; $k = 500\,\Omega$

$$\text{and} \quad f_0 = \sqrt{f_1 f_2} = 2.236\,\text{kHz}$$

$$B\omega = f_2 - f_1 = 4\,\text{kHz}$$

$$L_1 = \frac{k(f_2 - f_1)}{\pi f_1 f_2} = \frac{500 \times 4 \times 10^3}{\pi \times 1 \times 10^3 \times 5 \times 10^3} = 0.127\,\text{H}$$

$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{1}{4\pi \times 500 \times (4 \times 10^3)} = 3.971 \times 10^{-8}\,\text{F}$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)} = \frac{500}{4 \times \pi \times 4 \times 10^3} = 9.94\,\text{mH}$$

$$C_2 = \frac{1(f_2 - f_1)}{\pi k(f_2\,f_1)} = \frac{4 \times 10^3}{\pi \times 500 \times 10^3 \times 5 \times 10^3} = 0.5\,\mu\text{F}$$

Each of the two series arms of the constant K, T-section filter is given by $\frac{L_1}{2} = 63.5\,\text{mH}$; $2C_1 = 0.08\,\mu\text{F}$ and the shunt arm elements of the network are
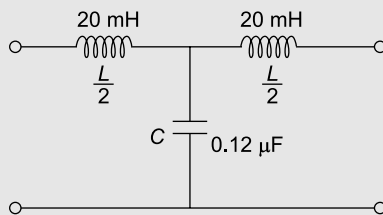
$$L_2 = 9.9\,\text{mH};\ C_2 = 0.5\,\mu\text{F}$$

For constant K, π section filter, the elements of the series arm are $L_1 = 127$ mH; $C_1 = 0.04\,\mu\text{F}$ and elements of the shunt arm are

$$2L_2 = 19.8\,\text{mH};\ \frac{C_2}{2} = 0.25\,\mu\text{F}$$

Problem 17.4   *A T-section filter is shown is Fig. 17.62. Calculate the value of cut-off frequency and determine the iterative impedance and the phase shift of the network at 1.5 kHz.*

Fig. 17.62

Solution   we have $\dfrac{L}{2} = 20\text{ mH} \Rightarrow L = 40\text{ mH}$

$$C = 0.12\ \mu\text{F}$$

The cut-off frequency

$$f_c = \frac{1}{\pi\sqrt{Lc}} = \frac{`1}{\pi\sqrt{40\times10^{-3}\times0.12\times10^{-6}}}$$

$$f_c = 4.6\,\text{kHz}$$

The iterative impedance is given by

$$Z_{oT} = \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$$= 577\sqrt{1-\left(\frac{1.5}{4.6}\right)^2} = 545.5\ \Omega$$

Phase shift $\beta = 2\sin^{-1}\left(\dfrac{f}{f_c}\right) = 2\sin^{-1}\left(\dfrac{1.5}{4.6}\right) = 38°$

Problem 17.5   *Find the frequency at which a prototype π-section low pass filter having a cut-off frequency f_c has an attenuation of 20 dB.*

Solution   we have $\alpha = 20\,\text{dB} = \dfrac{20}{8.696}$ nepers

$$= 2.23\text{ nepers.}$$

If f is the desired frequency for 20 dB, then

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

$$2.23 = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

$$\cosh(1.115) = \frac{f}{f_c}$$

$$f = f_c\cosh 1.115 = 1.689\,f_c$$

The frequency at which low pass π section filter has an attenuation of 20 dB will be 1.689 times the cut-off frequency.

Problem 17.6   *Design an m-derived LPF (T-section) having a cut-off frequency of 6 kHz and a design impedance of 500 Ω. The frequency of infinite attenuation should be 1.75 times the cut-off frequency.*

Solution   We have $f_c = 6000$ Hz; $k = 500$ Ω, and $f_\infty = 1.75 f_c$.

For the prototype lowpass section $L = \dfrac{k}{\pi f_c}$

$$= \frac{500}{\pi \cdot 6000} = 26.525 \text{ mH}$$

and $C = \dfrac{1}{\pi k f_c} = \dfrac{1}{\pi \times 500 \times 6000} = 0.106 \text{ μF}$

For an *m*-derived section the value of $m = \sqrt{1 - \left(\dfrac{f_c}{f_\infty}\right)^2}$

$$= \sqrt{1 - \left(\frac{6000}{1.75 \times 6000}\right)^2} = 0.820$$

Now each of the series element of lowpass *T*-section is given by
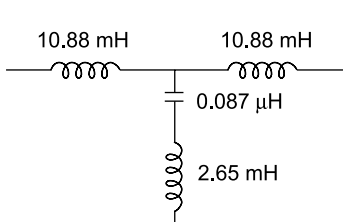
10.88 mH   10.88 mH

0.087 μH

2.65 mH

Fig. 17.63

$m\dfrac{L}{2} = \dfrac{0.820 \times 26.525 \times 10^{-3}}{2} = 10.68$ mH

The   shunt   arm   elements   are
$mC = 0.82 \times 0.106 \times 10^{-6} = 0.087$ μF

and

$\dfrac{1-m^2}{4m} \times L = \dfrac{1-(0.82)^2}{4 \cdot 0.82} \times (26.525 \times 10^{-3})$

$= 2.65$ mH

The required *m*-derived network is shown in Fig. 17.63

Problem 17.7   *A π-section filter network consists of a series arm inductance of 10 mH and two shunt arm capacitances of 0.16 μF each. Calculate the cut-off frequency and attenuation and phase shift at 12 kHz. What is the value of nominal impedance in the pass band.*

Solution   The given filter is shown in Fig. 17.64, it is a low pass filter; given $L = 10$ mH; $C/2 = 0.16$ μF; $C = 0.32$ μF.
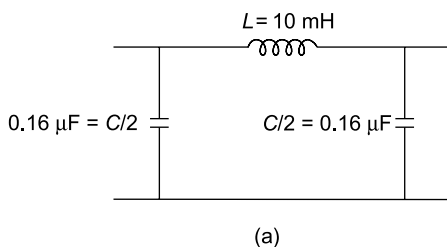
For π-section lowpass filter

(a)

Fig. 17.64

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$= \frac{1}{\pi\sqrt{10\times10^{-3}\times0.32\times10^{-6}}}$$

$$= 5.627 \text{ kHz}$$

Nominal terminating impedance is given by

$$k = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{10\times10^{-3}}{0.32\times10^{-6}}} = 176.77 \ \Omega$$

The attenuation constant $= 2\cosh^{-1}\left(\dfrac{\omega}{\omega_c}\right)$ nepers

$$= 2\cosh^{-1}\left(\frac{f}{f_c}\right) = 2\cosh^{-1}\left(\frac{12\times10^3}{5.627\times10^3}\right) = 2.78 \text{ nepers}$$

The phase shift introduced by the LPF will be π rad in the attenuation band.

Problem 17.8   *Each of the two series elements of a T-type low pass filter consists of an inductance of 30 mH having negligible resistance and a shunt element having capacitance of 0.16 μF. Calculate the value of cut-off frequency and determine the iterative impedance and the phase shift of the network at 2 kHz.*

Solution   We have $L/2 = 30$ mH $\Rightarrow L = 60$ mH, $C = 0.16$ μF

The cut-off frequency $f_c = \dfrac{1}{\pi\sqrt{LC}}$

$$= \frac{1}{\pi\sqrt{60\times10^{-3}\times0.16\times10^{-6}}}$$

$$f_c = 3.24 \text{ kHz}$$

The characteristic impedance is given by

$$Z_{0T} = \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}$$

$$= \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$$= \sqrt{\frac{60 \times 10^{-3}}{0.16 \times 10^{-6}}} \sqrt{1 - \frac{2 \times 10^3}{3.248 \times 10^3}} = (612)(0.619) = 379.05 \ \Omega$$

Since $f < f_c$ the attenuation $\alpha = 0$ and the phase shift in the pass band is given by

$$\beta = 2 \sin^{-1} \left(\frac{\omega}{\omega_c} \right) = 2 \sin^{-1} \left(\frac{2}{3.248} \right) = 76°$$

Problem 17.9 *Design the full series equalizer shown in Fig. 17.65. The design resistance $R_0 = 600 \ \Omega$ and attenuation of 12 dB at 800 Hz. Compute the elemental values.*

Fig. 17.65

Solution $D = 10 \log N$

$12 = 10 \log N$

$N = \text{Antilog} \left(\frac{12}{10} \right)$

$= 15.85$

We know that $N = 1 + \dfrac{\omega^2 L_1^2}{R_0^2}$

$L_1 = \dfrac{R_0 \sqrt{N - 1}}{\omega}$

$L_1 = \dfrac{600 \times \sqrt{15.58 - 1}}{2\pi \times 800} = 0.46 \text{ henry}$

$\dfrac{L_1}{C_1} = R_0^2$

$C_1 = \dfrac{L_1}{R_0^2} = \dfrac{0.46}{600 \times 600} = 1.28 \ \mu\text{F}$

Problem 17.10 *Design the full shunt equalizer shown in Fig. 17.66 for a design resistance $R_0 = 600 \ \Omega$ and attenuation of 10 dB at 600 Hz. Calculate the elemental values.*
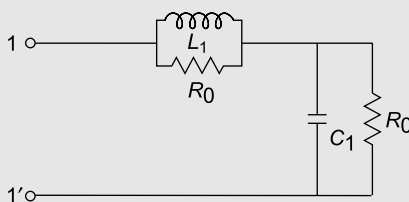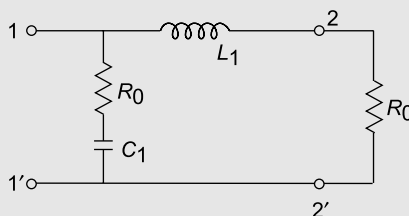

Fig. 17.66

Solution $D = 10 \log N$

$D = 10$ dB

$N = \text{Antilog } 1 = 10$

$$N = 1 + \frac{X_1^2}{R_0^2} = 1 + \frac{R_0^2}{X_1^2}$$

$X_1 = R_0 \sqrt{N-1}$

$\omega L_1 = R_0 \sqrt{N-1}$

$$L_1 = \frac{R_0 \sqrt{N-1}}{2\pi f} = \frac{600\sqrt{10-1}}{2\pi \times 600}$$

$L_1 = 0.48$ H

$$X_2 = \frac{R_0}{\sqrt{N-1}} = \frac{600}{3}$$

$$\frac{1}{\omega C_1} = \frac{600}{3}$$

$$C_1 = \frac{3}{2\pi \times 600 \times 600} = 1.33 \ \mu F$$

Problem 17.11   *Design a constant resistance lattice attenuation equalizer shown in Fig. 17.67. The series arm consists of $R_1 = 2$ kΩ in series with $L_1 = 30$ mH. If $R_2 = 300$ Ω, calculate the values of R_0 and capacitance C_1 of the shunt arm.*
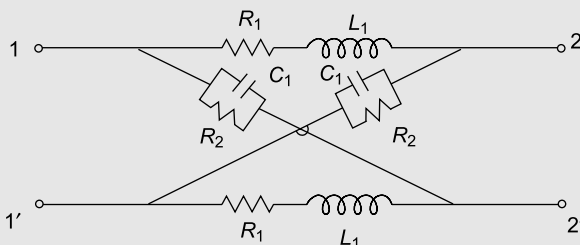
Fig. 17.67

Solution $R_1 = 2000 \ \Omega$ $L_1 = 30$ mH

$R_2 = 300 \ \Omega$

$R_1 R_2 = R_0^2$

$R = \sqrt{R_1 R_2} = 774.6 \ \Omega$

$$C_1 = \frac{L_1}{R_0^2} = \frac{0.03}{(774.6)^2} = 0.049 \ \mu F$$

Problem 17.12 Determine the series arm of a constant resistance lattice attenuation equalizer shown in Fig. 17.68 having design impedance of 2 Ω, the shunt arm consists of $R_2 = 2\ \Omega$ in series with a capacitor $C_2 = 0.1$ F.
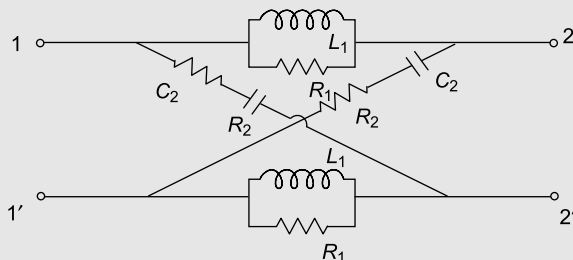
Fig. 17.68

Solution The shunt arm values are given as follows

$$R_2 = 2\ \Omega$$

$$C_2 = 0.1\ \text{F}$$

$$R_0 = 2\ \Omega$$

$$R_1 R_2 = \frac{L_1}{C_2} = R_0^2$$

$$R_1 = \frac{4}{2} = 2\ \Omega$$

$$L_1 = C_2\, R_0^2$$
$$= (0.1)\,(2)^2 = 0.4\ \text{H}$$

Problem 17.13 Obtain the inverse network for the network shown in Fig. 17.69.
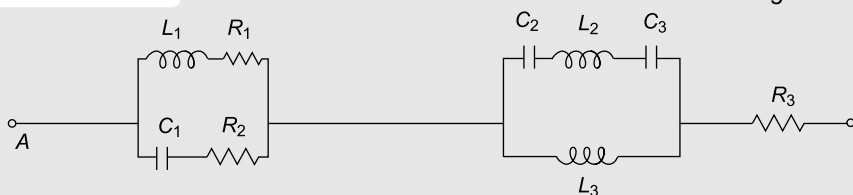
Fig. 17.69

Solution The elements of the inverse network are given by

$$C_1^1 = \frac{L_1}{R_0^2} \quad C_3^1 = \frac{L_3}{R_0^2} \quad L_2^1 = C_2 R_0^2 \quad R_1^1 = \frac{R_0^2}{R_1}$$

$$C_2^1 = \frac{L_2}{R_0^2} \quad L_1^1 = C_1 R_0^2 \quad L_3^1 = C_3 R_0^2 \quad R_2^1 = \frac{R_0^2}{R_2} \quad R_3^1 = \frac{R_0^2}{R_3}$$

The inverse network is shown in Fig. 17.70.
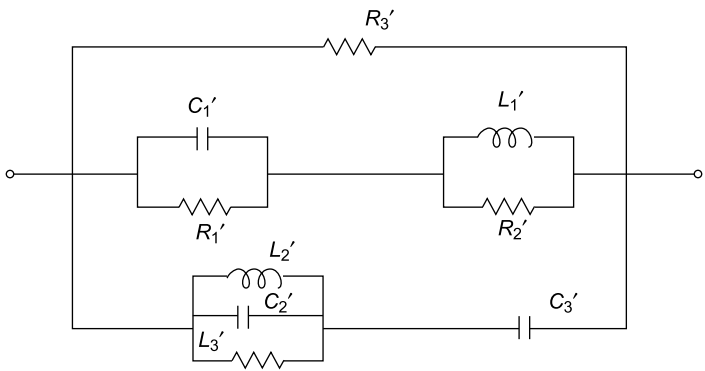
Fig. 17.70

PSpice Problems

Problem 17.1   *Determine the response of Twin T-band stop filter shown in Fig. 17.71(a) using PSpice.*
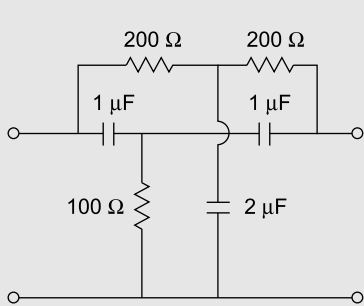
Fig. 17.71 (a)
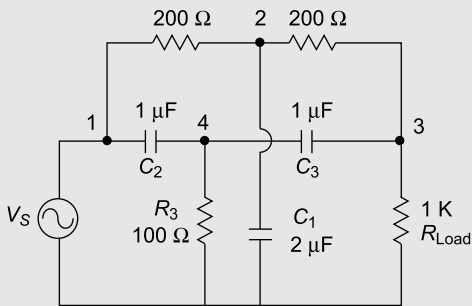
Fig. 17.71 (b)

TWIN-T BANDSTOP FILTER

V1	1	0	AC 1 0
R1	1	2	200
C1	2	0	2 U
R2	2	3	200
C2	1	4	1 U
R3	4	0	100
C3	4	3	1 U

Twin-*T* bandstop filter

(C) 16-1 (active)

Twin *T* band stop filter response

Log (*V*(3)/*V*(1))

□ V(3)/V(1)

1.0 Hz   10 Hz   100 Hz   1.0 kHz   10 kHz

Log (Frequency)
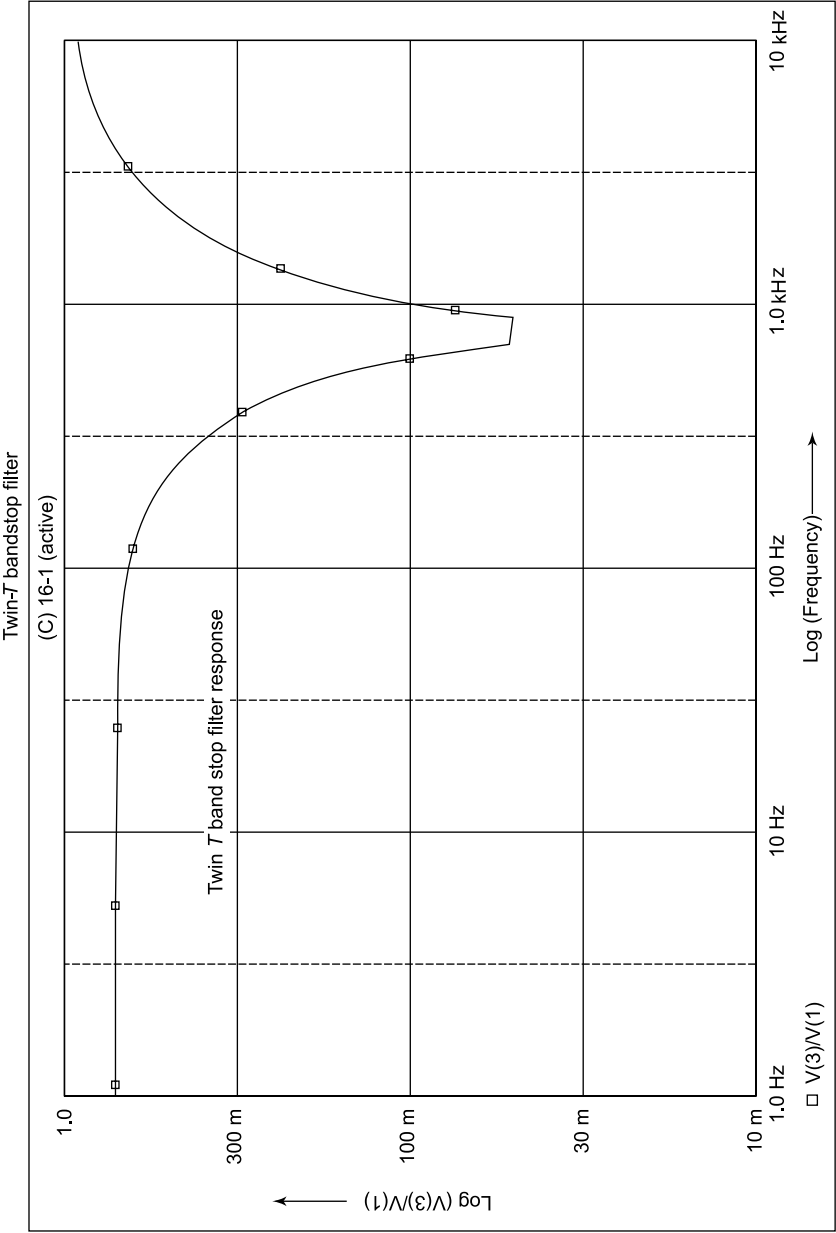
Fig. 17.72

```
RLOAD 3 0 1K
.AC DEC 20 1 1000K
.PLOT AC V(3)
.PROBE
.END
```

Result

FREQ	V(3)

| (*)--------- | 1.0000E – 04 | 1.0000E – 03 | 1.0000E – 02 | 1.0000E – 01 |
1.0000E + 00

```
                  - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E + 00    7.143E – 01.      .       .       .           *.
1.122E + 00    7.143E – 01.      .       .       .           *.
1.259E + 00    7.143E – 01.      .       .       .           *.
1.413E + 00    7.143E – 01.      .       .       .           *.
1.585E + 00    7.143E – 01.      .       .       .           *.
1.778E + 00    7.143E – 01.      .       .       .           *.
1.995E + 00    7.143E – 01.      .       .       .           *.
2.239E + 00    7.143E – 01.      .       .       .           *.
2.512E + 00    7.142E – 01.      .       .       .           *.
2.818E + 00    7.142E – 01.      .       .       .           *.
3.162E + 00    7.142E – 01.      .       .       .           *.
3.548E + 00    7.142E – 01.      .       .       .           *.
3.981E + 00    7.142E – 01.      .       .       .           *.
4.467E + 00    7.142E – 01.      .       .       .           *.
3.162E + 02    3.926E – 01.      .       .       .       *   .
3.548E + 02    3.483E – 01.      .       .       .      *    .
3.981E + 02    3.019E – 01.      .       .       .    *      .
4.467E + 02    2.539E – 01.      .       .       .   *       .
5.012E + 02    2.047E – 01.      .       .       .  *        .
5.623E + 02    1.547E – 01.      .       .       . *         .
6.310E + 02    1.039E – 01.      .       .       .*          .
7.079E + 02    5.265E – 02.      .       .      *.           .
7.943E + 02    8.235E – 04.      *.      .       .           .
8.913E + 02    5.159E – 02.      .       .      *.           .
1.000E + 03    1.047E – 01.      .       .       .*          .
1.122E + 03    1.585E – 01.      .       .       . *         .
1.259E + 03    2.132E – 01.      .       .       .  *        .
1.413E + 03    2.687E – 01.      .       .       .   *       .
1.585E + 03    3.249E – 01.      .       .       .    *      .
1.778E + 03    3.815E – 01.      .       .       .     *     .
1.995E + 03    4.381E – 01.      .       .       .      *    .
2.239E + 03    4.943E – 01.      .       .       .       *   .
                  - - - - - - - - - - - - - - - - - - - - - - - -
```

Problem 17.2 *Using PSpice, determine the response of LC low pass filter shown in Fig. 17.73(a).*
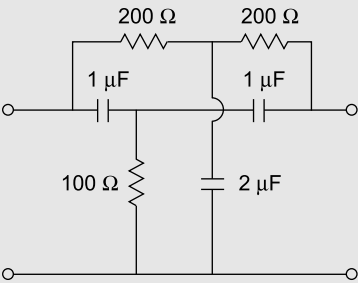
Fig. 17.73 (a)

Fig. 17.73 (b)

LC LOWPASS FILTER

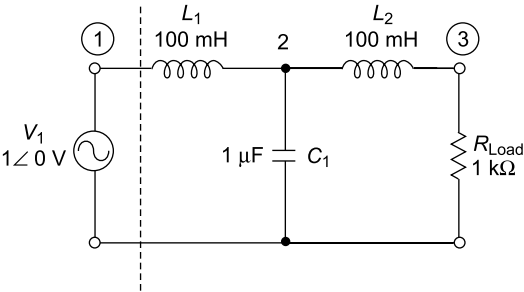| V1	1	0	AC 1 0
L1	1	2	100 M
C1	2	0	1 U
L2	2	3	100 M

RLOAD 3 0 1K
.AC LIN 20 100 1.5 K
.PLOT AC V(3)
.PROBE
.END

Result

```
    FREQ          V(3)
    (*)--------   1.0000E – 04     1.0000E – 02    1.0000E + 001.0000E + 02
1.0000E + 04

                  - - - - - - - - - - - - - - - - - - - - - - -
    1.000E + 01   1.000E + 00 .          .       .*       .         .
    5.358E + 02   3.117E + 00 .          .      .*        .         .
    1.062E + 03   2.620E – 01 .          .  *    .         .         .
    1.587E + 03   8.366E – 02 .          . *     .         .         .
    2.113E + 03   3.761E – 02 .          . *     .         .         .
    2.639E + 03   2.005E – 02 .          .*      .         .         .
    3.165E + 03   1.190E – 02 .         .*       .         .         .
    3.691E + 03   7.626E – 03 .         *        .         .         .
    4.216E + 03   5.170E – 03 .       *.         .         .         .
    4.742E + 03   3.662E – 03 .     *   .         .         .         .
    5.268E + 03   2.687E – 03 .    *    .         .         .         .
    5.794E + 03   2.028E – 03 .   *     .         .         .         .
    6.319E + 03   1.568E – 03 .   *     .         .         .         .
    6.845E + 03   1.237E – 03 .   *     .         .         .         .
    7.371E + 03   9.930E – 04 .  *      .         .         .         .
    7.897E + 03   8.090E – 04 .  *      .         .         .         .
    8.423E + 03   6.677E – 04 . *       .         .         .         .
    8.948E + 03   5.574E – 04 . *       .         .         .         .
```
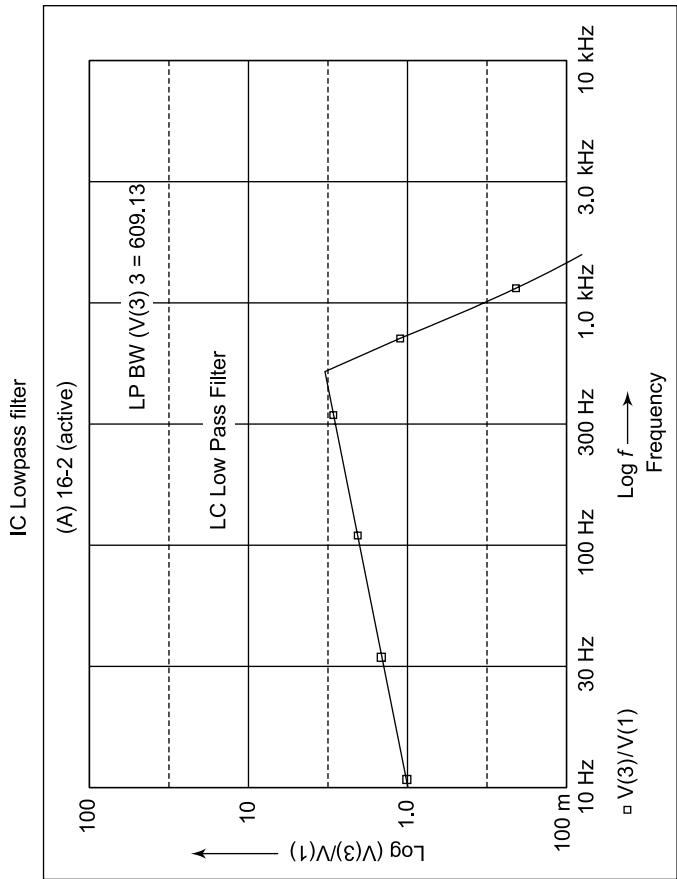
Fig. 17.74

```
9.474E + 03      4.701E – 04 . *        .      .      .        .
1.000E + 04      4.001E – 04 . *        .      .      .        .
- - - - - - - - - - - - - - - - - - - - - - - - -
```

Practice Problems

17.1 Design a low pass *T*-section filter having a cut-off frequency of 1.5 kHz to operate with a terminated load resistance of 600 Ω.

17.2 A *T*-section low pass filter has series inductance 80 mH and a shunt capacitance of 0.022 µF. Determine the cut-off frequency and nominal design impedance. Obtain the equivalent π-section.

17.3 Design a high pass filter with a cut-off frequency of 1 kHz with a terminated design impedance of 800 Ω.

17.4 Design an *m* derived high pass filter having a design impedance of $500\,\Omega$ and a cut-off frequency of 1 kHz. Take $m = 0.2$.

17.5 Design a *m*-derived high pass filter with a cut-off frequency of 10 kHz, design impedance of $600\,\Omega$ and $m = 0.3$.

17.6 For a π section filter network shown in Fig. 17.75, calculate the cut-off frequency and the value of nominal impedance in the pass band.
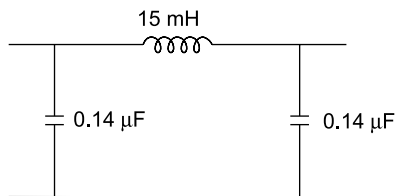

15 mH

0.14 µF   0.14 µF

Fig. 17.75

17.7 Determine the cut-off frequency and design impedance for the *T*-section shown in Fig. 17.76.

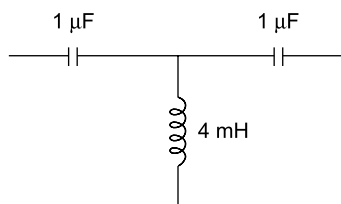**17.8** Determine the band width and cut-off frequency for the filter shown in Fig. 17.77.

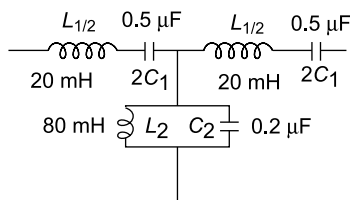1 µF   1 µF

4 mH

Fig. 17.76

$L_{1/2}$ 0.5 µF $L_{1/2}$ 0.5 µF

20 mH $2C_1$ 20 mH $2C_1$

80 mH L_2 C_2 0.2 µF

Fig. 17.77

17.9 Design a prototype band pass filter both T and π sections having cut-off frequencies of 3000 Hz and 6000 Hz and nominal characteristic impedance of $600\,\Omega$. Also, find the resonant frequency of shunt arm or series arm.

17.10 A π-section filter network is shown in Fig. 17.78. Calculate the cut-off frequency and phase shift at 10 kHz. What is the value of nominal impedance in the pass band.
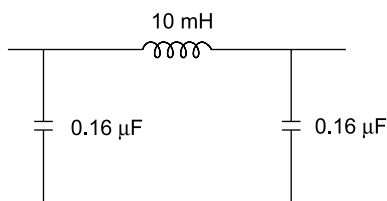

10 mH

0.16 µF   0.16 µF

Fig. 17.78

17.11 Design a prototype band stop filter section having cut-off frequency of 2000 Hz and 5000 Hz and design resistance of 600 Ω.

17.12 An attenuator is composed of symmetrical π-section having series arm of 275 Ω and shunt arm each of 450 Ω. Find

   (i)   The characteristic impedance of the network
   (ii) Attenuation per section

17.13 Design full series equalizer for a design resistance $R_0 = 600\ \Omega$ and attenuation of 20 dB at 400 Hz. Calculate the attenuation M at 1000 MHz.

17.14 Design the full shunt equalizer, for design resistance $R_0 = 600\ \Omega$ and attenuation at frequencies of 600 Hz and 1200 Hz.

17.15 Design a constant resistance lattice attenuation equalizer to produce an attenuation of 20 dB at 50 Hz and 3 dB at 3000 Hz. Calculate its loss at 500 Hz. The equalizer is working between two impedances of 500 Ω each.

17.16 Using PSpice, find the bandwidth and center frequency of the band stop filter of Fig. 17.79.

17.17 Using PSpice, determine center frequency and band width of the band pass filter shown in Fig. 17.80 (a) and (b).
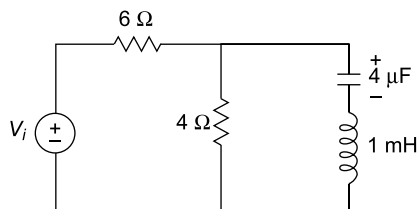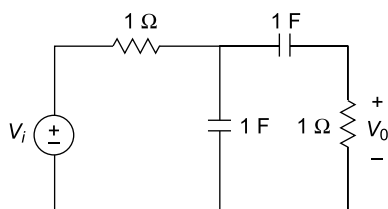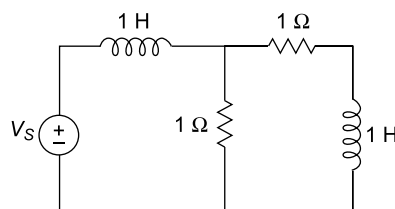

Fig. 17.79

(a)

(b)

Fig. 17.80

17.18 A low quality factor, double tuned band pass filter is shown in Fig. 17.81. Use PSpice, to generate the magnitude plot of $V_0\ (w)$.
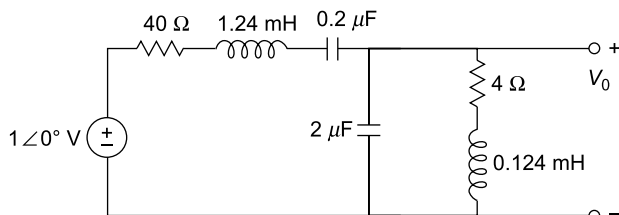

Fig. 17.81

Answers to Practice Problems

17.1 $L = 0.127$ H;
  $C = 0.35\ \mu$F
17.2 $f_c = 7.587$ kHz;
  $R_0 = 1.907$ KΩ
17.3 0.09 μF; 0.06 H

Fig. 17.82

17.4
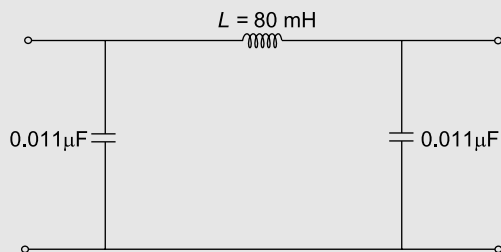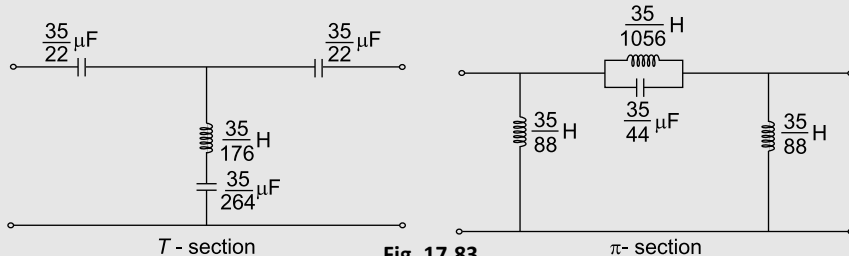

T - section   π- section

Fig. 17.83

17.5 For *T*-section; series arm component 6.66×10^{-9} F; shunt arm 0.015 mH, 1.3×10^{-9} F

For π-section series arm 6.19 mH: 3.33×10^{-9} F; shunt arm 0.031 H
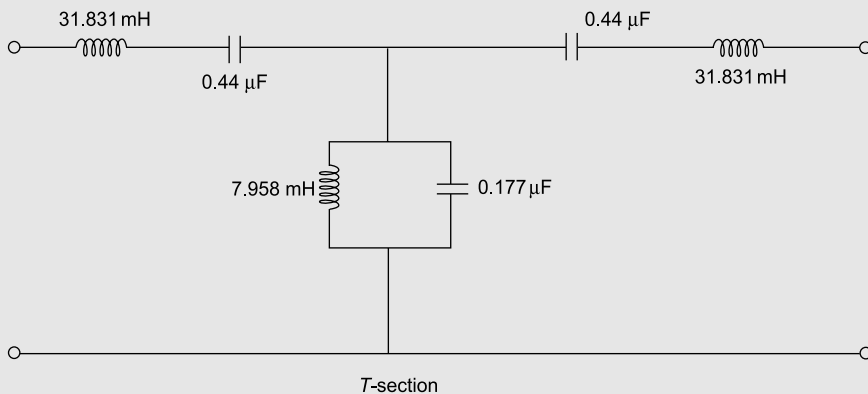
17.7 1.779 kHz; 89.44 Ω

17.9 $f_0 = 4242.7$ Hz

T-section

Fig. 17.84

π - section

Fig. 17.85

17.11

28.648 mH

0.088 µF

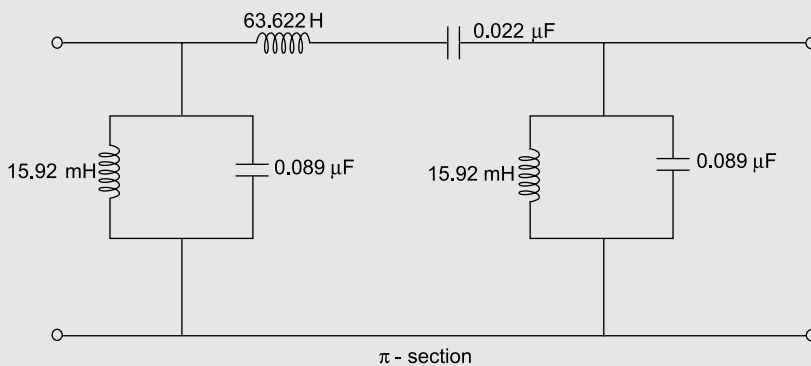28.648 mH

0.088 µF
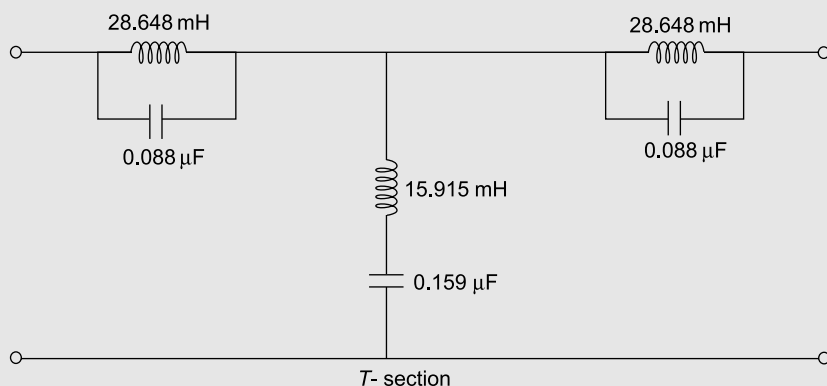
15.915 mH

0.159 µF

T- section

Fig. 17.86

17.12 (i) $R_0 = 217.731\ \Omega$

(ii) 9.17 dB

Objective-Type Questions

17.1 A low pass filter is one which

(a) passes all low frequencies

(b) attenuates all high frequencies

(c) passes all frequencies up to cut-off frequency, and attenuates all other frequencies

17.2 A high pass filter is one which

(a) passes all high frequencies

(b) attenuates all low frequencies

(c) attenuates all frequencies below a designated cut-off frequency, and passes all frequencies above cut-off

17.3 A band pass filter is one which

(a) attenuates frequencies between two designated cut-off frequencies and passes all other frequencies

(b) passes frequencies between two designated cut-off frequencies, and attenuates all other frequencies

(c) passes all frequencies

17.4 An ideal filter should have

(a) zero attenuation in the pass band

(b) infinite attenuation in the pass band

(c) zero attenuation in the attenuation band

17.5 The propagation constant of a symmetrical *T*-section and π-section are the same.

    (a) true   (b) false

17.6 The values of *L* and *C* for a low pass filter with cut-off frequency of 2.5 kHz to operate with a terminated load resistance of 450 ohms are given by

    (a) 57.32 mH; 0.283 μF   (b) 28.66 mH; 0.14 μF

    (c) 114.64 μH; 0.566 μF

17.7 The attenuation is sharp in the stop band for K-type filter.

    (a) true   (b) false

17.8 The attenuation is not sharp in the stop band for an *m*-derived filter.

    (a) true   (b) false

17.9 In the *m*-derived low pass filters, the resonant frequency is to be chosen so that it is

    (a) above the cut-off frequency   (b) below the cut-off frequency

    (c) none of the above

17.10 In the *m*-derived high pass filters, the resonant frequency is to be chosen so that it is

    (a) above the cut-off frequency   (b) below the cut-off frequency

    (c) none of the above

17.11 A band pass filter may be obtained by using a high pass filter followed by a low pass filter

    (a) true   (b) false

17.12 A band elimination filter is one

    (a) which attenuates all frequencies less than lower cut-off frequency f_1

    (b) which attenuates all frequencies greater than upper cut-off frequency f_2

    (c) frequencies lying between f_1 and f_2 are attenuated and all other frequencies are passed

Answers to Objective-Type Questions

17.1 (c)	**17.2** (c)	**17.3** (b)	**17.4** (a)	**17.5** (a)
17.6 (a)	**17.7** (b)	**17.8** (b)	**17.9** (a)	**17.10** (b)
17.11 (b)	**17.12** (c)			

and Synthesis of One-port Networks

CHAPTER 18

As stated in Chapter 15, the poles of the stable system must lie on the left half of the s-plane. Any network function can be written as the ratio of two polynomials, and is given by

$$Z(s) = \frac{P(s)}{Q(s)}$$

A polynomial must satisfy the following conditions.

(a) $Z(s)$ must be a real function of s

$$Z(s) = \frac{P(s)}{Q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}$$

where all the quotients a_i, b_j are real, and hence $Z(s)$ is real if s is real.

(b) All the roots of $P(s)$ must have zero real parts, or negative real parts.

Hurwitz polynomials have the following properties.

1. All the quotients in the polynomial

   $$P(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n$$

   are positive. A polynomial may not have any missing terms between the highest and the lowest order unless all even or all odd terms are missing. For example, the polynomial $P(s) = s^5 + 3s^3 + 5s^2 + 2s + 1$ is not Hurwitz as the term s^4 is missing. At the same time, the polynomial $P(s) = s^3 + 3s$ is Hurwitz because all quotient terms are positive and all even terms are missing.

2. The roots of the odd and even parts of a Hurwitz polynomial $P(s)$ lie on the $j\omega$ axis. Consider the polynomial $P(s)$ having odd and even parts $o(s)$ and $e(s)$, respectively; then

   $$P(s) = o(s) + e(s)$$

   Both have roots on the $j\omega$ axis.

3. If the polynomial $P(s)$ is either even or odd, the roots of $P(s)$ lie on the $j\omega$ axis.

4. All the quotient terms are positive in the continued fraction expansion of the ratio of the odd to even, or even to odd parts of the polynomial $P(s)$. Consider a polynomial

   $$P(s) = s^4 + s^3 + 6s^2 + 3s + 4$$

   The even parts of the polynomial, $e(s) = s^4 + 6s^2 + 4$

   The odd parts of the polynomial $o(s) = s^3 + 3s$