ANSWER KEY

CIA - III

14E304 - ELECTROMAGNETIC THEORY

II EEE

PART - A

A1: FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION:

The Electromagnetic Force (e.m.f) induced in a closed path is proportional to rate of change of magnetic Flux enclosed by the closed path.

Faraday's law can be stated as

$$e = -N\frac{d\phi}{dt} \quad \text{Volts.}$$

A2: POYNTING VECTOR:

The power density is given by

$$\vec{P} = \vec{E} \times \vec{H}$$

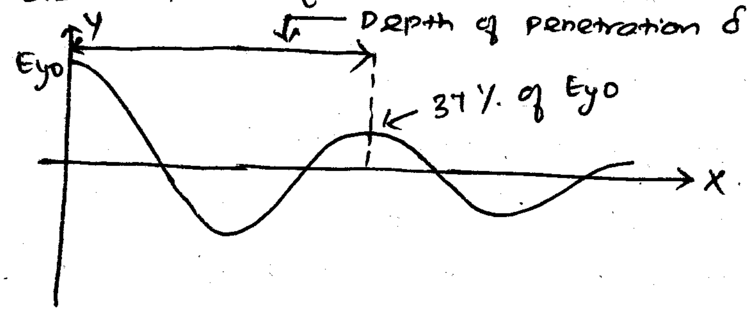
where, $\vec{P}$ is called poynting vector.

A3: TRANSVERSE ELECTROMAGNETIC WAVE:

Transverse Electromagnetic wave is a wave for which electric field ($\bar{E}$), magnetic field ($\bar{H}$) and the direction of propagation are prependicular to each other.

A4: SKIN DEPTH:

The skin depth is defined as that depth in which the wave has been attenuated to $1/e$ ie, approximately 37 % of its original value. The skin depth is also called as depth of penetration.



A5: **BREWSTER ANGLE:**

Brewster angle is the angle of the incident wave for which there is no reflection.

Brewster angle for,

Parallel polarization $\theta_1 = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

Prependicular polarization: No such angle.

A6: **LENZ LAW:**

The direction of induced e.m.f. is such that it opposes the cause producing it ie, changes in the magnetic flux.

A7: **STANDING WAVE:**

If there are two waves, one incident in forward direction, while other reflected back in backward direction, then the standing waves are said to be produced along the line.

A8: **INTRINSIC IMPEDENCE:**

Intrinsic Impedence is defined as the ratio of magnitudes of $\vec{E}$ and $\vec{H}$ in the medium.

$$\eta = \frac{E_m}{H_m} = 120\pi = 377\ \Omega.$$

A9: **POLARIZATION:** (1)

The polarization of uniform plane waves is defined as time varying behaviour of the electric field intensity Vector $\vec{E}$ at some fixed point in space, along the direction of propagation.

CLASSIFICATIONS: (1)

1) Linear polarization 2) Elliptical polarization 3) Circular Polarization.

A10: REFLECTION COEFFICIENT:

The reflection coefficient is defined as the ratio of reflected waves to incident waves. It is denoted by Γ.

$$\boxed{\Gamma = \frac{E_r}{E_i}}$$

PART B:

B1) a) i) MAXWELL'S EQUATION - POINT & INTEGRAL FORM - TIME VARYING FIELDS:

① Maxwell's Equation Derived from Faraday's law: (2)

Consider Faraday's law,

$$\oint \vec{E} \cdot \overline{dL} = -\int_S \frac{\partial \overline{B}}{\partial t} \, \overline{ds} \quad \text{--- ①}$$

Eq. ① is the maxwell's equation derived from faraday's law expressed in Integral Form.

using stokes theorem for eq ①, we get

$$\int_S (\nabla \times \overline{E}) \, \overline{ds} = -\int_S \frac{\partial \overline{B}}{\partial t} \, \overline{ds} \quad \text{--- ②}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- ③}$$

Eq ③ is the point form derived from faradays law - maxwells eq.

② Maxwell's Equation Derived from Ampere's circuital law: (2)

Amp. Ckt. law

$$\oint \overline{H} \cdot \overline{dl} = I_{enclosed} = \int_S \vec{J} \cdot \vec{ds}$$

$$\oint \overline{H} \cdot \overline{dL} = \int_S \vec{J} \cdot \vec{ds}$$

$$\oint \vec{H}\cdot \vec{dL} = \int_S \left[\vec{J_c} + \frac{\partial \vec{D}}{\partial t}\right] \vec{ds}$$

Integral foam of Maxwell's eq. derived from Amp. Ckt. low.

Applying Stokes theorem to above equation,

$$\oint \vec{H}\cdot \vec{dL} = \int_S (\nabla\times\vec{H})\, \vec{ds} = \int_S \left[\vec{J_c} + \frac{\partial \vec{D}}{\partial t}\right]\cdot \vec{ds}$$

$$\nabla\times\vec{H} = \vec{J_c} + \frac{\partial \vec{D}}{\partial t}$$

Point foam of maxwells eq. derived from Amp. Ckt. low.

③ Maxwell's equation Derived from Gauss's law -

Electrostatic field: (1)

Gauss low,

$$\oint \vec{D}\cdot \vec{ds} = Q_{enclosed}$$

$$\oint \vec{D}\cdot \vec{ds} = \int_V \rho_v \, dv$$

The above eq. is the Integral foam of M.E derived from gauss law for electric field.

Applying divergence to above eq.,

$$\int_V (\nabla\cdot\vec{D})\, dv = \int_V \rho_v \, dv$$

$\boxed{\nabla\cdot\vec{D} = \rho_v}$ Point foam of Maxwell's eq. derived from gauss low for electric field.

④ Magnetostatic field: (1)

Gauss law for magnetic field

$$\oint_S \vec{B}\cdot \vec{ds} = 0 \rightarrow \text{Integral foam}$$

Taking divergence to above equation

$$\int_V (\nabla\cdot\vec{B})\cdot dv = 0$$

$$\nabla\cdot\vec{B} = 0 \rightarrow \text{Point foam.}$$

B1) a) ii) AVERAGE POWER DENSITY

To find Average Power Density (P_{avg}), Let us integrate power density in z direction over one cycle and divide by the period T. of one cycle.

$$P_{avg} = \frac{1}{T}\int_0^T \frac{E_m^2}{\eta}\cos^2(\omega t - \beta z)\,dt \qquad (2)$$

$$= \frac{E_m^2}{\eta T}\int_0^T \frac{1+\cos 2(\omega t - \beta t)}{2}\,dt$$

$$= \frac{E_m^2}{\eta T}\left[\frac{t}{2} + \frac{\sin 2(\omega t - \beta t)}{(2\omega)\,2}\right]_0^T \qquad (1)$$

$$= \frac{E_m^2}{\eta T}\left[\frac{T}{2} + \frac{\sin(4\pi T - 2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$$

$$= \frac{E_m^2}{\eta T}\left[\frac{T}{2} - \frac{\sin 2\beta z}{4\omega} + \frac{\sin 2\beta z}{4\omega}\right]$$

$$= \frac{E_m^2 T}{2\eta T} = \frac{E_m^2}{2\eta}$$

$$\boxed{P_{avg} = \frac{E_m^2}{2\eta}}\ \mathrm{W/m^2} \qquad (2)$$

B1) a) iii) ELECTRIC CIRCUIT AND MAGNETIC CIRCUIT - COMPARISON

(4) (Any four points)

ELECTRIC CIRCUIT	MAGNETIC CIRCUIT
1. The Path traced by the current is called electric circuit.	1. The path traced by the magnetic flux is called magnetic circuit
2. E.M.F is the driving force.	2. M.M.F is the driving force.

ELECTRIC CIRCUIT	MAGNETIC CIRCUIT
3. Resistance R opposes the flow of current.	3. Reluctance $\mathbb{R}$ is opposed by the magnetic path.
4. Conductivity σ	4. Permeability μ.
5. Field Intensity E.	5. Field Intensity H.
6. Current density J.	6. Flux density. B
7. Conductance.	7. Permeance.
8. Ohms low $e = IR$	8. Ohms Low $e_m = \phi R$
9. Kirchoffs low $\Sigma I = 0$ $\Sigma EMF = 0$	9. Kirchoffs Low $\Sigma \phi = 0$ $\Sigma MMF = \Sigma \phi s = \Sigma H.l$

B1) b) i) EFFECTS OF EMI AND EMC:

Electromagnetic Interference is the dissruption of operation of an electronic device when it is the vicinity of an electromagnetic field in the radio frequency spectrum that is caused by another device.

The internal circuit of personal computer generate Electromagnetic fields in the RF range. Also, cathode ray tube display generate EM energy over a wide band of frequencies. These emission can interfere with the performance of sensitive wireless receivers near by. High powered wireless transmitters can produce EM fields strong enough to upset the operation of electronic equipment near by. (3)

Electromagnetic compatibility is the branch of electrical sciences which studies the unintentional generation propogation and reception of electromagnetic energy with reference to unwanted effects.

It is the concept of enabling different electronic devices to operate without mutual interference. All electronic circuits have the possibility of radiating or picking up wanted electrical interference which can compromise the operation of one or other of the circuits. (2)

B1) b) ii) GENERAL WAVE EQUATION FOR MAGNETIC FIELDS:

Assume that the medium obeys the Ohm's law ie $\vec{J} = \sigma \vec{E}$. Then the maxwell's equation is given by

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ; \quad B = \mu H \Rightarrow \quad \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{—} (1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \begin{matrix} \vec{D} = \varepsilon \bar{E} \\ \bar{J} = \sigma \bar{E} \end{matrix} \Rightarrow \nabla \times \vec{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \quad \text{—} (2)$$

$$\nabla \cdot \bar{B} = 0 \quad ; \quad \nabla \cdot \vec{H} = 0 \quad \text{—} (3)$$

$$\nabla \cdot \vec{D} = 0 \quad ; \quad \nabla \cdot \bar{E} = 0 \quad \text{—} (4)$$

(1)

To obtain wave equation of $\bar{H}$, take curl on both sides of (2) we get

$$\nabla \times (\nabla \times \bar{H}) = \nabla \times \sigma \bar{E} + \varepsilon \nabla \times \frac{\partial \bar{E}}{\partial t}$$

$$\nabla \times \nabla \times \bar{H} = \sigma (\nabla \times \bar{E}) + \varepsilon \frac{\partial}{\partial t} (\nabla \times \bar{E}) \quad \text{—} (5)$$

sub eq (1) in eq (5)

$$\nabla \times \nabla \times \bar{H} = \sigma \left(-\mu \frac{\partial \bar{H}}{\partial t} \right) + \varepsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial \bar{H}}{\partial t} \right)$$

$$= -\mu \sigma \frac{\partial \bar{H}}{\partial t} - \mu \varepsilon \frac{\partial^2 \bar{H}}{\partial t^2} \quad \text{—} (6)$$

using vector identity

$$\nabla \times \nabla \times \bar{H} = \nabla \cdot (\nabla \cdot \bar{H}) - \nabla^2 H \quad — (7) \quad (1)$$

sub eq (3) in (7)

$$\nabla \times \nabla \times \vec{H} = -\nabla^2 \bar{H} \quad — (8)$$

Equating (6) & (8)

$$-\nabla^2 \bar{H} = -\mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}} \quad — (9) \quad (2)$$

This is the wave equation for Magnetic field $\bar{H}$.
Now multiplying both sides by μ we get,

(9) $\times \mu$

$$\nabla^2 (\mu \bar{H}) = \mu\sigma \frac{\partial \mu \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mu \bar{H}}{\partial t^2}$$

$\mu H = B$

$$\boxed{\nabla^2 \bar{B} = \mu\sigma \frac{\partial \bar{B}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2}} \quad (2)$$

This is the wave equation for flux density $\bar{B}$.

B1) b) iii) PLANE WAVES - LOSSY DIELECTRIC

All the dielectric materials exhibit some conductivity ($\sigma \neq 0$). So during analysis of the uniform plane waves through dielectric with some amount of conductivity, we cannot neglect σ by assuming it to be zero. Due to certain conductivity, certain amount of loss in the medium takes place. Hence the wave travelling through such medium gets attenuated $(\alpha=0)$. Such dielectric is called lossy dielectric.

The wave equation for $\bar{E}$ is given by

$$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \varepsilon\mu \frac{\partial^2 \bar{E}}{\partial t^2} \qquad (2)$$

The waves travel in z-direction only, $\bar{E}$ in x direction

$$\frac{\partial^2 \bar{E}_x}{\partial z^2} = \mu\sigma \frac{\partial \bar{E}_x}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}_x}{\partial t^2}$$

$$\bar{E}_x = E_m e^{j\omega t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = (j\omega)(j\omega) E_m e^{j\omega t} = (j\omega)^2 E_x$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\sigma (j\omega) E_x + \mu\varepsilon (j\omega)^2 E_x \qquad (1)$$

$$= \left[j\omega\mu(\sigma + j\omega\varepsilon)\right] E_x$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x} \qquad (1)$$

B2) a) i) GENERAL WAVE EQUATION FOR ELECTRIC FIELD:

Assume that the medium obeys the Ohm's law ie $\bar{J} = \sigma\bar{E}$. Then the maxwell's equation is given by,

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \; ; \quad B = \mu H \quad \Rightarrow \quad \nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{— (1)}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \; ; \quad \bar{D} = \varepsilon\bar{E}, \; J = \sigma\bar{E} \quad \Rightarrow \quad \nabla \times \bar{H} = \sigma\bar{E} + \varepsilon\frac{\partial \bar{E}}{\partial t} \quad \text{— (2)}$$

$$\nabla \cdot \bar{B} = 0 \; ; \quad \nabla \cdot \bar{H} = 0 \quad \text{— (3)}$$

$$\nabla \cdot \bar{D} = 0 \; ; \quad \nabla \cdot \bar{E} = 0 \quad \text{— (4)}$$

(1)

To obtain wave equation of $\bar{E}$, taking curl on both sides of eq ①, we get,

$$\nabla \times (\nabla \times \bar{E}) = -\mu \left(\nabla \times \frac{\partial \bar{H}}{\partial t} \right) \quad \text{——} ⑤$$

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \bar{H}) \quad \text{——} ⑥$$

substituting eq ② in eq ⑥

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t} \left[\sigma \bar{E} + \varepsilon \frac{\partial E}{\partial t} \right]$$

$$= -\mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} \quad \text{——} ⑦$$

According to vector identity,

$$\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 E \quad \text{——} ⑧ \ (1)$$

Sub $\nabla \cdot \bar{E} = 0$ in eq ⑧

$$\nabla \times \nabla \times \bar{E} = -\nabla^2 \bar{E}$$

sub. in eq ⑦,

$$-\nabla^2 \bar{E} = -\mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}} \quad \text{——} ⑨ \ (2)$$

This is the wave equation for electric field $\bar{E}$.

multiplying eq ⑨ by ε on both sides,

$$\nabla^2 (\varepsilon \bar{E}) = \mu\sigma \frac{\partial (\varepsilon \bar{E})}{\partial t} + \mu\varepsilon \frac{\partial^2 (\varepsilon \bar{E})}{\partial t^2}$$

$$\boxed{\nabla^2 \bar{D} = \mu\sigma \frac{\partial \bar{D}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{D}}{\partial t^2}}$$

This is the wave equation for $\bar{D}$. (2)

B2) a) ii) POYNTING THEOREM:

Poynting Theorem states that, the net power flowing out of a given volume V is equal to the time rate of decrease in the energy stored within volume 'V' minus the ohmic power dissipated. (2)

Integral & Point form:

consider maxwell's equation,

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{——} (1)$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \quad \text{——} (2)$$

Dotting both the sides of eq (2) with $\bar{E}$, we get

$$\bar{E} \cdot (\nabla \times \bar{H}) = \bar{E} \cdot (\sigma \bar{E}) + \bar{E}\left(\varepsilon \frac{\partial E}{\partial t}\right) \quad \text{——} (3)$$

vector identity,

$$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A}(\nabla \times \bar{B}) \qquad (1)$$

substituting $\bar{A} = \bar{E}$ & $\bar{B} = \bar{H}$, we get,

$$\bar{H} \cdot (\nabla \times \bar{E}) - \bar{E}(\nabla \times \bar{H}) = \nabla(\bar{E} \times \bar{H})$$

$$\bar{H} \cdot (\nabla \times \bar{E}) - \nabla(\bar{E} \times \bar{H}) = \bar{E}(\nabla \times \bar{H})$$

sub eq (3) in above equation,

$$H \cdot (\nabla \times \bar{E}) - \nabla(\bar{E} \times \bar{H}) = \sigma E^2 + \bar{E}\left(\varepsilon \frac{\partial E}{\partial t^2}\right) \text{——} (4)$$

consider the first team of (4)

$$\bar{H} \cdot (\nabla \times \bar{E}) = \bar{H}\left(-\mu \frac{\partial \bar{H}}{\partial t}\right)$$

$$\bar{H} \cdot (\nabla \times \bar{E}) = -\mu H \cdot \frac{\partial \bar{H}}{\partial t} \quad \text{——} (i)$$

we know,

$$\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} H^2 \quad \text{—(ii)}$$

$$\bar{E} \cdot \frac{\partial \bar{E}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} E^2 \quad \text{—(iii)}$$

sub (i) (ii) (iii) in eq (4)

$$\vec{H} (\nabla \times \bar{E}) - \nabla \cdot (\bar{E} \times \bar{H}) = \sigma E^2 + \varepsilon \frac{\partial}{\partial t^2} E^2$$

$$-\frac{\mu}{2} \frac{\partial}{\partial t} H^2 - \nabla (\bar{E} \times \bar{H}) = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t^2}$$

$$-\nabla (\bar{E} \times \bar{H}) = \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t} [\mu H^2 + \varepsilon E^2]$$

$$\bar{E} \times \bar{H} = \bar{P}$$

$$\boxed{-\nabla \cdot (\bar{P}) = \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t} [\mu H^2 + \varepsilon E^2]} \quad \text{—(5)}$$

(2)

The above equation represents poynting Theorem in Point form.

If we integrate this power over a volume, we can get energy distribution as,

$$-\int_V \nabla \cdot \bar{P} \, dV = \int_V \sigma E^2 \, dv + \frac{\partial}{\partial t} \int_V \frac{1}{2} [\mu H^2 + \varepsilon E^2] \, dv$$

Applying divergence theorem to left of the above eq.

$$\boxed{-\oint \bar{P} \cdot \bar{ds} = \int_V \sigma E^2 \, dv + \frac{\partial}{\partial t} \int_V \frac{1}{2} [\mu H^2 + \varepsilon E^2] \, dv}$$

(1)

The above equation represents poynting theorem in integral form.

B2) a) iii) PROBLEM:

Given,

$f = 300$ Hz

$\sigma = 0$

$\mu_r = 1$

$\epsilon_r = 78$

To find,

Wavelength (λ)

Solution:

wavelength $\lambda = \frac{2\pi}{\beta}$ (1)

$\beta = \omega\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r} = 2\pi \times 300 \times \sqrt{4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 78}$

$\boxed{\beta = 5.54 \times 10^{-4} \text{ r/m}}$ (1)

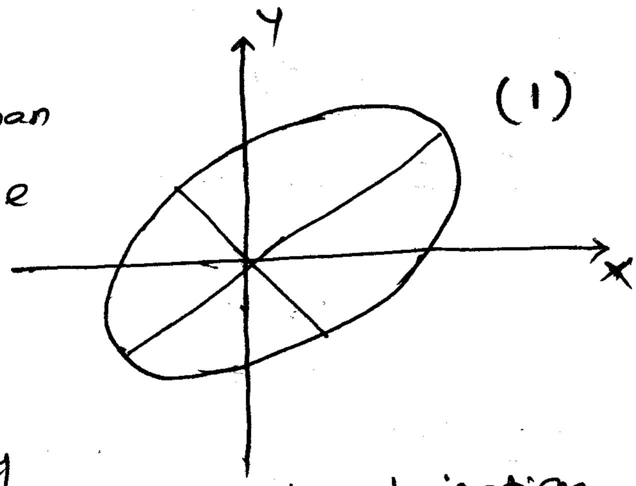
$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.5 \times 10^{-4}}$

$\boxed{\lambda = 1.13 \times 10^{4} \text{ m}}$ (1)

B2) b) i) ELLIPTICAL POLARIZATION:

Consider that the electric field $\bar{E}$ has both the components which are not having same amplitudes and are not in phase. As the wave propagates, $\overline{E_x}$ and $\overline{E_y}$ will have maximum and minimum amplitudes at different instants of time depending on the relative amplitudes of $\overline{E_x}$ and $\overline{E_y}$ at any instant of time. If the locus of the end points of $\bar{E}$ is traced, it is observed that $\bar{E}$ moves elliptically. Then such a wave is said to be elliptically polarized as shown in fig. (1)

When the phase difference between $\overline{E_x}$ and $\overline{E_y}$ is other than 90°, then the axes of the ellipse are inclined at an angle θ. (1)

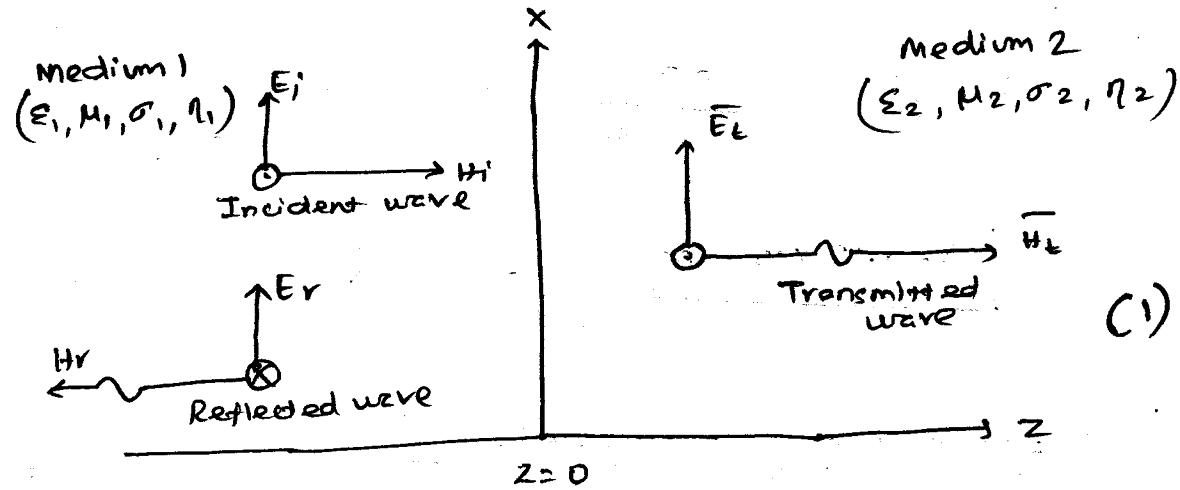


(1)

Elliptical polarization

When the phase difference between $\overline{E_x}$ and $\overline{E_y}$ is exactly 90°, then the axes of the ellipse lie along the co-ordinate axes. (1)

B2) b) ii) **REFLECTION AND TRANSMISSION COEFFICIENT — NORMALLY INCIDENT ON THE SURFACE OF A DIELECTRIC**



(1)

The total field in medium 1 is given by

$$\overline{E_1} = \overline{E_i} + \overline{E_r}$$

$$\overline{H_1} = \overline{H_i} + H_r$$

III'y for medium 2,

$$\overline{E_2} = \overline{E_t}$$

$$\overline{H_2} = \overline{H_t}$$

According to the boundry conditions, the tangential components of $\bar{E}$ and $\bar{H}$ must be continuous to the interface $z=0$

$\overline{E_1}\tan = \overline{E_2}\tan$

$\overline{H_1}\tan = \overline{H_2}\tan$

Thus at the interface, $z=0$,

$$\overline{E_i} + \overline{E_r} = \overline{E_t} \quad \text{—— } 1$$

$$\overline{H_i} + \overline{H_r} = \overline{H_t} \quad \text{—— } 2$$

w.k.t $E_i = \eta_1 H_i$

$E_r = -\eta_1 H_r$

$E_t = \eta_2 H_t$

In eq 2, putting the values of H_i, H_r & H_t

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$E_i - E_r = \frac{\eta_1}{\eta_2} E_t \quad \text{—— } 3 \qquad (1)$$

Adding 1 & 3

$$2E_i = \frac{\eta_1 + \eta_2}{\eta_2} E_t$$

$$E_t = \frac{2\eta_2}{\eta_1 + \eta_2} E_i$$

Transmission co-efficient (τ)

$$\boxed{\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}} \qquad (2)$$

Eliminating E_t from eq 1 & 3, ie 1/3

$$E_i + E_r = \frac{\eta_2}{\eta_1} E_i - E_r$$

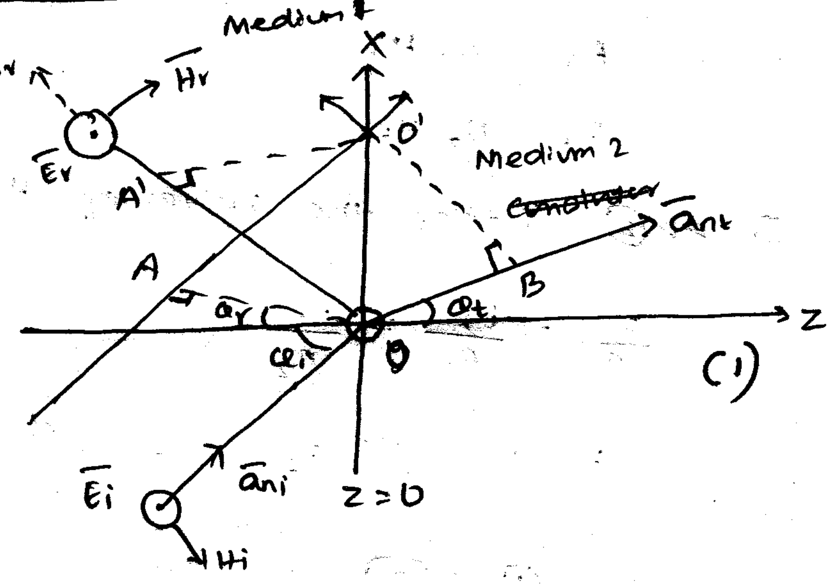
$$E_i(\eta_1 - \eta_2) = -E_r(\eta_1 + \eta_2)$$

$$E_r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_i$$

Reflection coefficient Γ

$$\boxed{\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad (2)$$

B2) b) iii) **WAVE INCIDENT OBLIQUELY TO THE SURFACE OF PERFECT INSULATOR:**



(1)

The distance $\overline{OA'}$ and $\overline{AO'}$ must be same

$$\overline{OO'} \sin \theta_i = \overline{OO'} \sin \theta_r$$

ie, $\theta_r = \theta_i$ ——— (1)

angle of incidence is equal to angle of reflection.
This is snell's law of reflection. (1)

$$\frac{\overline{OB}}{V_2} = \frac{\overline{AO'}}{V_1}$$

$V_1 = V_2 =$ velocity of the wave in medium 1 & 2

$$\frac{\overline{OB}}{\overline{AO'}} = \frac{\overline{OO'} \sin \theta_t}{\overline{OO'} \sin \theta_i} = \frac{V_2}{V_1}$$

Simplifying above eqration,

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{V_2}{V_1} = \frac{(\omega/\beta_2)}{(\omega/\beta_1)} = \frac{\beta_1}{\beta_2} \quad \text{—— (1)}$$

Index of reflection, $n = \frac{c}{v} = \frac{\text{Velocity of light in free space}}{\text{Velocity of light in medium}}$ (1)

Then for medium 1, index of reflection is given by

$$n_1 = \frac{c}{V_1}$$

|||'ly

$$n_2 = \frac{c}{V_2}$$

then

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{n_1}{n_2}$$

But $V_1 = \frac{1}{\sqrt{\mu_0 \varepsilon_1}}$ $\quad V_2 = \frac{1}{\sqrt{\mu_0 \varepsilon_2}}$

Putting V_1, V_2 in eq (1)

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{1/\sqrt{\mu_0 \varepsilon_2}}{1/\sqrt{\mu_0 \varepsilon_1}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \quad \text{—— (2)}$$

Also w.r.t $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}$ $\quad \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$

Rearranging eq (2),

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{\sqrt{\mu_0 \varepsilon_1}}{\sqrt{\mu_0 \varepsilon_2}} = \frac{\sqrt{\mu_0/\varepsilon_2}}{\sqrt{\mu_0/\varepsilon_1}} = \frac{\eta_2}{\eta_1}$$

Thus,

$$\boxed{\frac{\sin \theta_t}{\sin \theta_i} = \frac{V_2}{V_1} = \frac{\beta_1}{\beta_2} = \frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}} \quad (2)$$