Networks and Signals

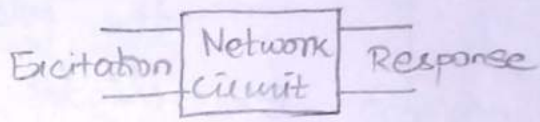
Networks :

What is Network ?

When a number of impedances are connected together to form a system that consists of set(s) interconnected circuits performing specific functions it is called a network or circuit.

An electrical network is a combination of numerous network elements. (eg. R, L, C etc).

The input to a network is excitation and output is response.



Network Elements:

A network element is a component of the circuit having different characteristics as given below.

1. Linear and Non-Linear Elements

Linear → Shows linear characteristics of Voltage vs current.

Ex: Resistances, Inductors & capacitors.

Their Resistance, Inductance & capacitance do not change with a change in applied voltage or the circuit current.

Non-linear → Current passing through it does not change linearly with the linear change in applied voltage at a particular frequency.

Ex: Semi conductor devices

2. Active and passive Elements

Active ⇒ If a circuit element has the capability of enhancing the energy level of a signal passing through it.

Ex: Vacuum tubes & semiconductor devices

passive ⇒ They do not have any intrinsic means of signal boosting.

Ex: Resistors, Inductors, capacitors, Thermistors etc.

3. unilateral and Bilateral Elements

unilateral ⇒ Magnitude of the current passing through an element is affected due to change in the polarity of an applied voltage.

Ex: diodes, transistors

Bilateral ⇒ current Magnitude remains the same even if the applied voltage polarity is changed.

Ex: Resistor, Inductor and capacitor.

Classification of Network:

Resistors, Inductors and capacitors are called as Network parameters. & they are in the form of lumped and distributed.

1. Linear circuits: parameters remain constant with change in applied voltage or current.

Ex: Resistance, Inductance & capacitance

2. Non-linear circuits: parameters change with change in voltage or current.

Ex: Semiconductor devices.

3. Unilateral circuit: By changing the polarity, properties of circuit May change.
Ex: diode, transistor etc.

4. Bilateral circuit: By changing the polarity, properties of circuit May not change

5. Active network: One or More than one source of emf.

6. passive network: does not contain any source of emf. It serves as load terminal (input) & May have output terminals

7. Lumped Network: physically separate network elements like R, L or C.

Distributed Network: No physical separation of network elements. Ex: transmission lines

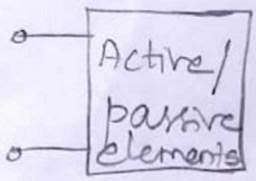
8. Recurrent Network: when a large circuit consists of similar networks connected one after another. Also called as cascaded or ladder network.

Non-Recurrent Network: A single network.

Port in Network:

1. one port Network:

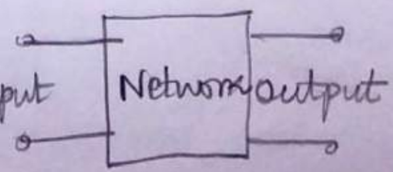
Any active or passive network having only two terminals



Ex: Antenna, strain guage, oscillator tank circuit etc.

2. Two port Network:

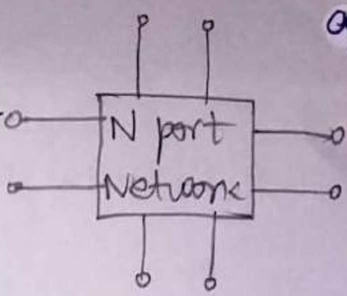
A network consists of two pair of terminals, where one pair is input to the other pair is output.



Ex: Transmission lines

3. n port Network:

Network consists of 'n' number of pairs of terminals.



UNIT- 1

Two Port Network

I. Ports (or) Terminal pairs :

An arbitary network made up of passive elements are considered and are represented by using an rectangular box.

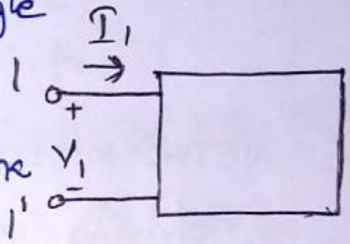
There are two types of networks based upon the terminal pairs or the ports. They are

i) one port Network

ii) Two port Network

1. one port Network :

* It is a rectangular box with a single pair of terminals.



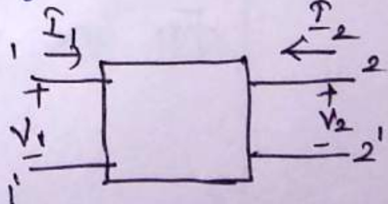
* one Voltage, one current & one network function is defined

* A driving source is connected to the pair of terminals

2. Two port Network :

* It is a network / electrical network inside a black box and the network has two pairs of terminals.

Ex : Electronic systems, Communication systems Transmission & Distribution systems etc.



* Fig consists of two pair of terminals 1-1' and 2-2' are called ports.

* Two voltages and currents. (V_1, V_2, I_1, I_2)

* A driving source is connected across one port and the load is connected across another port.

II NETWORK FUNCTIONS OF ONE PORT AND TWO PORT NETWORKS

The network functions are formed by finding the system equation, using Mesh, Nodal analysis or Network reduction techniques.

i) Network Functions

1. Driving point Impedance (Z)
2. Driving point Admittance (Y)

ii) Transfer functions

1. Voltage transfer ratio
2. Current transfer ratio
3. Transfer impedance
4. Transfer admittance

NETWORK FUNCTIONS OF ONE PORT NETWORK

1. Driving point Impedance (Z)

Driving point Impedance of any port Network is given by the ratio of voltage and current in S-domain.

$$Z(s) = \frac{V(s)}{I(s)}$$

For one port Network, $Z_{11}(s) = \dfrac{V_1(s)}{I_1(s)}$

2. Driving point Admittance

Driving point admittance of any port network is the reciprocal of driving point impedance and its unit is mho ($\mho$)

$$\therefore Y(s) = \frac{1}{Z(s)} \mho$$

It is defined as ratio of current and voltage in the s-domain.

$$Y(s) = \frac{I(s)}{V(s)} \mho$$

For one port Network, $Y_{11}(s) = \dfrac{I_1(s)}{V_1(s)}$

NETWORK FUNCTIONS OF TWO PORT NETWORKS

1. Driving point Impedance

For a two port network, it is defined as the ratio of transform voltage at port 1,2 to the transform current at port 1,2

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \text{ and } Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

2. Driving point Admittance

It is defined as the ratio of transform current at any one port to the transform voltage at that respective port.

$$Y_{11}(s) = \frac{1}{Z_{11}(s)} = \frac{I_1(s)}{V_1(s)} \text{ and}$$

$$Y_{22}(s) = \frac{1}{Z_{22}(s)} = \frac{I_2(s)}{V_2(s)}$$

TRANSFER FUNCTION OF NETWORK

1. Voltage transfer Ratio

It is the ratio of transform voltage at one port to the transform voltage at another port.

It is denoted by $G(s)$ and is a dimensionless quantity.

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)} \text{ and } G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

2. Current transfer Ratio

It is the ratio of transform current at one port to the transform current at another port. It is denoted by $\alpha(s)$.

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)} \text{ and } \alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

3. Transfer Impedance

It is the ratio of voltage transform at one port to the current transform at other port & denoted by $Z(s)$.

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)} \text{ and } Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

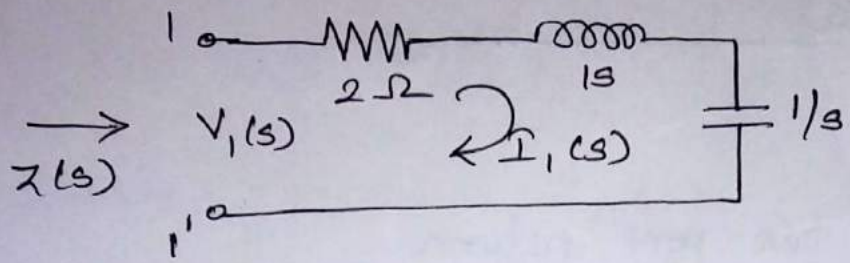
4. Transfer Admittance

It is the ratio of current transform at one port to the voltage transform at other port & is denoted by $Y(s)$

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)} \text{ and } Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

PROBLEMS

1. For the network shown in figure, obtain the driving point impedance.



Solution:

The given network is a one port network

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

$$V_1(s) = 2I_1(s) + 1s\,I_1(s) + \frac{1}{s}I_1(s)$$

$$= I_1(s)\left[2 + s + \frac{1}{s}\right]$$

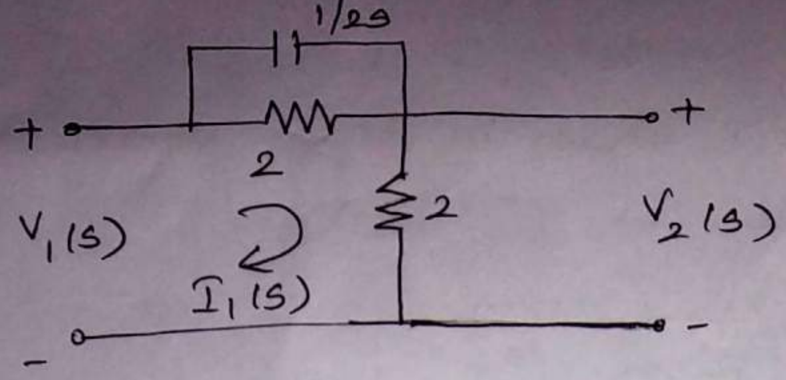
$$= I_1(s)\left[\frac{2s + s^2 + 1}{s}\right]$$

$$Z_{11}(s) = \frac{I_1(s)\left[\frac{2s+s^2+1}{s}\right]}{I_1(s)}$$

$$= \frac{2s + s^2 + 1}{s}$$

Driving point impedance, $Z_{11}(s) = Z(s) = \dfrac{2s+s^2+1}{s}$

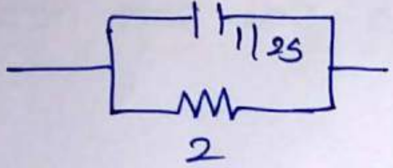
2. For the network shown in the figure, obtain the transfer functions $G_{21}(s)$, $Z_{21}(s)$ and driving point impedance $Z_{11}(s)$



Solution:

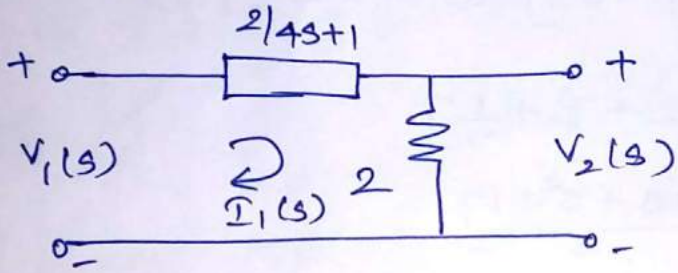
It is a two port network

Consider the parallel combination of R & C



$$\Rightarrow Z_{eq} = \frac{1}{2s + 1/2} = \frac{2}{4s+1}$$

The given network can be modified as



Apply KVL at input side,

$$V_1(s) = I_1(s)\frac{2}{4s+1} + 2I_1(s)$$

$$= I_1(s)\left[\frac{2}{4s+1} + 2\right]$$

$$= I_1(s)\left[\frac{8s+4}{4s+1}\right]$$

Apply KVL at output side,

$$V_2(s) = 2I_1(s)$$

i) Voltage transfer Ratio, $G_{21}(s)$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2 I_1(s)}{\left(\frac{8s+4}{4s+1}\right) I_1(s)}$$

$$G_{21}(s) = 2\left(\frac{4s+1}{8s+4}\right)$$

ii) Transfer Impedance, $Z_{21}(s)$

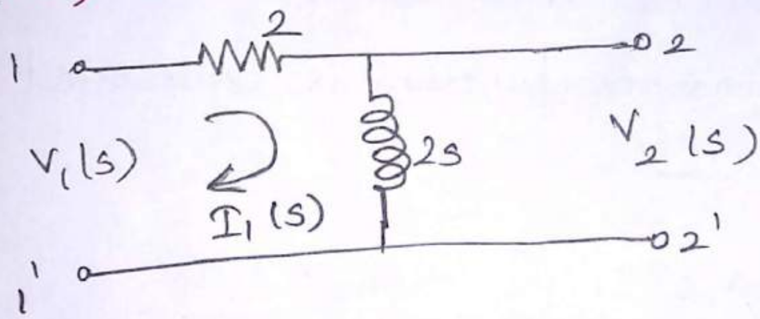
$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{2 I_1(s)}{I_1(s)}$$

$$Z_{21}(s) = 2$$

iii) Driving point Impedance, $Z_{11}(s)$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = \frac{8s+4}{4s+1}$$

3. For the circuit shown in the figure obtain $G_{21}(s)$, $Z_{21}(s)$ and $Z_{11}(s)$.



Solution:

Apply KVL at input side,

$$V_1(s) = 2 I_1(s) + 2s\, I_1(s)$$
$$= I_1(s)\,(2+2s)$$

Apply KVL at output side,

$$V_2(s) = 2s\, I_1(s)$$

1. Voltage transfer Ratio $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$

$$G_{21}(s) = \frac{2s\,I_1(s)}{(2+2s)\,I_1(s)}$$

$$= \frac{2s}{2+2s} = \frac{2s}{2(s+1)} = \frac{s}{s+1}$$

2. Transfer Impedance $Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$

$$Z_{21}(s) = \frac{2s\,I_1(s)}{I_1(s)}$$

$$Z_{21}(s) = 2s$$

3. Driving point Impedance $Z_{11}(s)$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = \frac{I_1(s)\,(2+2s)}{I_1(s)}$$

$$Z_{11}(s) = 2+2s = 2(s+1)$$

III POLES AND ZEROS OF A NETWORK FUNCTION

Generally a network function is represented as $N(s)$ (ie) $N(s) = \frac{P(s)}{Q(s)}$

$$\therefore N(s) = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}$$

where,

$a_0, a_1, \ldots, a_n \Rightarrow$ Coefficients of $P(s)$

$b_0, b_1, \ldots, b_m \Rightarrow$ coefficients of $Q(s)$

$P(s)$ & $Q(s)$ are the polynomial functions.

* these coefficients are real and positive for a passive network.

* When the numerator and denominator polynomial are factorised, the network function can be written as,

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 (s-z_1)(s-z_2) \cdots (s-z_n)}{b_0 (s-p_1)(s-p_2) \cdots (s-p_m)}$$

where $z_1, z_2, \ldots z_n \Rightarrow$ n roots for $P(s)=0$

$p_1, p_2, \ldots p_m \Rightarrow$ m roots for $Q(s)=0$

Scaling factor, $H = \frac{a_0}{b_0}$ which is constant

* $z_1, z_2, \ldots, z_n$ are called as zeros of the transfer function and denoted as "o"
* $p_1, p_2, \ldots, p_m$ are the poles of transfer function and denoted as "x".
* A Network function is completely defined by its poles and zeros.
* $N(s) \to 0$, when anyone of s becomes zero

$N(s) \to \infty$, when anyone of s is equal to any one pole.

TYPES OF POLES AND ZEROS :

There are two types,

* simple poles (or) zeros and
* Multiple poles (or) zeros

* when the poles (or) zeros are not repeated then the function is said to be having simple poles (or) simple zeros.
* when the poles (or) zeros are repeated then the function is said to be having multiple poles (or) multiple zeros.

* If $n > m$, then $(n-m)$ number of zeros are located at $s=\infty$

If $n < m$, then $(m-n)$ number of poles are located at $s=\infty$

Stable $\Rightarrow$ A network is said to be stable, when the real part of poles and zeros are negative and hence all the poles and zeros must lie in the left half of s-plane.

Example:

Find the poles and zeros ~~of~~ for the network function given below, $N(s) = \frac{(s+1)^2 (s+5)}{(s+2)(s+3+j2)(s+3-j2)(s+4)^2}$

Solution:

If numerator polynomial $= 0$, we get zeros

If denominator polynomial $= 0$, we get poles

$\therefore$ from $(s+1)^2 \rightarrow$ Double zeros at $s=-1$

$(s+5) \rightarrow$ Simple zero at $s=-5$

$(s+2) \rightarrow$ Simple pole at $s=-2$

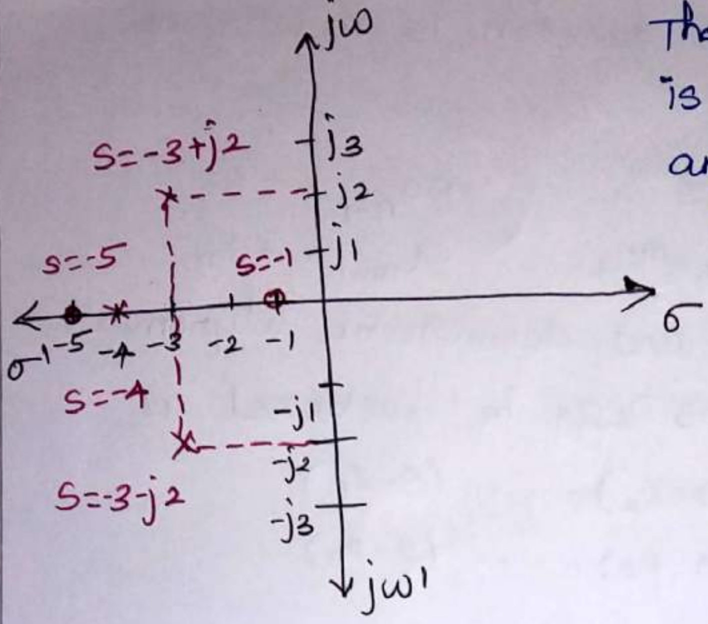
$(s+3+j2) \rightarrow$ Simple pole at $s = -(3+j2)$

$(s+3-j2) \rightarrow$ Simple pole at $s = -(3-j2)$

$(s+4)^2 \rightarrow$ Double poles at $s=-4$

$\therefore$ 2 zeros and 4 poles are there for the given network function.

Pole-zero plot:



The given network function is stable, since the poles and zeros lie in the left half of s-plane.

Significance of poles and zeros:

At poles ⇒ Network functions become infinite

At zeros ⇒ Network functions become zero

At other complex frequency ⇒ Network function has a finite non-zero value

a) Driving point Impedance

$$Z(s) = \frac{V(s)}{I(s)}$$

pole of $Z(s)$ ⇒ $I(s) = 0$ for a finite voltage which means an open circuit

zero of $Z(s)$ ⇒ $V(s) = 0$ for a finite current which means an short circuit

b) Driving point Admittance

$$Y(s) = \frac{I(s)}{V(s)}$$

pole of $Y(s)$ ⇒ $V(s) = 0$ for a finite current which means an short circuit

zero of $Y(s)$ ⇒ $I(s) = 0$ for a finite voltage which means an open circuit.

PROPERTIES OF DRIVING POINT FUNCTIONS

a) the deiving point function is a ratio of polynomials in s.

Let $P(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n$

$Q(s) = b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m$

be the numerator and denominator polynomials.

The above functions can be factorized as,

$P(s) = (s - z_1)(s - z_2) \dots (s - z_n)$

$Q(s) = (s - p_1)(s - p_2) \dots (s - p_n)$

$z_1, z_2 \dots, z_n$ = zeros of $N(s) = 0$

$p_1, p_2, \dots, p_m$ = poles of $N(s) = \infty$

A function $N(s)$ have a pole at ∞, $N\left(\frac{1}{s}\right)$ has a pole at $s=0$

" " " zero at 0, $N(1/s)$ has a zero at $s=\infty$

b) i) $N(s)$ be a deiving point impedance (ie) $Z(s)$

$$N(s) = Z(s) = \frac{V(s)}{I(s)}$$

zero of $N(s) \Rightarrow V(s) = 0 \Rightarrow$ short circuit

pole of $N(s) \Rightarrow I(s) = 0 \Rightarrow$ open circuit

ii) consider deiving point impedance

$$\cancel{N(s)} = Y(s) = \frac{I(s)}{V(s)}$$

zero of $Y(s) \Rightarrow I(s) = 0 \Rightarrow$ open circuit

pole of $Y(s) \Rightarrow V(s) = 0 \Rightarrow$ short circuit

c) since all the elements are real positive quantities the coefficients $a_0, a_1, \dots, a_n$ & $b_0, b_1, \dots, b_m$ are real and positive.

Any zeros or poles, if complex must occur in conjugate pairs.

d) Real parts of all zeros & poles must be negative or zero.

A network function whose response is finite for all t, for a given finite input is said to be stable.

Driving point impedance z(s) is stable if all the poles lie in the negative half of s-plane

e) poles or zeros lying on the jw axis must be simple.

If it is not simple then the time function which is a pole contains the term $t^k e^{j\omega t}$ which tends to infinity at t tends to infinite. the function becomes unstable.

f) The degree of p(s) and Q(s) may differ by zero or one only.

g) The lowest degree terms in p(s) and Q(s) may differ in degree by zero or one only.

h) p(s) & Q(s) cannot have missing terms unless all even or all odd degree terms are absent.

Necessary conditions for a driving point function:

1. The coefficients of P(s) & N(s) must be real and positive
2. Complex poles and zeros must occur in conjugate pairs
3. a. The real part of all zeros & poles must be zero or negative
   b. If the real part is zero, then the pole and zero must be simple

4. The polynomials p(s) & Q(s) May not have any missing terms between the highest and the lowest degrees, unless all even or all odd terms are missing

5. The degree of p(s) & Q(s) May differ by zero or one only.

6. The lowest degree in p(s) & Q(s) May differ in degree by at the most one.

PROPERTIES OF TRANSFER FUNCTIONS

a) The transfer function is a ratio of polynomials in s

b) the coefficients of p(s) & Q(s) Must be real. Therefore all poles and zeros if complex must occur in conjugate pairs.

c) The real parts of all poles Must be negative and any pole on the jw axis Must be simple

d) since poles of the transfer function are zeros of Q(s), it follows that the zeros of Q(s) must lie in the negative half plane.

e) for G(s) & α(s), degree of p(s) is less than or equal to degree of Q(s).

f) The degree of p(s) of $Z_{21}(s)$ or $Y_{21}(s)$ is less than or equal to the degree of the denominal polynomial plus one.

Necessary conditions for Transfer functions:

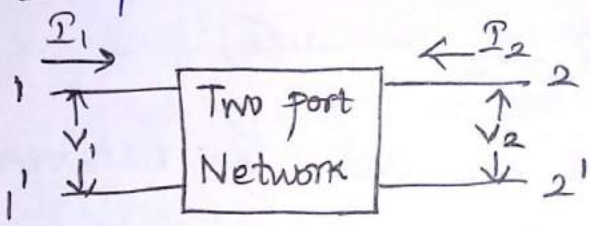
1. a) the coefficients of $p(s)$ & $N(s)$ Must be real.
   b) the coefficients of $Q(s)$ Must be positive, but some of the coefficients on $p(s)$ May be negative
2. Complex or Imaginary poles and zeros must occur in conjugate pairs
3. The real part of poles Must be negative or zero. If real part is zero, then the pole must be simple.
4. $Q(s)$ May not have any missing terms between the highest and the lowest degree, unless all even or all odd terms are Missing
5. $p(s)$ May have Missing terms between the lowest and the highest degree.
6. The degree of $p(s)$ May be as small as zero, Independent of the degree of $Q(s)$
7. a) for $G(s)$ & $\alpha(s)$, the Maximum degree of $p(s)$ Must equal the degree of $Q(s)$
   b) for $Z_{21}(s)$ & $Y_{21}(s)$, the Maximum degree of $p(s)$ Must equal the degree of $Q(s)$ plus one

TWO PORT PARAMETERS

* In two port network, if the two ports containing no sources in their branch, then it is called as passive ports.

Eg: power transmission lines, transformers etc.

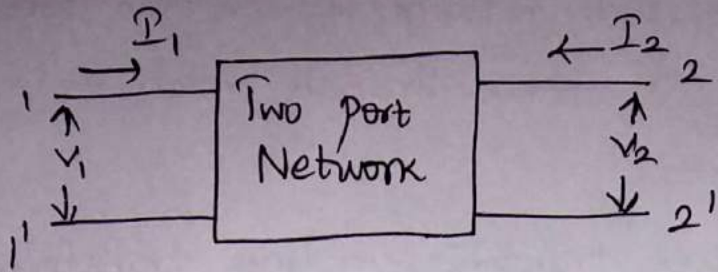
* If the two ports containing sources in their branch, then it is called as active ports.
* In two port network, current and voltages are defined at both the ports.
* I_1, I_2 and V_1, V_2 are the currents and voltages in the network. Among these any of the two variables will be dependent variables and the other two variables will be independent variables.



* Depending upon the dependent and independent variables, the two port parameters are classified as follows:

i) Z parameters (or) open circuit impedance parameters
ii) Y parameters (or) short circuit admittance parameters
iii) Hybrid (h) parameters
iv) Inverse hybrid parameters (g)
v) Transmission (ABCD) parameters
vi) Inverse Transmission (A'B'C'D') parameters

i) Z parameters (or) open Circuit Impedance parameters



* The above figure shows a general linear two-port network which does not contain any independent source.

* The Z parameters of a 2 port network will be defined for positive direction of voltage and current.

* Here V_1 and V_2 => dependent Variables

I_1 and I_2 => Independent Variables

$$\therefore V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (2)}$$

Z_{11}, Z_{12}, Z_{21} and Z_{22} are the network functions called as impedance (Z) parameters.

These parameters May be represented by Matrices.

$$[V] = [Z]\,[I] \qquad \left(\text{from } Z = \frac{V}{I}\right)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

V & I => column Matrix

Z => square Matrix

* Individual Z parameters can be obtained by setting each port current to zero.

∴ For finding, Z_{11} and Z_{21}, the port 2-2' should be open-circuited.

$$\therefore Z_{11} = \frac{V_1}{I_1} \text{ at } I_2 = 0$$

where Z_{11} - driving point impedance at 1-1' port with 2-2' port open-circuited. Therefore it is called as open circuit input impedance

Similarly,

$$Z_{21} = \frac{V_2}{I_1} \text{ at } I_2 = 0$$

where, Z_{21} - transfer impedance at 1-1' port with 2-2' port open circuited. It is called as Open circuit forward transfer impedance.

∴ For finding, Z_{12} and Z_{22}, the port 1-1' should be open-circuited.

$$\therefore Z_{12} = \frac{V_1}{I_2} \text{ at } I_1 = 0$$

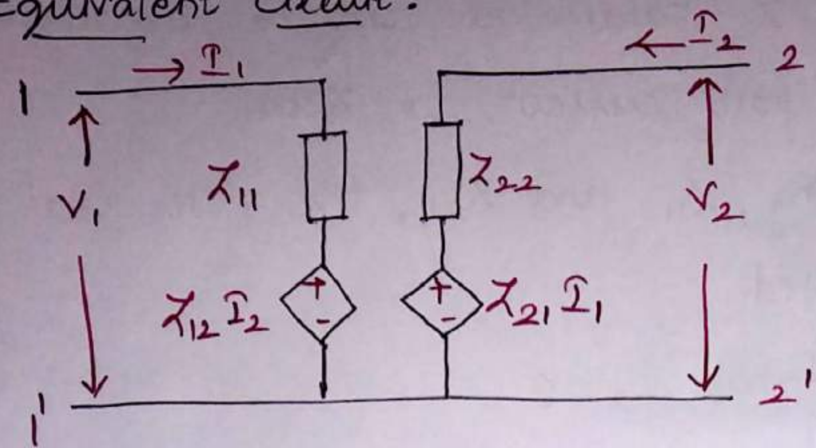
where, Z_{12} = transfer impedance at port 2-2' with port 1-1' open circuited. It is called as open circuit reverse transfer impedance

Similarly,

$$Z_{22} = \frac{V_2}{I_2} \text{ at } I_1 = 0$$

where, Z_{22} = driving point impedance at port 2-2' with port 1-1' open circuited. It is called as open circuit output impedance

Equivalent circuit:



If the network is reciprocal or bilateral, then in accordance with the reciprocity principle

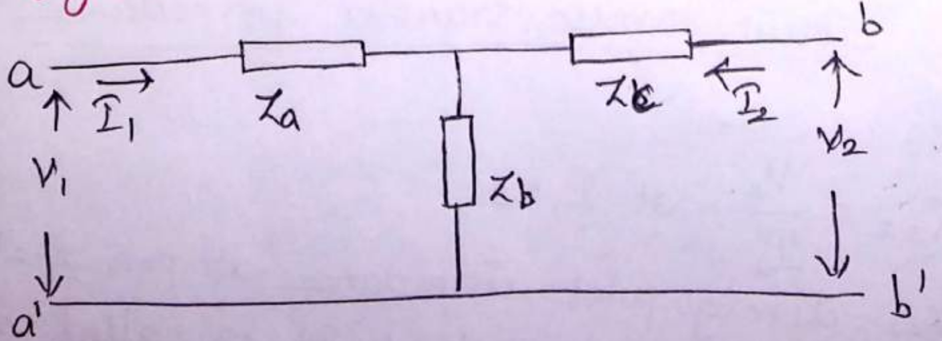
$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

(or) $Z_{21} = Z_{12}$

* All the parameters have the dimensions of impedance.
* Individual parameters are specified only when the current in one of the port is zero. This corresponds to one of the ports being open circuited from which the Z parameters also derive the name open circuit impedance parameters.

Problems:

1. Find the Z-parameters for the circuit shown in the figure below.



Solution:

Voltage equations for Z parameters are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

i) when the port (b-b') is open circuited,

$$Z_{11} = \frac{V_1}{I_1} \text{ at } I_2 = 0$$

Apply KVL at input side,

$$V_1 = Z_a I_1 + Z_b I_1 = I_1 (Z_a + Z_b)$$

$$\therefore Z_{11} = \frac{(Z_a + Z_b) I_1}{I_1}$$

$$Z_{11} = Z_a + Z_b$$

$$Z_{21} = \frac{V_2}{I_1} \text{ at } I_2 = 0$$

Apply KVL at output side

$$V_2 = I_1 Z_b$$

$$\therefore Z_{21} = \frac{Z_b I_1}{I_1}$$

$$Z_{21} = Z_b$$

ii) when the port (a-a') is open circuited

$$Z_{22} = \frac{V_2}{I_2} \text{ at } I_1 = 0$$

Apply KVL at ~~output~~ input side,

$$V_2 = Z_c I_2 + Z_b I_2 = I_2 (Z_c + Z_b)$$

$$\therefore Z_{22} = \frac{I_2 (Z_c + Z_b)}{I_2} = (Z_c + Z_b)$$

$Z_{12} = \frac{V_1}{I_2}$ at $I_1 = 0$

Apply KVL at output side

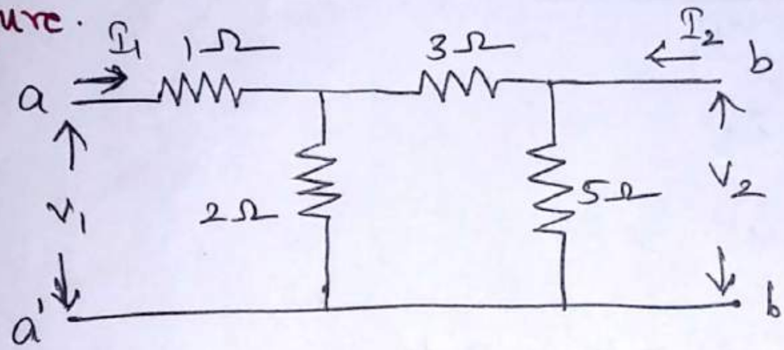
$V_1 = I_2 Z_b$

$Z_{12} = \frac{I_2 Z_b}{I_2} = Z_b$

$Z_{12} = Z_b$

$\therefore Z_{12} = Z_{21}$ according to reciprocity principle, the network is bilateral.

2. find the Z-parameters for the circuit shown in figure.



Solution:

i) when the port (b-b') is open circuited (we can find Z_{11} & Z_{21})

$Z_{11} = \frac{V_1}{I_1}$ at $I_2 = 0$

$V_1 = I_1 R_{eq_1}$

$R_{eq_1} = [(3+5) \| 2] + 1 = \frac{16}{10} + 1$

$R_{eq_1} = 2.6\ \Omega$

$V_1 = I_1 (2.6)$

$\therefore Z_{11} = \frac{2.6 I_1}{I_1}$

$\boxed{Z_{11} = 2.6\Omega}$

$$Z_{21} = \frac{V_2}{I_1} \text{ at } I_2 = 0$$

$V_2 = I R$

Let us assume current through 5Ω be I_x.

$$I_x = I_1 \times \frac{2}{2+3+5} = \frac{I_1}{5}$$

$$V_2 = \frac{I_1}{5} \times 5$$

$V_2 = I_1$

$$\therefore Z_{21} = \frac{I_1}{I_1}$$

$$\boxed{Z_{21} = 1\Omega}$$

ii) when the port (a-a') is open circuited
(we can find Z_{12} & Z_{22})

$$Z_{22} = \frac{V_2}{I_2} \text{ at } I_1 = 0$$

$V_2 = I_2 \, Req_2$

$Req_2 = (3+2) \| 5 = 2.5\Omega$

$V_2 = 2.5 \, I_2$

$$\therefore Z_{22} = \frac{2.5 I_2}{I_2}$$

$$\boxed{Z_{22} = 2.5\Omega}$$

$$Z_{21} = \frac{V_1}{I_2} \text{ at } I_1 = 0$$

$V_1 = I R$

Let us assume current through 2Ω be I_y.

$$I_y = I_2 \times \frac{5}{5+3+2} = \frac{I_2}{2}$$

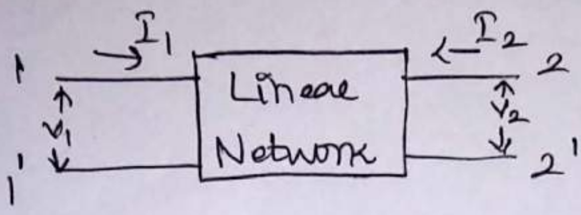
$$V_1 = I_y R = \frac{I_2}{2} \times 2 = I_2$$

$$Z_{21} = \frac{I_2}{I_2}$$

$$\boxed{Z_{21} = 1\,\Omega}$$

$Z_{12} = Z_{21} = 1\,\Omega \Rightarrow$ Bilateral network

ii) Y- Parameters (or) Short circuit Admittance Parameters



* Y parameters for the positive directions of voltages and currents may be defined by expressing the port currents I_1 and I_2 in terms of the voltages V_1 and V_2.

* $I_1, I_2 \Rightarrow$ Dependent Variables

  $V_1, V_2 \Rightarrow$ Independent Variables

$$\therefore I_1 = Y_{11}V_1 + Y_{12}V_2 \quad — (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad — (2)$$

where Y_{11}, Y_{12}, Y_{21} & Y_{22} are called as network functions (or) admittance parameters.

* In Matrix Representation (from $Y = \frac{I}{V}$)

$$[I] = [Y]\,[V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

* Individual Y parameters for a given network can be obtained by setting each port voltage to zero.

* By short circuiting the port (2-2') then $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$

Y_{11} ⇒ Driving point admittance at (1-1') port with (2-2') port short circuited. It is called as Short circuit input admittance

$$Y_{21} = \frac{I_2}{V_1} \text{ at } V_2 = 0$$

Y_{21} ⇒ Transfer admittance at (1-1') port with (2-2') port short circuited. It is called as short circuit forward transfer admittance

* By Short circuiting the port (1-1') then $V_1 = 0$

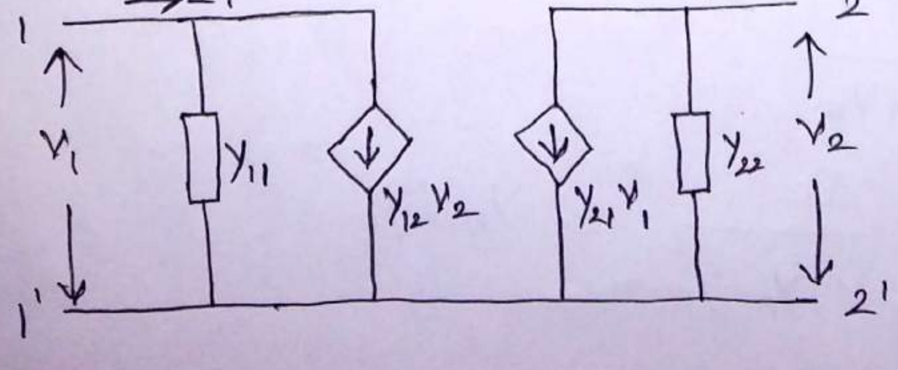
$$Y_{12} = \frac{I_1}{V_2} \text{ at } V_1 = 0$$

Y_{12} ⇒ Transfer admittance at port (2-2') with port (1-1') short circuited. It is called as short circuit Reverse transfer admittance

$$Y_{22} = \frac{I_2}{V_2} \text{ at } V_1 = 0$$

Y_{22} ⇒ Driving point admittance at port (2-2') with port (1-1') short circuited. It is called as short circuit output admittance

Equivalent circuit:



If the network is reciprocal or bilateral then

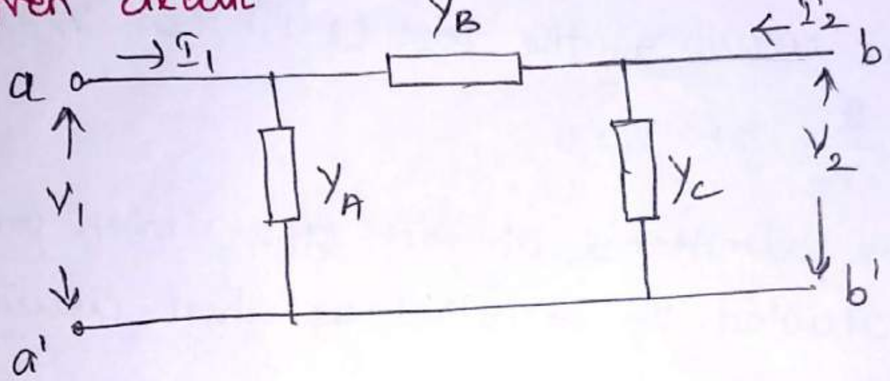
$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

(or) $Y_{12} = Y_{21}$

All parameters are obtained by short circuiting the ports so it is called as short circuit admittance parameters.

Problems:

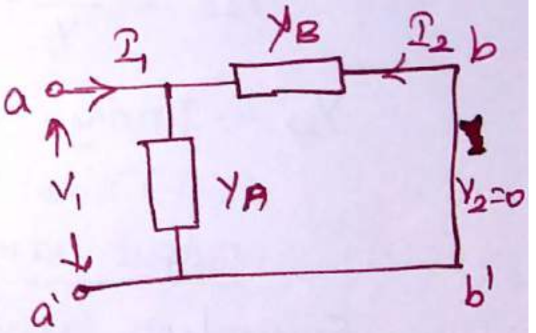
1. find the short circuit admittance parameters for the given circuit.



Solutions:

i) The port (b-b') is short circuited (we can find Y_{11} & Y_{21})

$$Y_{11} = \frac{I_1}{V_1} \text{ at } V_2 = 0$$



Apply ohm's law,

$$V_1 = \frac{I_1}{Y_{eq}} \quad \text{where } Y_{eq} = Y_A + Y_B$$

$$V_1 = \frac{I_1}{Y_A + Y_B}$$

$$\therefore Y_{11} = \frac{I_1}{I_1 / Y_A + Y_B} = Y_A + Y_B$$

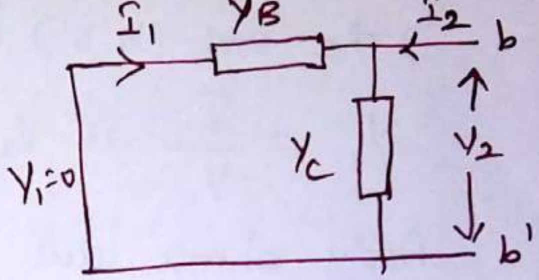
$Y_{21} = \frac{I_2}{V_1}$ at $V_2 = 0$

Apply ohm's law, $-I_2 = V_1 Y_B$

$Y_{21} = \frac{-V_1 Y_B}{V_1}$

$Y_{21} = -Y_B$

ii) The port (a-a') is short circuited (find Y_{12} & Y_{22})



$Y_{22} = \frac{I_2}{V_2}$ at $V_1 = 0$

Apply ohm's law, $I_2 = V_2 Y_{eq}$

$Y_{eq} = Y_B + Y_C$

$I_2 = V_2 (Y_B + Y_C)$

$\therefore Y_{22} = \frac{V_2 (Y_B + Y_C)}{V_2}$

$Y_{22} = (Y_B + Y_C)$

$Y_{12} = \frac{I_1}{V_2}$ at $V_1 = 0$

Apply ohm's law, $-I_1 = V_2 Y_B$

$Y_{12} = \frac{-V_2 Y_B}{V_2}$

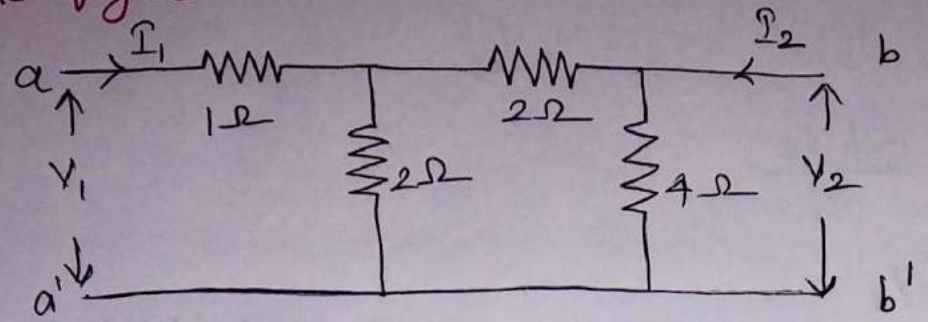
$Y_{12} = -Y_B$

$Y_{12} = Y_{21} \Rightarrow$ Bilateral network

$I_1 = (Y_A + Y_B) V_1 - Y_B V_2$

$I_2 = (-Y_B) V_1 + (Y_B + Y_C) V_2$

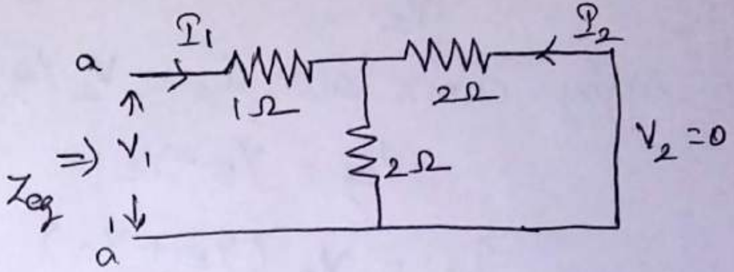
2. find the y parameters for the network shown in the figure.



Solution:

i) The port (b-b') is short circuited for finding Y_{11} & Y_{21}

$$Y_{11} = \frac{I_1}{V_1} \text{ at } V_2 = 0$$



Apply ohm's law

$$V_1 = I_1 Z_{eq}$$

$$Z_{eq} = (2 \| 2) + 1 = 2\Omega$$

$$V_1 = 2I_1$$

$$Y_{11} = \frac{I_1}{2I_1} = \frac{1}{2} \mho$$

$$Y_{21} = \frac{I_2}{V_1} \Big| V_2 = 0$$

Apply Current Division Rule } $-I_2 = I_1 \times \frac{2}{4}$

$$-I_2 = \frac{I_1}{2} \quad \left(\text{sub } I_1 = \frac{V_1}{2}\right)$$

$$-I_2 = \frac{V_1}{4}$$

$$Y_{21} = \frac{-V_1}{4V_1} = -\frac{1}{4}$$

$$Y_{21} = \frac{-1}{4} \mho$$

ii) The port (a-a') is short circuited for finding Y_{12} & Y_{22}

$$Y_{22} = \frac{I_2}{V_2} \text{ at } V_1 = 0$$

Apply ohm's law,

$$V_2 = I_2 Z_{eq}$$

$$Z_{eq} = [(1\|2)+2]\|4$$

$$Z_{eq} = \frac{8}{5}\,\Omega$$

$$V_2 = \frac{8}{5} I_2$$

$$\therefore Y_{22} = \frac{I_2}{(8/5) I_2}$$

$$Y_{22} = \frac{5}{8}\,\mho \qquad Y_{12} = \frac{I_1}{V_2} \text{ at } V_1 = 0$$

Apply current division Rule, $-I_1 = I_2 \times \frac{2}{(1+2+2)}$

$$-I_1 = I_2 \times \frac{2}{5} \quad \left(\text{sub } I_2 = \frac{5V_2}{8}\right)$$

$$-I_1 = \frac{5V_2 \times 2}{5 \times 8} = \frac{1}{4} V_2$$

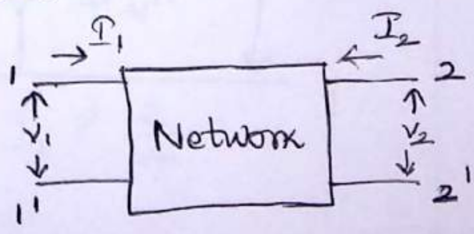
$$\therefore Y_{12} = \frac{-1 \times V_2}{4 \times V_2} = \frac{-1}{4}\,\mho$$

$$\therefore I_1 = 0.5 V_1 - 0.25 V_2$$

$$I_2 = -0.25 V_1 + 0.625 V_2$$

iii) Hybrid (H) parameters:

* Hybrid parameters are used in transistor circuits
* Here Voltage of one port and current of other port are taken as independent Variables.
* If the Voltage at port 1-1' and current at port 2-2' are taken as dependent Variables (V_1 & I_2)



$$\therefore V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

where h_{11}, h_{12}, h_{21} & h_{22} are hybrid parameters.

* In Matrix form, $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

* when $V_2 = 0$, the port (2-2') is short circuited

$h_{11} = \frac{V_1}{I_1}$ at $V_2 = 0$ ⇒ short circuit input impedance $\left(\frac{1}{Y_{11}}\right)$

$h_{21} = \frac{I_2}{I_1}$ at $V_2 = 0$ ⇒ short circuit forward current gain $\left(\frac{Y_{21}}{Y_{11}}\right)$

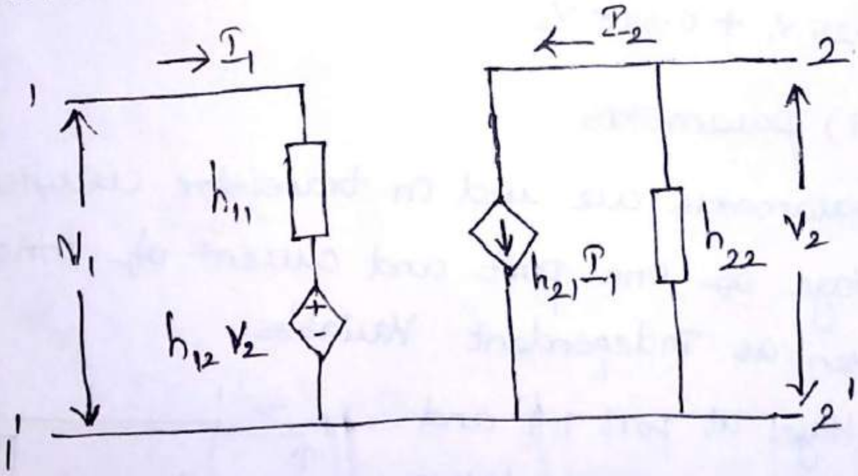
* when (1-1') port is open circuit, $I_1 = 0$

$h_{12} = \frac{V_1}{V_2}$ at $I_1 = 0$ ⇒ Open circuit reverse voltage gain $\left(\frac{Z_{12}}{Z_{22}}\right)$

$h_{22} = \frac{I_2}{V_2}$ at $I_1 = 0$ ⇒ open circuit output admittance $\left(\frac{1}{Z_{22}}\right)$

Since h parameters represent dimensionally an impedance, an admittance, a voltage gain and a current gain, these are called hybrid parameters.

Equivalent Circuit:



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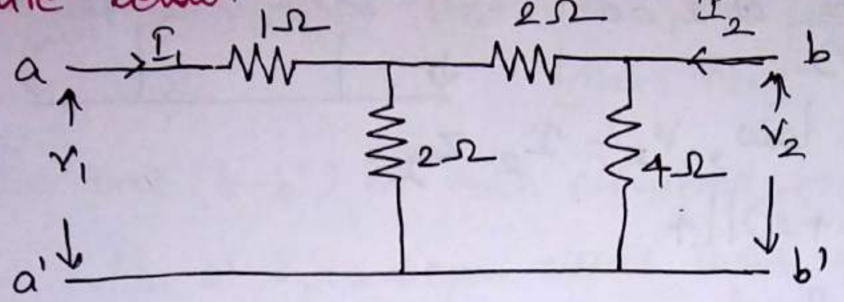
Problems:

1. find the h parameters of the network shown in figure below:



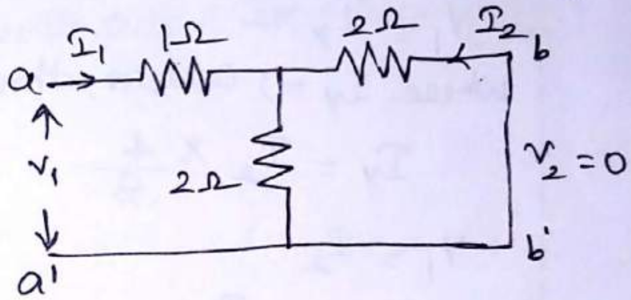
Solution:

The general equation of h parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

i) when the port (b-b') is short circuited for finding h_{11} & h_{21}



$$h_{11} = \frac{V_1}{I_1} \text{ at } V_2 = 0$$

Apply ohm's law, $V_1 = I_1 Z_{eq}$

$$Z_{eq} = (2\|2)+1$$

$$= 2\Omega$$

$$V_1 = 2 I_1$$

$$\therefore h_{11} = \frac{2 I_1}{I_1}$$

$$h_{11} = 2\Omega$$

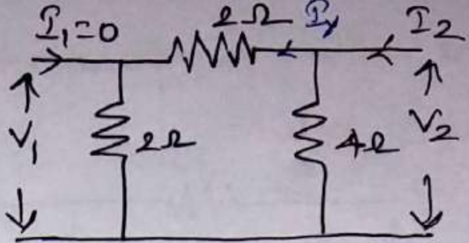
$$h_{21} = \frac{I_2}{I_1} \text{ at } V_2 = 0$$

Apply current division Rule, $-I_2 = I_1 \times \frac{2}{4}$

$$h_{21} = \frac{-2 I_1}{4 I_1}$$

$$h_{21} = \frac{-1}{2}$$

ii) The port (a-a') is open circuited, for finding h_{12} & h_{22}



$$h_{22} = \frac{I_2}{V_2} \text{ at } I_1 = 0$$

Apply ohm's law, $V_2 = I_2 Z_{eq}$

$$Z_{eq} = (2+2)||4$$
$$= 2\Omega$$

$$V_2 = 2I_2$$

$$h_{22} = \frac{I_2}{2I_2} = \frac{1}{2} \ \mho$$

$$\therefore h_{12} = \frac{V_1}{V_2} \text{ at } I_1 = 0$$

$$V_1 = I_y \times 2$$

where I_y => current through 2Ω Resistor

$$I_y = I_2 \times \frac{4}{8}$$

$$V_1 = I_2$$

$$\therefore h_{12} = \frac{I_2}{2I_2} = \frac{1}{2}$$

The equations,

$$V_1 = 2I_1 + 0.5V_2$$
$$I_2 = -0.5I_1 + 0.5V_2$$

iv) Inverse hybrid (g) parameters:

The current at input port I_1 and the Voltage at output port V_2 can be expressed interms of V_1 & I_2.

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

where g_{11}, g_{12}, g_{21} & g_{22} are the inverse hybrid parameters.

* In Matrix form, $\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

It can be verified that $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$

* The port (b-b') is open circuited, for finding g_{11} & g_{21}

$g_{11} = \frac{I_1}{V_1}$ at $I_2 = 0$ ⇒ open circuit input admittance ($1/z_{11}$)

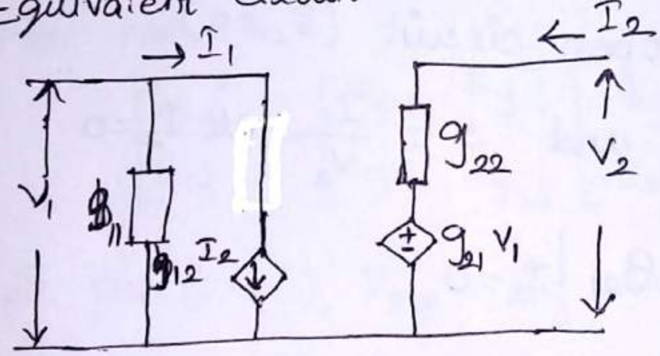
$g_{21} = \frac{V_2}{V_1}$ at $I_2 = 0$ ⇒ open circuit Voltage gain

* The port (a-a') is short circuited

$g_{12} = \frac{I_1}{I_2}$ at $V_1 = 0$ ⇒ Short circuit Reverse current gain

$g_{22} = \frac{V_2}{I_2}$ at $V_1 = 0$ ⇒ Short circuit output impedance $\left(\frac{1}{Y_{22}}\right)$

Equivalent circuit :



V) Transmission (ABCD) parameters

* Also called as chain parameters or general circuit parameters
* It is used in transmission line theory and Cascade networks.
* The input variables V_1 & I_1 at port (1-1') called the sending end are expressed in terms of the output variables V_2 & I_2 at port (2-2') called the receiving end.

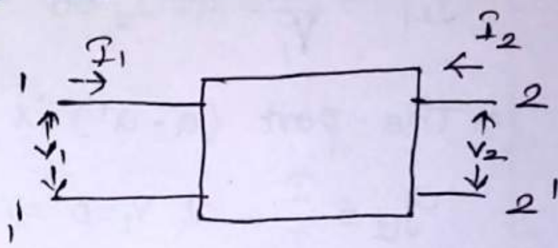
* It provides the direct relationship between input and output, as expressed by

$$V_1 = AV_2 - BI_2 \quad — ①$$

$$I_1 = CV_2 - DI_2 \quad — ②$$

Here port current I_2 is denoted by negative sign and is not for the parameters B & D.

* The parameters A, B, C & D are called as transmission parameters.



* In Matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

⇓

Transmission Matrix

* With port (2-2') open circuit ($I_2 = 0$)

$$A = \frac{V_1}{V_2} \text{ at } I_2 = 0 \quad \text{and} \quad C = \frac{I_1}{V_2} \text{ at } I_2 = 0$$

$$\frac{1}{A} = \frac{V_2}{V_1}\Big|_{I_2=0} = g_{21}\Big|_{I_2=0}$$

where $1/A$ is called open circuit voltage gain

$$\frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_{21}$$

where $1/C$ is called open circuit transfer impedance

* with port (2-2') short circuit ($V_2 = 0$)

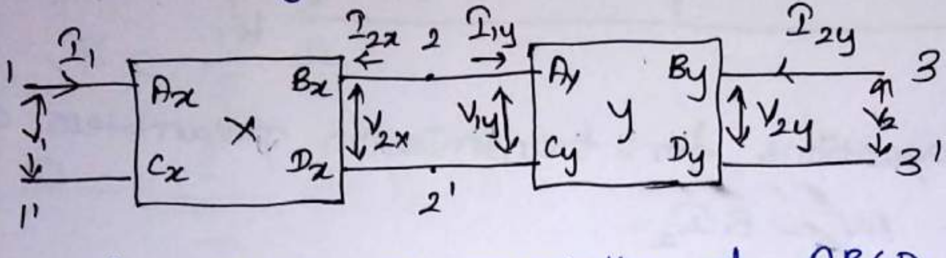
$$-B = \frac{V_1}{I_2} \text{ at } V_2 = 0 \quad \text{and} \quad -D = \frac{I_1}{I_2} \text{ at } V_2 = 0$$

$$\frac{-1}{B} = \frac{I_2}{V_1}\Big|_{V_2=0} = Y_{21} \Rightarrow \text{Short circuit transfer admittance}$$

$$-\frac{1}{D} = \frac{I_2}{I_1}\Big|_{V_2=0} = \alpha_{21} \Rightarrow \text{Short circuit current gain}$$

Cascade Connection :

Consider two two-port Networks N_x and N_y connected in cascade with port voltages and currents as shown in figure.



* The Matrix representation of ABCD parameters for the network x is

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix}$$

for network y,

$$\begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

At port (2-2'), $V_{2x} = V_{1y}$ and $I_{2x} = -I_{1y}$

combining the two matrices,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

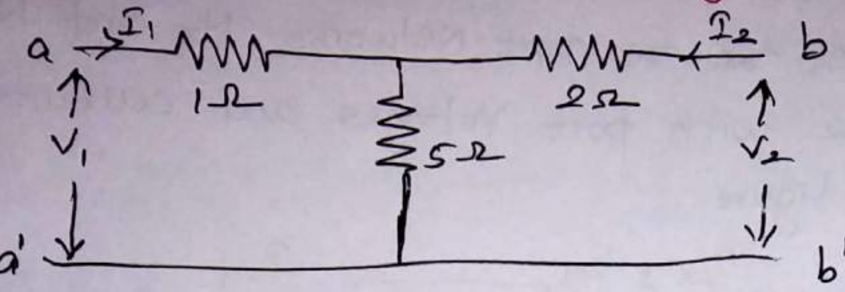
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

* Transmission Matrix = product of transmission matrices of the individual two port networks.

* This property is used in the design of telephone systems, Microwave networks, radar etc.

Problems:

1. find the transmission or general circuit parameters for the circuit shown in the figure.



Solution:

The equations for transmission parameters are

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

i) when (b-b') is open circuit, $I_2 = 0$

$$A = \frac{V_1}{V_2} \text{ at } I_2 = 0$$

Apply KVL at input side, $V_1 = 1I_1 + 5I_1$

$V_1 = 6I_1$

Apply KVL at output side, $V_2 = 5I_1$

$$A = \frac{6I_1}{5I_1} = \frac{6}{5}$$

|||ly,

$$C = \frac{I_1}{V_2} \text{ at } I_2 = 0$$

$$C = \frac{I_1}{5I_1} = \frac{1}{5}\ \mho$$

ii) when (b-b') is short circuit, $V_2 = 0$

$$B = \frac{-V_1}{I_2} \text{ at } V_2 = 0$$

Apply current division Rule, $-I_2 = I_1 \times \frac{5}{7}$

$$I_1 = \frac{V_1}{Z_{eq}} \Rightarrow Z_{eq} = (2||5) + 1 = \frac{17}{7}$$

$$\therefore I_1 = \frac{V_1}{17/7} = \frac{7}{17} V_1$$

Sub in I_2,

$$-I_2 = \frac{7}{17} V_1 \times \frac{5}{7}$$

$$-I_2 = \frac{5}{17} V_1$$

$$B = \frac{-V_1}{5/17\, V_1} = \frac{17}{5}\ \Omega$$

$$B = \frac{17}{5}\ \Omega$$

$$D = \frac{-I_1}{I_2} \text{ at } V_2 = 0$$

$$= \frac{-7/17\, V_1}{-5/17\, V_1} = \frac{7}{5}$$

$$D = \frac{7}{5}$$

The equations are,

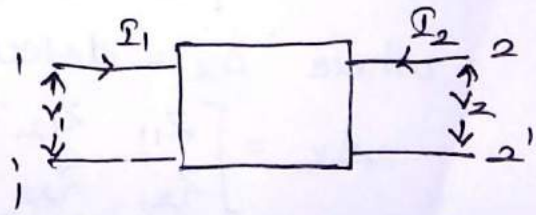
$$V_1 = \frac{6}{5} V_2 - \frac{17}{5} I_2$$

$$I_1 = \frac{1}{5} V_2 - \frac{7}{5} I_2$$

vi) Inverse Transmission ($A'B'C'D'$) parameters

The voltage and current at port (2-2') is expressed interms of voltage and current at port (1-1') is

$$V_2 = A'V_1 - B'I_1$$
$$I_2 = C'V_1 - D'I_1$$



where A', B', C' & D' are called Inverse transmission parameters.

* $A'B'C'D'$ parameters have properties similar to ABCD.

* when port (1-1') is open circuit, $I_1 = 0$

$$A' = \frac{V_2}{V_1} \text{ at } I_1 = 0 \quad , \quad C' = \frac{I_2}{V_1} \text{ at } I_1 = 0$$

* when port (2-1') is short circuit, $V_1 = 0$

$$B' = \frac{-V_2}{I_1} \text{ at } V_2 = 0 \quad , \quad D' = -\frac{I_2}{I_1} \text{ at } V_1 = 0$$

INTER RELATIONSHIPS OF DIFFERENT PARAMETERS

1. Expression of Y parameters in terms of Z parameters

From Z parameters (V_1 & V_2) and Y parameters (I_1 & I_2) it is easy to derive the relation between the open circuit impedance and short circuit admittance parameters by means of two matrix equations of the respective parameters.

Z parameters $\Rightarrow$ $V_1 = Z_{11} I_1 + Z_{12} I_2$ — (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (2)

By solving eq (1) & (2) for I_1 & I_2,

$$I_1 = \begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix} \Big/ \Delta z \quad \text{and} \quad I_2 = \begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix} \Big/ \Delta z$$

where Δz = determinant of Z Matrix

$$\Delta z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$I_1 = \frac{Z_{22}}{\Delta z} V_1 - \frac{Z_{12}}{\Delta z} V_2 \quad — (3)$$

$$I_2 = \frac{Z_{11}}{\Delta z} V_2 - \frac{Z_{21}}{\Delta z} V_1 \quad — (4)$$

Y parameters $\Rightarrow$ $I_1 = Y_{11} V_1 + Y_{12} V_2$ — (5)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ — (6)

Comparing Eq (3) & (4) with (5) & (6)

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

2. Expression of Z parameters in terms of Y parameters

In similar to previous section, Z parameters may be expressed in terms of the admittance parameters by solving eq (5) & (6) for V_1 & V_2

$$V_1 = \begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix} \Big/ \Delta y \quad \text{and} \quad V_2 = \begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix} \Big/ \Delta y$$

where Δy = Determinant of the Y Matrix

$$\Delta y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}$$

$$V_1 = \frac{Y_{22}}{\Delta y} I_1 - \frac{Y_{12}}{\Delta y} I_2 \quad — (7)$$

$$V_2 = \frac{-Y_{21}}{\Delta y} I_1 + \frac{Y_{11}}{\Delta y} I_2 \quad — (8)$$

Comparing Eq (7) & (8) with (1) & (2)

$$Z_{11} = \frac{Y_{22}}{\Delta y}, \quad Z_{12} = \frac{-Y_{12}}{\Delta y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta y}, \quad Z_{22} = \frac{Y_{11}}{\Delta y}$$

Problems:

1. For a given, $z_{11} = 3\,\Omega$, $z_{12} = 1\,\Omega$, $z_{21} = 2\,\Omega$ and $z_{22} = 1\,\Omega$. find the admittance Matrix and the product of Δy and Δz.

Solution:

The admittance Matrix $= \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$

$$= \begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$$

Given, $z = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$$\therefore \Delta z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = 3-2$$

$$\Delta z = 1$$

$$\therefore \Delta y = \begin{bmatrix} \frac{1}{1} & \frac{-1}{1} \\ \frac{-2}{1} & \frac{3}{1} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = 3-2 = 1$$

$$\Delta y = 1$$

$$\therefore (\Delta y)(\Delta z) = 1$$

3. ABCD parameters in terms of Z parameters

The ABCD parameter is

$$V_1 = AV_2 - BI_2 \quad — (1)$$

$$I_1 = CV_2 - DI_2 \quad — (2)$$

The Z parameter is, $V_1 = Z_{11} I_1 + Z_{12} I_2 \quad — (3)$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad — (4)$$

The equation for ABCD parameter is

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad B = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}, \quad D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

Sub $I_2 = 0$ in eq (3) & (4)

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11}}{Z_{21}} = A$$

Sub $I_2 = 0$ in eq (4)

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_{21}} = C$$

Sub $V_2 = 0$ in eq (3) & (4) & solve for I_2

$$I_2 = \frac{-V_1 Z_{21}}{\Delta Z}$$

$$\therefore B = \frac{-V_1}{I_2} = \frac{-V_1 \times \Delta Z}{-V_1 Z_{21}} = \frac{\Delta Z}{Z_{21}} = B$$

Sub $V_2 = 0$ in eq (4),

$$D = \frac{-I_1}{I_2} = \frac{Z_{22}}{Z_{21}} = D$$

The determinant of the transmission Matrix is

$AD - BC$

Sub the impedance parameters in A, B, C, D

$$AD - BC = \frac{Z_{11}}{Z_{21}}\frac{Z_{22}}{Z_{21}} - \frac{\Delta_Z}{Z_{21}}\frac{1}{Z_{21}}$$

$$= \frac{Z_{11}Z_{22}}{(Z_{21})^2} - \frac{\Delta_Z}{(Z_{21})^2}$$

$$AD - BC = \frac{\cancel{Z_{11}Z_{22}} - \cancel{Z_{11}Z_{22}} + Z_{12}Z_{21}}{(Z_{21})^2}$$

$$AD - BC = \frac{+Z_{12}}{Z_{21}}$$

for a bilateral network, $Z_{12} = Z_{21} \Rightarrow \frac{AD-BC}{C} = \frac{1}{C}$

$\therefore AD - BC = +1$

* Z parameters in terms of h-parameters

The Z parameter equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2)$$

h-parameter equations

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -(3)$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -(4)$$

from (4),

$$V_2 = \frac{I_2 - h_{21} I_1}{h_{22}} \quad -(5)$$

Sub (5) in (3)

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{I_2 - h_{21} I_1}{h_{22}}\right]$$

$$= \left[h_{11} - \frac{h_{21}h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2 \quad — (6)$$

Compare (6) with (1)

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} \quad \&$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Let, $\Delta h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = h_{11}h_{22} - h_{12}h_{21}$

then $Z_{11} = \dfrac{\Delta h}{h_{22}}$

from (4), $V_2 = \dfrac{I_2 - h_{21} I_1}{h_{22}}$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad — (7)$$

Compare (7) with (2)

$$Z_{21} = \frac{-h_{21}}{h_{22}} \quad \& \quad Z_{22} = \frac{1}{h_{22}}$$

$$\boxed{Z_{11} = \frac{\Delta h}{h_{22}} \quad Z_{12} = \frac{h_{12}}{h_{22}} \quad Z_{21} = \frac{-h_{21}}{h_{22}} \quad Z_{22} = \frac{1}{h_{22}}}$$

5. h parameters in terms of Z-parameters:

The h parameter equations are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad — (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad — (2)$$

Z - parameter equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad — (3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad — (4)$$

from (4),

$$I_2 = \frac{V_2 - Z_{21} I_1}{Z_{22}} \quad -(5)$$

Sub (5) in (3)

$$V_1 = Z_{11} I_1 + Z_{12} \left[\frac{V_2 - Z_{21} I_1}{Z_{22}} \right]$$

$$V_1 = \left[Z_{11} - \frac{Z_{21} Z_{12}}{Z_{22}} \right] I_1 + \frac{Z_{12} V_2}{Z_{22}}$$

$$V_1 = \left[\frac{Z_{11} Z_{22} - Z_{21} Z_{12}}{Z_{22}} \right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

Let $\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = Z_{11} Z_{22} - Z_{21} Z_{12}$

$$\therefore V_1 = \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$\therefore \boxed{h_{11} = \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}}$$

from (4),

$$I_2 = \frac{-Z_{21} I_1 + V_2}{Z_{22}}$$

$$I_2 = \frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2$$

$$\therefore \boxed{h_{21} = \frac{-Z_{21}}{Z_{22}} \quad \& \quad h_{22} = \frac{1}{Z_{22}}}$$

5. Y- parameters in terms of h- parameters

Y- parameter equations are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad -(1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad -(2)$$

h - parameter equations are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -(3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -(4)$$

from eq (3),

$$I_1 = \frac{V_1 - h_{12} V_2}{h_{11}}$$

$$I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2 \quad -(5)$$

Compare (5) with (1)

$$\boxed{Y_{11} = \frac{1}{h_{11}} \quad \& \quad Y_{12} = \frac{-h_{12}}{h_{11}}}$$

sub (5) in (4)

$$I_2 = h_{21}\left[\frac{V_1 - h_{12} V_2}{h_{11}}\right] + h_{22} V_2$$

$$= \frac{h_{21}}{h_{11}} V_1 + \left[h_{22} - \frac{h_{12} h_{21}}{h_{11}}\right] V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left[\frac{h_{22} h_{11} - h_{12} h_{21}}{h_{11}}\right] V_2$$

Let $\Delta h = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$\therefore I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta Z}{h_{11}} V_2 \quad -(6)$$

Compare (6) with (2)

$$\boxed{Y_{21} = \frac{h_{21}}{h_{11}}, \quad Y_{22} = \frac{\Delta h}{h_{11}}}$$

h parameters interms of Y parameters

h parameter equations are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -(1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -(2)$$

Y parameter equations are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad -(3)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad -(4)$$

from eq (3),

$$V_1 = \frac{I_1 - Y_{12} V_2}{Y_{11}}$$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2 \quad -(5)$$

Compare (5) with (1)

$$\boxed{h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = \frac{-Y_{12}}{Y_{11}}}$$

sub (5) in (4)

$$I_2 = Y_{21}\left[\frac{I_1}{Y_{11}} - \frac{Y_{12} V_2}{Y_{11}}\right] + Y_{22} V_2$$

$$= \frac{Y_{21}}{Y_{11}} I_1 + \left[\frac{Y_{22} Y_{11} - Y_{12} Y_{21}}{Y_{11}}\right] V_2$$

Let $\Delta Y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}$

$$\therefore I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2 \quad -(6)$$

Compare (6) with (2)

$$\boxed{h_{21} = \frac{Y_{21}}{Y_{11}}, \quad h_{22} = \frac{\Delta Y}{Y_{11}}}$$

8. Z parameters in terms of ABCD parameters

Z parameter equations are

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad - (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad - (2)$$

ABCD equations are

$$V_1 = AV_2 - BI_2 \quad - (3)$$

$$I_1 = CV_2 - DI_2 \quad - (4)$$

from (4),

$$V_2 = \frac{I_1 + DI_2}{C} \quad - (5)$$

sub (5) in (3)

$$V_1 = A\left[\frac{I_1 + DI_2}{C}\right] - BI_2$$

$$V_1 = \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2 \quad - (6)$$

Compare (6) with (1)

$$\boxed{Z_{11} = \frac{A}{C}, \quad Z_{12} = \frac{AD - BC}{C}}$$

Compare (5) with (2)

$$\boxed{Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}}$$

9. ABCD parameters in terms of h-parameter

ABCD equations are

$$V_1 = AV_2 - BI_2 \quad - (1)$$

$$I_1 = CV_2 - DI_2 \quad - (2)$$

h parameter equations are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad - (3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad - (4)$$

from (4),

$$I_1 = \frac{I_2 - h_{22} V_2}{h_{21}} \quad - (5)$$

Sub (5) in (3)

$$V_1 = h_{11}\left[\frac{I_2 - h_{22} V_2}{h_{21}}\right] + h_{12} V_2$$

$$V_1 = \frac{h_{11}}{h_{21}} I_2 + \left[h_{12} - \frac{h_{11} h_{22}}{h_{21}}\right] V_2$$

$$= \frac{h_{11}}{h_{21}} I_2 - \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{21}}\right] V_2$$

Let $\Delta h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix}$

$$\therefore V_1 = \frac{h_{11}}{h_{21}} I_2 - \frac{\Delta h}{h_{21}} V_2 \quad - (6)$$

Compare (6) with (1)

$$\boxed{A = \frac{-\Delta h}{h_{21}}, \quad B = \frac{-h_{11}}{h_{21}}}$$

compare (5) with (2)

$$\boxed{C = \frac{-h_{22}}{h_{21}}, \quad D = \frac{-1}{h_{21}}}$$

10. h parameters in terms of ABCD parameters

h parameters,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -(1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -(2)$$

ABCD,

$$V_1 = A V_2 - B I_2 \quad -(3)$$

$$I_1 = C V_2 - D I_2 \quad -(4)$$

from eq (4),

$$I_2 = \frac{-I_1 + C V_2}{D} \quad -(5)$$

$$I_2 = \frac{-1}{D} I_1 + \frac{C}{D} V_2 \quad -(6)$$

Compare (6) with (2)

$$\boxed{h_{21} = \frac{-1}{D}, \quad h_{22} = \frac{C}{D}}$$

Sub eq (5) in (3)

$$V_1 = \cancel{h_{11}} A V_2 - B \left[\frac{-I_1 + C V_2}{D} \right]$$

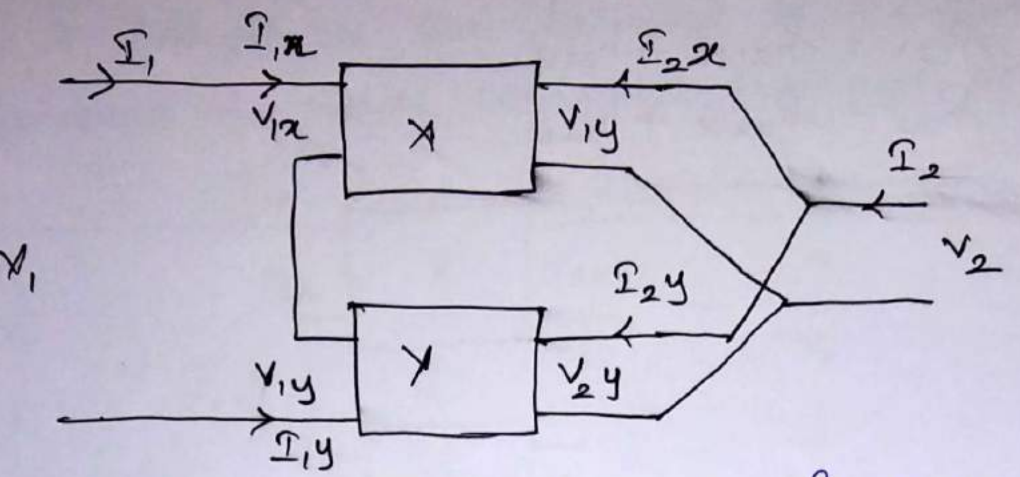
$$V_1 = \frac{+B}{D} I_1 + \left[\frac{AD - BC}{D} \right] V_2 \quad -(7)$$

Compare (7) with (1)

$$\boxed{h_{11} = \frac{B}{D}, \quad h_{12} = \frac{AD - BC}{D}}$$

INTERCONNECTION OF TWO PORT NETWORK

i) Series - parallel connection of two port network



Let $h_{11x}, h_{12x}, h_{21x}$ & h_{22x} are h-parameters of network x & $h_{11y}, h_{12y}, h_{21y}$ & h_{22y} are the h-parameters of network y.

The h- parameter equations of x is

$$V_{1x} = h_{11x} I_{1x} + h_{12x} V_{2x}$$

$$I_{2x} = h_{21x} I_{1x} + h_{22x} V_{2x}$$

and h parameter of y is

$$V_{1y} = h_{11y} I_{1y} + h_{12y} V_{2y}$$

$$I_{2y} = h_{21y} I_{1y} + h_{22y} V_{2y}$$

$I_1 = I_{1x} = I_{1y}$ and $V_1 = V_{1x} + V_{1y}$

$I_2 = I_{2x} + I_{2y}$ and $V_2 = V_{2x} = V_{2y}$

$$\therefore V_1 = h_{11x} I_{1x} + h_{12x} V_{2x} + h_{11y} I_{1y} + h_{12y} V_{2y}$$

$$V_1 = (h_{11x} + h_{11y}) I_1 + (h_{12x} + h_{12y}) V_2 \text{ \&}$$

$$I_2 = h_{21x} I_{1x} + h_{22x} V_{2x} + h_{21y} I_{1y} + h_{22y} V_{2y}$$

$$= (h_{21x} + h_{21y}) I_1 + (h_{22x} + h_{22y}) V_2$$

$\therefore$

$$\therefore h_{11} = h_{11x} + h_{11y}$$

$$h_{12} = h_{12x} + h_{12y}$$

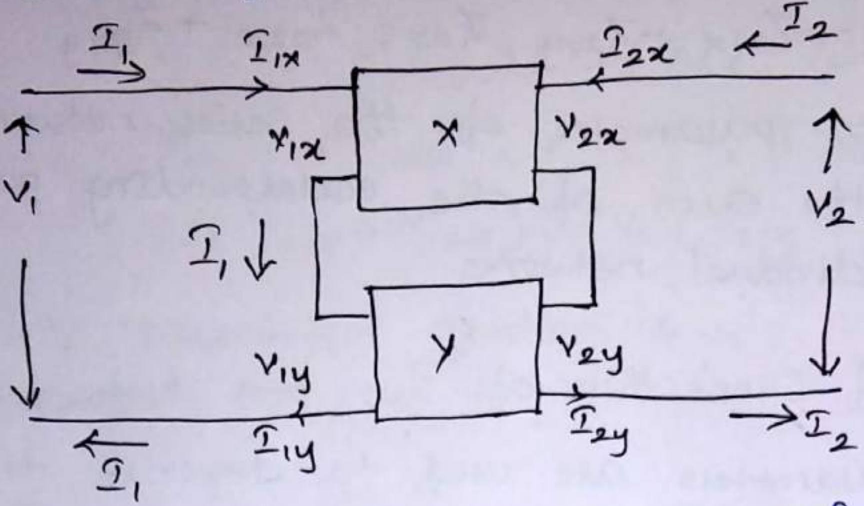
$$h_{21} = h_{21x} + h_{21y}$$

$$h_{22} = h_{22x} + h_{22y}$$

INTERCONNECTION OF TWO PORT NETWORKS

ii) Series Connection of a Two Port Network

Z parameters can be used to describe the parameters of series connected two port networks.



* Each port has a common reference node for its input and output
* If these references are connected together then the equations of the networks x and y in terms of Z parameters are

$$V_{1x} = Z_{11x} I_{1x} + Z_{12x} I_{2x}$$
$$V_{2x} = Z_{21x} I_{1x} + Z_{22x} I_{2x}$$
$$V_{1y} = Z_{11y} I_{1y} + Z_{12y} I_{2y}$$
$$V_{2y} = Z_{21y} I_{1y} + Z_{22y} I_{2y}$$

From the interconnection of networks,

$$I_1 = I_{1x} = I_{1y}, \quad I_2 = I_{2x} = I_{2y}$$
$$V_1 = V_{1x} + V_{1y}, \quad V_2 = V_{2x} + V_{2y}$$
$$\therefore V_1 = Z_{11x} I_1 + Z_{12x} I_2 + Z_{11y} I_1 + Z_{12y} I_2$$
$$= I_1 (Z_{11x} + Z_{11y}) + I_2 (Z_{12x} + Z_{12y})$$
$$V_2 = Z_{21x} I_1 + Z_{22x} I_2 + Z_{21y} I_1 + Z_{22y} I_2$$
$$= I_1 (Z_{21x} + Z_{21y}) + I_2 (Z_{22x} + Z_{22y})$$

The Z parameter for two port network is, (15

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

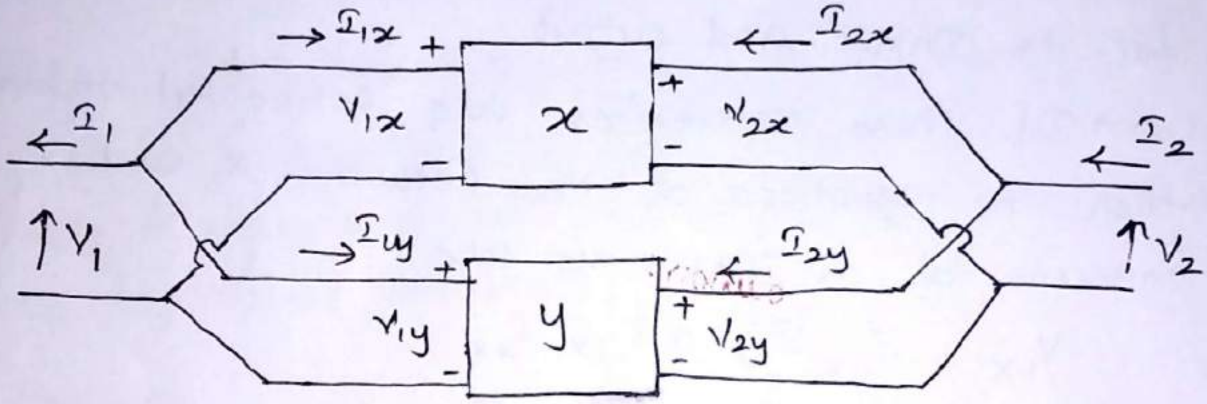
where $Z_{11} = Z_{11x} + Z_{11y}$, $Z_{12} = Z_{12x} + Z_{12y}$

$Z_{21} = Z_{21x} + Z_{21y}$, $Z_{22} = Z_{22x} + Z_{22y}$

Each Z parameter of the series network is given as the sum of the corresponding parameters of the individual networks

iii) Parallel connection of Two port Networks

Y parameters are used to describe the parallel connection.



If each two port has a reference node that is common to its input and output port and if the two ports are connected so that they have a common reference node, then the equations of X & Y in terms of Y parameters are given by

$$I_{1x} = Y_{11x} V_{1x} + Y_{12x} V_{2x}$$
$$I_{2x} = Y_{21x} V_{1x} + Y_{22x} V_{2x}$$
$$I_{1y} = Y_{11y} V_{1y} + Y_{12y} V_{2y}$$
$$I_{2y} = Y_{21y} V_{2y} + Y_{22y} V_{2y}$$

From the interconnection of the networks,

$$V_1 = V_{1x} = V_{1y} \; ; \; V_2 = V_{2x} = V_{2y}$$

$$I_1 = I_{1x} + I_{1y} \, , \; I_2 = I_{2x} + I_{2y}$$

$$\therefore I_1 = Y_{11x} V_1 + Y_{12x} V_2 + Y_{11y} V_1 + Y_{12y} V_2$$

$$= V_1 (Y_{11x} + Y_{11y}) + V_2 (Y_{12x} + Y_{12y})$$

$$I_2 = Y_{21x} V_1 + Y_{22x} V_2 + Y_{21y} V_2 + Y_{22y} V_2$$

$$= V_1 (Y_{21x} + Y_{22x}) + V_2 (Y_{21y} + Y_{22y})$$

The Y parameters equation is

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

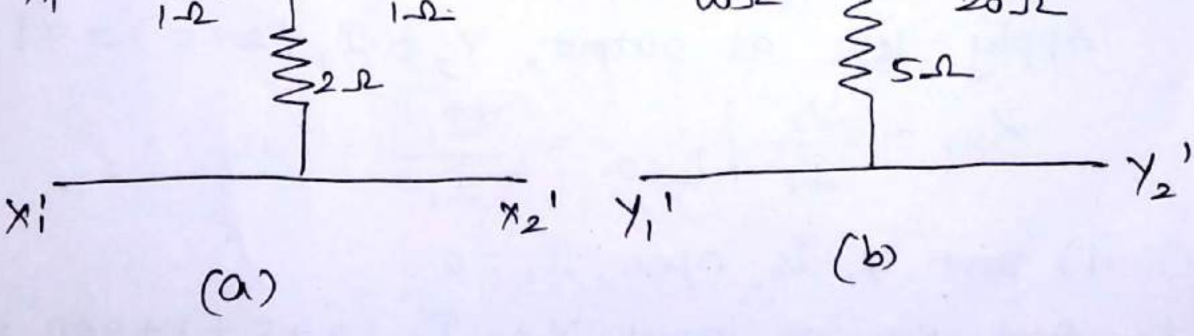
where $Y_{11} = Y_{11x} + Y_{11y}$, $Y_{12} = Y_{12x} + Y_{12y}$

$Y_{21} = Y_{21x} + Y_{21y}$, $Y_{22} = Y_{22x} + Y_{22y}$

Each Y parameter of the parallel network is given as the sum of the corresponding parameters of the individual networks.

Problems:

1. Two port networks shown in fig are connected in series. obtain the Z parameters of the combination. Also verify by direct calculation.



Solution:

The z parameters of the network in fig (a)

$$Z_{11x} = Z_a + Z_c = 1 + 2 = 3\,\Omega$$

$$Z_{12x} = Z_{21x} = Z_c = 2 = 2\,\Omega$$

$$Z_{22x} = Z_b + Z_c = 2 + 1 = 3\,\Omega$$

The z parameters of the network in fig (b)

$$Z_{11y} = Z_a + Z_c = 10 + 5 = 15\,\Omega$$

$$Z_{12y} = Z_{21y} = Z_c = 5 = 5\,\Omega$$

$$Z_{22y} = Z_b + Z_c = 20 + 5 = 25\,\Omega$$

The z parameters of the combined network

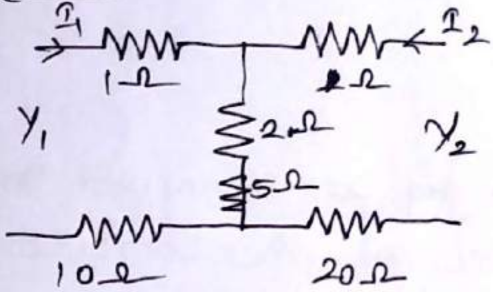
$$Z_{11} = Z_{11x} + Z_{11y} = 3 + 15 = 18\,\Omega$$

$$Z_{12} = Z_{12x} + Z_{12y} = 2 + 5 = 7\,\Omega$$

$$Z_{21} = Z_{21x} + Z_{21y} = 2 + 5 = 7\,\Omega$$

$$Z_{22} = Z_{22x} + Z_{22y} = 3 + 25 = 28\,\Omega$$

Check: Network is connected as single network



i) port V_2 is open, $I_2 = 0$

Apply KVL at input,

$$V_1 = I_1(2+5+10+1)$$

$$V_1 = 18\,I_1$$

$$\therefore Z_{11} = \left.\frac{18 I_1}{I_1}\right|_{I_2=0} = 18\,\Omega$$

Apply KVL at output, $V_2 = I_1(2+5) = 7I_1$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{7I_1}{I_1} = 7\,\Omega$$

ii) port V_1 is open, $I_1 = 0$

Apply KVL at input, $V_2 = I_2(2+5+1+20) = 28I_2$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{28I_2}{I_2} = 28\,\Omega$$

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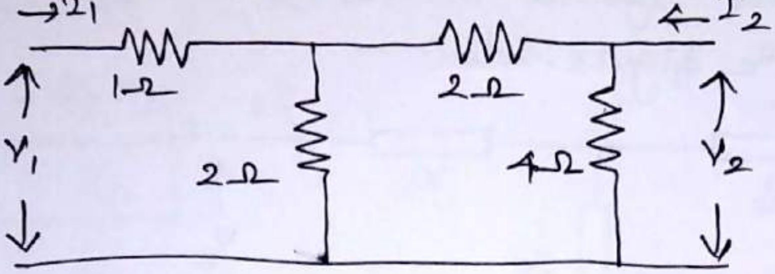
Apply KVL at output, $V_1 = I_2 (2+5) = 7 I_2$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{7 I_2}{I_2} = 7\,\Omega$$

2. Two identical sections of the network shown in figure are connected in parallel. Obtain the Y parameters of the combination.



solution:

(Refer Y parameter problem in page no)

$Y_{11} = \frac{1}{2}\,\mho$, $Y_{21} = \frac{-1}{4}\,\mho$, $Y_{22} = \frac{5}{8}\,\mho$, $Y_{12} = \frac{-1}{4}\,\mho$

If two networks are connected in parallel,

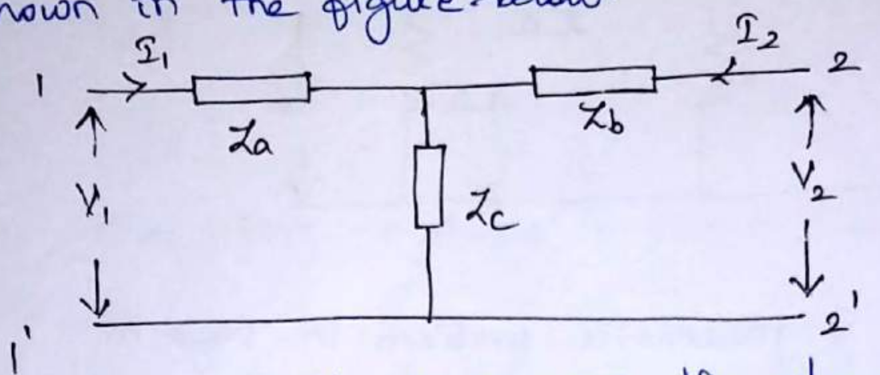
$Y_{11} = \frac{1}{2} + \frac{1}{2} = 1\,\mho$, $Y_{12} = \frac{-1}{4} \times 2 = \frac{-1}{2}\,\mho$

$Y_{21} = \frac{-1}{4} \times 2 = \frac{-1}{2}\,\mho$, $Y_{22} = \frac{5}{8} \times 2 = \frac{5}{4}\,\mho$

T REPRESENTATION

* A two port network with any number of elements may be converted into a two-port three element network.

* Thus a two port network may be represented by an equivalent T network (ie) three impedances are connected together in the form of a T as shown in the figure below.



* It is possible to express the elements of the T-network in terms of Z parameters or ABCD parameters as explained below.

Z parameters:

* when (2-2') is open circuit, $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$V_1 = I_1 Z_a + I_1 Z_c = I_1(Z_a + Z_c) \quad (KVL)$$

$$\boxed{Z_{11} = (Z_a + Z_c)}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$V_2 = I_1 Z_c \quad (KVL)$$

$$Z_{21} = \frac{I_1 Z_c}{I_1}$$

$$\boxed{Z_{21} = Z_c}$$

* when (1-1') is open circuit, $I_1 = 0$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$V_2 = I_2(Z_b + Z_c)$ (KVL)

$$\boxed{Z_{22} = (Z_b + Z_c)}$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$V_1 = I_2 Z_c$

$$\boxed{Z_{12} = Z_c}$$

From the above relations, T- parameters can be found as

$Z_{12} = Z_{21} = Z_c \Rightarrow Z_c = Z_{12} = Z_{21}$

$Z_b = Z_{22} - Z_{12}$

$Z_a = Z_{11} - Z_{21}$

ABCD parameters:

* when (2-2') is open circuit, $I_2 = 0$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0}$$

$V_1 = I_1(Z_a + Z_c)$

$V_2 = I_1 Z_c$

$$\boxed{A = \frac{Z_a + Z_c}{Z_c}}$$

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

$$C = \frac{I_1}{I_1 Z_c} = \frac{1}{Z_c}$$

$$\boxed{C = \frac{1}{Z_c}}$$

* when (2-2') is ~~open~~ short circuit, $V_2 = 0$

$$B = \frac{-V_1}{I_2}\bigg|_{V_2=0}$$

$$-I_2 = \frac{I_1 \times Z_c}{Z_b + Z_c}$$

where $I_1 = \frac{V_1}{Z_{eq}} = \frac{V_1 (Z_b + Z_c)}{Z_b Z_c + Z_a Z_b + Z_a Z_c}$

$$-I_2 = \frac{V_1 (Z_b + Z_c) \times Z_c}{(Z_b + Z_c)(Z_b Z_c + Z_a Z_b + Z_a Z_c)}$$

$$\therefore \boxed{B = \frac{Z_b Z_c + Z_a Z_b + Z_a Z_c}{Z_c}}$$

$$D = \frac{-I_1}{I_2}\bigg|_{V_2=0}$$

$$= \frac{-I_1 \times (Z_b + Z_c)}{-I_1 \times Z_c}$$

$$\boxed{D = \frac{Z_b + Z_c}{Z_c}}$$

from the above relations, T- parameters can be found as

$$\therefore \frac{Z_a}{Z_c} = A - 1 \Rightarrow Z_a = \frac{A-1}{C}$$

$$Z_b = \frac{D-1}{C}$$

$$Z_c = \frac{1}{C}$$

Problems:

1. The Z parameters of a two port network are $Z_{11} = 10\,\Omega$, $Z_{22} = 15\,\Omega$, $Z_{12} = Z_{21} = 5\,\Omega$. Find the equivalent T network and ABCD parameters.

Solution:

Z parameters,

$$Z_a = Z_{11} - Z_{21} = 10 - 5 = 5\,\Omega$$

$$Z_b = Z_{22} - Z_{12} = 15 - 5 = 10\,\Omega$$

$$Z_c = Z_{12} = Z_{21} = 5\,\Omega$$

The ABCD parameters of the network,

$$A = \frac{Z_a}{Z_c} + 1 = \frac{5}{5} + 1 = 2$$

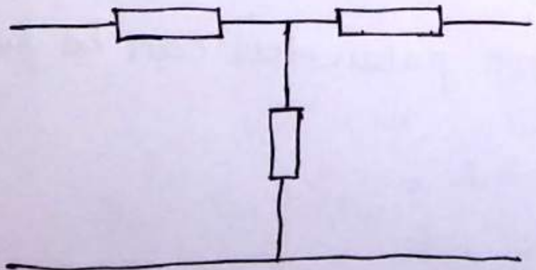
$$B = \frac{Z_b Z_c + Z_a Z_b + Z_a Z_c}{Z_c} = \frac{50 + 50 + 25}{5} = \frac{125}{5} = 25\,\Omega$$

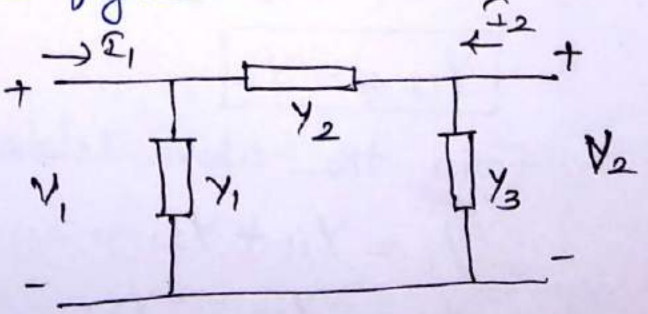
$$C = \frac{1}{Z_c} = \frac{1}{5} = 0.2\,\mho$$

$$D = \frac{Z_b + Z_c}{Z_c} = \frac{10}{5} + 1 = 3$$

π REPRESENTATION

As like T Network, a two port network may be represented by an equivalent π-network (i.e) three impedances or admittances are connected together in the form of π as shown in the figure.





* It is possible to express the elements in terms of y or ABCD parameters.

Y parameters:

* when (2-2') port short circuit, $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

$$V_1 = \frac{I_1}{Y_{eq}} = \frac{I_1}{Y_1+Y_2}$$

$$\boxed{Y_{11} = Y_1 + Y_2}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$$-I_2 = V_1 \times Y_2$$

$$\boxed{Y_{21} = -Y_2}$$

* when (1-1') short circuit, $V_1 = 0$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

$$V_2 = \frac{I_2}{Y_{eq}} = \frac{I_2}{Y_2+Y_3}$$

$$\boxed{Y_{22} = Y_2 + Y_3}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$-I_1 = V_2 Y_2$$

$$\boxed{Y_{12} = -Y_2}$$

From the above relations, π parameters can be found as

$$Y_1 = Y_{11} + Y_{21}$$

$$Y_2 = -Y_{12} = -Y_{21}$$

$$Y_3 = Y_{22} + Y_{12}$$

ABCD parameters:

$$A = \frac{-Y_{22}}{Y_{21}} = \frac{Y_3 + Y_2}{Y_2}$$

$$B = \frac{-1}{Y_{21}} = \frac{1}{Y_2}$$

$$C = \frac{-\Delta y}{Y_{21}} = \frac{Y_1 Y_2 + Y_2 Y_3 + Y_1 Y_3}{Y_2}$$

$$D = \frac{-Y_{11}}{Y_{21}} = \frac{Y_1 + Y_2}{Y_2}$$

From the above relations, ABCD parameters can be found as

$$Y_1 = \frac{D-1}{B}$$

$$Y_2 = \frac{1}{B}$$

$$Y_3 = \frac{A-1}{B}$$

Problems:

1. The port currents of a two port network are given by
$I_1 = 2.5V_1 - V_2$, $I_2 = -V_1 + 5V_2$
find the equivalent π Network.

Solution:

Y parameters of the network, $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 2.5\,\mho$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -1\,\mho$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -1\,\mho, \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 5\,\mho$$

$$\therefore Y_1 = Y_{11} + Y_{21} = 2.5 - 1 = 1.5\,\mho$$

$$Y_2 = -Y_{12} = -Y_{21} = +1\,\mho$$

$$Y_3 = Y_{22} + Y_{12} = 5 - 1 = 4\,\mho$$

BRIDGE NETWORK

The network which have more than three branches is called Bridge Network.

Two Types i) Lattice Network
ii) Laddee Network

LATTICE NETWORK :

* Lattice Network is one of the common faue teeminal two-port network which is shown in figure below.
* It is used in filtee sections and also used as attenuators.

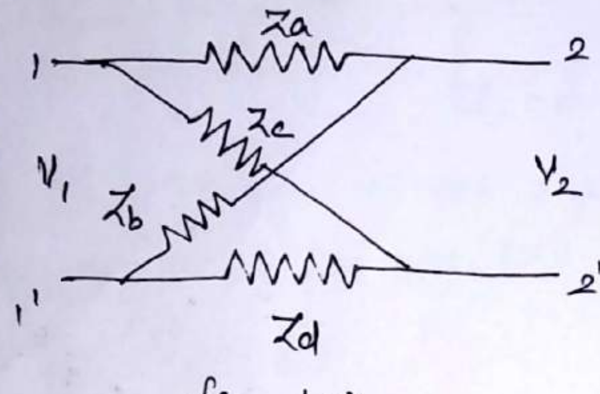


fig (a)

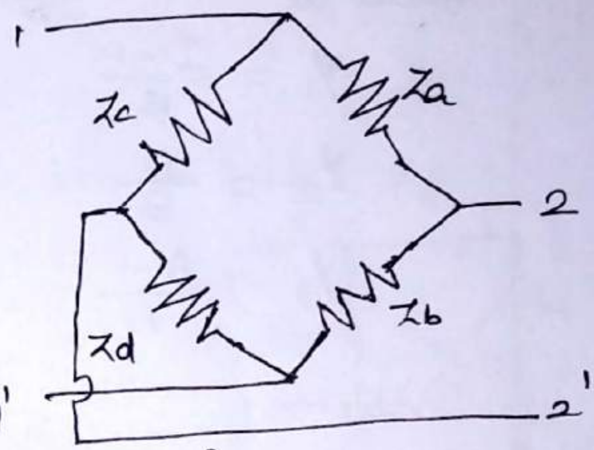


fig (b)

* In fig (a), the impedances Za & Zd - Series aems
  Zb & Zc - Diagonal aems
* If Zd = 0, lattice Network becomes a π network (fig a)
* The lattice network in fig (a) can be redrawn as in fig (b) and fig (c)

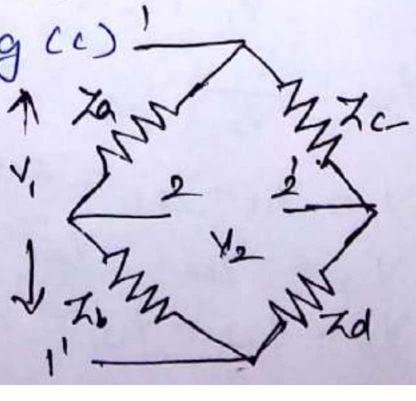


fig (c)

The z parameters are

i) $z_{11} = \dfrac{V_1}{I_1}\Big|_{I_2=0}$ (2-2' open circuit)

$V_1 = I_1 z_{eq}$

$$z_{eq} = (z_a+z_b) \parallel (z_c+z_d) = \frac{(z_a+z_b)(z_c+z_d)}{z_a+z_b+z_c+z_d}$$

$$\therefore V_1 = \frac{I_1 (z_a+z_b)(z_c+z_d)}{z_a+z_b+z_c+z_d} \quad — (1)$$

$$z_{11} = \frac{(z_a+z_b)(z_c+z_d)}{z_a+z_b+z_c+z_d}$$

If the network is symmetric, then $z_a = z_d$ & $z_b = z_c$

$$\boxed{z_{11} = \frac{z_a+z_b}{2}}$$

ii) $z_{21} = \dfrac{V_2}{I_1}\Big|_{I_2=0}$

$V_2 \Rightarrow$ voltage across 2-2'. It can be found by using KVL in fig (d)

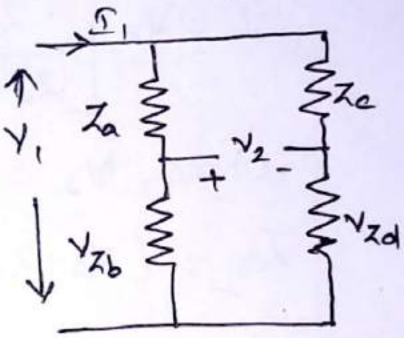


fig (d)

$V_2 = V_{zb} - V_{zd}$

$$= \frac{V_1 z_b}{z_a+z_b} - \frac{V_1 z_d}{z_d+z_c} \quad \text{(voltage division Rule)}$$

$$V_2 = V_1 \left[\frac{z_b}{z_a+z_b} - \frac{z_d}{z_d+z_c}\right]$$

from (1),

$$V_2 = \left[\frac{I_1 (z_a+z_b)(z_c+z_d)}{z_a+z_b+z_c+z_d}\right]\left[\frac{z_b}{z_a+z_b} - \frac{z_d}{z_d+z_c}\right]$$

$$= I_1 \left[\frac{(z_a+z_b)(z_c+z_d)}{(z_a+z_b+z_c+z_d)}\right]\left[\frac{z_b(z_d+z_c) - z_d(z_a+z_b)}{(z_a+z_b)(z_c+z_d)}\right]$$

$$= I_1 \left[\frac{z_b z_c + z_b z_d - z_d z_a - z_d z_b}{z_a+z_b+z_c+z_d}\right]$$

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$$V_2 = I_1 \left[\frac{Z_b Z_c - Z_a Z_d}{Z_a + Z_b + Z_c + Z_d} \right]$$

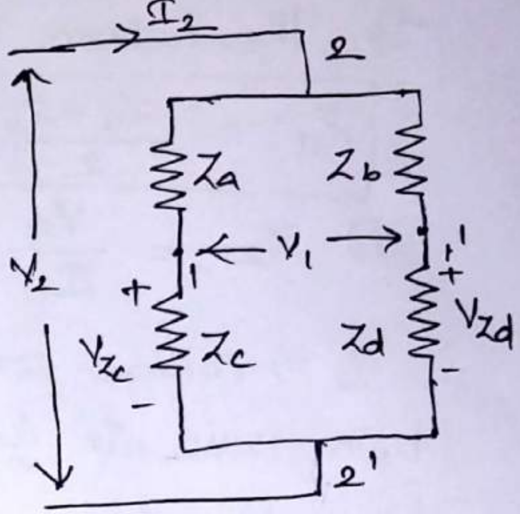
$$\therefore Z_{21} = \frac{Z_b Z_c - Z_a Z_d}{Z_a + Z_b + Z_c + Z_d}$$

If the network is symmetric, $Z_a = Z_d$ & $Z_b = Z_c$

$$\boxed{Z_{21} = \frac{Z_b - Z_a}{2}}$$

iii) $Z_{12} = \frac{V_1}{I_2} \Big|_{I_1 = 0}$

for finding this, redraw the network as shown in fig (e)



V_1 is the Voltage across 1-1'.

Apply KVL, $V_1 = V_{Zc} - V_{Zd}$

$$V_1 = V_2 \frac{Z_c}{Z_a + Z_c} - V_2 \frac{Z_d}{Z_b + Z_d}$$

$$V_1 = V_2 \left[\frac{Z_c}{Z_a + Z_c} - \frac{Z_d}{Z_b + Z_d} \right]$$

where, $V_2 = I_2 Z_{eq}$

$$Z_{eq} = (Z_a + Z_c) \| (Z_b + Z_d) = \frac{(Z_a + Z_c)(Z_b + Z_d)}{Z_a + Z_b + Z_c + Z_d}$$

$$\therefore V_2 = I_2 \left[\frac{(Z_a + Z_c)(Z_b + Z_d)}{Z_a + Z_b + Z_c + Z_d} \right] \quad — ②$$

sub V_2 in V_1

$$V_1 = I_2 \left[\frac{\cancel{(Z_a + Z_c)(Z_b + Z_d)}}{Z_a + Z_b + Z_c + Z_d} \right] \left[\frac{Z_c (Z_b + Z_d) - Z_d (Z_a + Z_c)}{\cancel{(Z_a + Z_c)(Z_b + Z_d)}} \right]$$

$$= I_2 \left[\frac{Z_c Z_b + \cancel{Z_c Z_d} - Z_a Z_d - \cancel{Z_c Z_d}}{Z_a + Z_b + Z_c + Z_d} \right]$$

$$\therefore Z_{12} = \frac{Z_c Z_b - Z_a Z_d}{Z_a + Z_b + Z_c + Z_d}$$

If the network is symmetrical, $Z_a = Z_d$, $Z_b = Z_c$

$$\therefore \boxed{Z_{12} = \frac{Z_b - Z_a}{2}}$$

iv) $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

from ⑤,

$$Z_{22} = \frac{(Z_a+Z_c)(Z_b+Z_d)}{Z_a+Z_b+Z_c+Z_d}$$

If the network is symmetric, $Z_a = Z_d$, $Z_b = Z_c$

$$\boxed{\therefore Z_{22} = \frac{Z_a+Z_b}{2}}$$

$\therefore$ from the above equations,

$$Z_{11} = Z_{22} = \frac{Z_a+Z_b}{2}$$

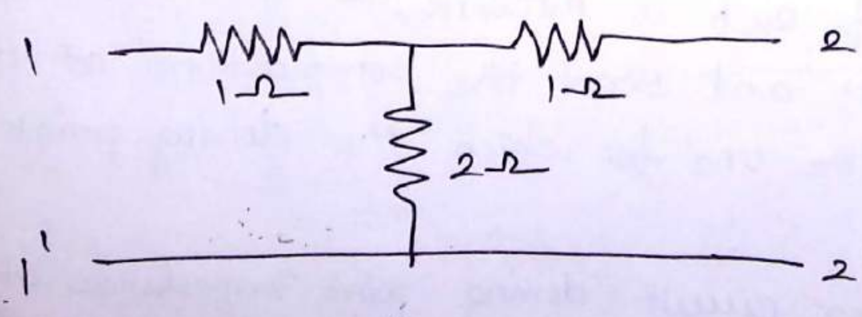
$$Z_{12} = Z_{21} = \frac{Z_b-Z_a}{2}$$

$\therefore$ from Z parameters, Lattice parameters can be found as

$$\boxed{\begin{aligned} Z_a = Z_d &= Z_{11} - Z_{12} \\ Z_b = Z_c &= Z_{21} + Z_{12} \end{aligned}}$$

Problem

1. obtain the lattice Equivalent of a Symmetrical T network shown in figure below.



Solution:

The Z parameters of the network,

$Z_{11} = Z_a + Z_c = 1 + 2 = 3\,\Omega$

$Z_{12} = Z_{21} = Z_c = 2\,\Omega$

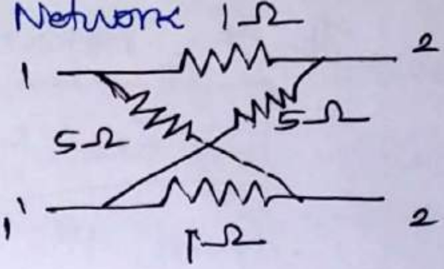
$Z_{22} = Z_b + Z_c = 1 + 2 = 3\,\Omega$

Since, $Z_{11} = Z_{22}$ & $Z_{12} = Z_{21}$, the given network is Symmetrical and reciprocal

∴ The parameters of the lattice Network

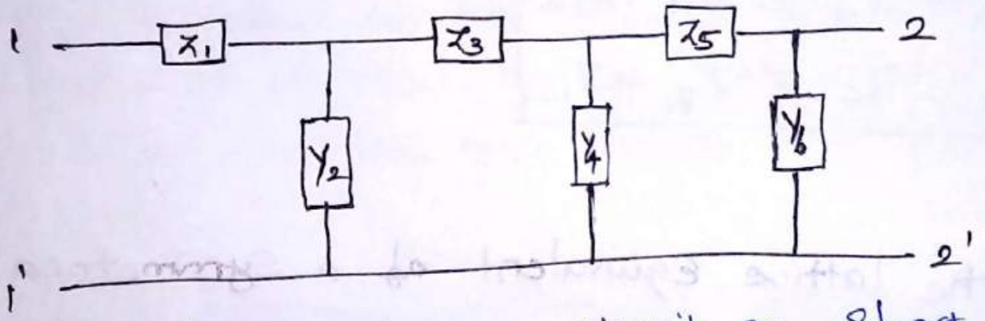
$Z_a = Z_d = Z_{11} - Z_{12} = 3 - 2 = 1\,\Omega$

$Z_b = Z_c = Z_{11} + Z_{12} = 3 + 2 = 5\,\Omega$



LADDER NETWORK

If each of the arms (ie Z_1, Y_2, Z_3, Y_4 etc) consists of only one of the passive parameters (ie R or L or C), the network is known as a simple ladder network.



* To find an open circuit or short circuit parameters of such a network, we open or short the appropriate port and start the computations at the port other than the one for which the driving point impedance is required.

∴ The open circuit driving point impedance at port 1 is

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2 = 0}$$

$$V_1 = I_1 Z_{eq}$$

Here Z_{eq} can be easily found out by continued fraction.

$$\therefore Z_{11} = Z_{eq} = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}}$$

* The above equation represents continued fraction

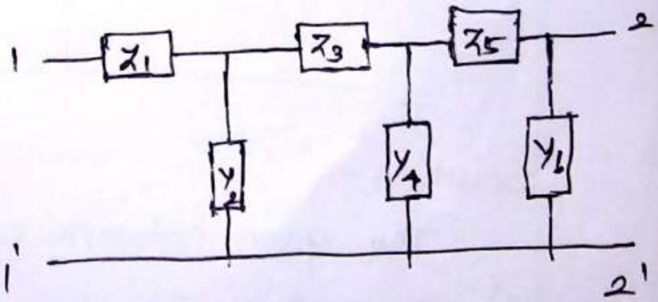
* To find out the transfer functions of a ladder network, we assume a unit output voltage (ie) $V_2 = 1$ and find the corresponding input through the successive by using Kirchoff's law.

* from the values of V_1 or I_1 for a unit V_2, we find the required transfer function.

Problems:

1. In the figure below, each series impedance consists of an inductance of 1 H and shunt branch consists of a capacitor of 1F. There are 3 series elements and 3 shunt elements. Find the open circuit driving point impedance at port 1.

Solution:



Given: $Z_1 = Z_3 = Z_5 = s$

$Y_1 = Y_4 = Y_6 = s$

$$\therefore Z_{11} = s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s}}}}} \Rightarrow s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{\frac{s^2+1}{s}}}}}$$

$$Z_{11} = s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{s}{s^2+1}}}} = s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{1}{\cfrac{s^3+s+s}{s^2+1}}}}$$

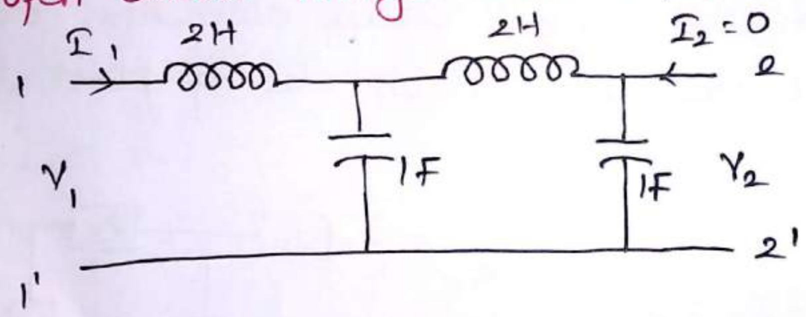
$$= s + \cfrac{1}{s + \cfrac{1}{s + \cfrac{s^2+1}{s^3+2s}}} = s + \cfrac{1}{s + \cfrac{1}{\cfrac{s^4+2s^2+s^2+1}{s^3+2s}}}$$

$$= s + \cfrac{1}{s + \cfrac{s^3+2s}{s^4+3s^2+1}} = s + \cfrac{1}{\cfrac{s^5+3s^3+s+s^3+2s}{s^4+3s^2+1}}$$

$$= s + \frac{s^4+3s^2+1}{s^5+4s^3+3s} = \frac{s^6+4s^4+3s^2+s^4+3s^2+1}{s^5+4s^3+3s}$$

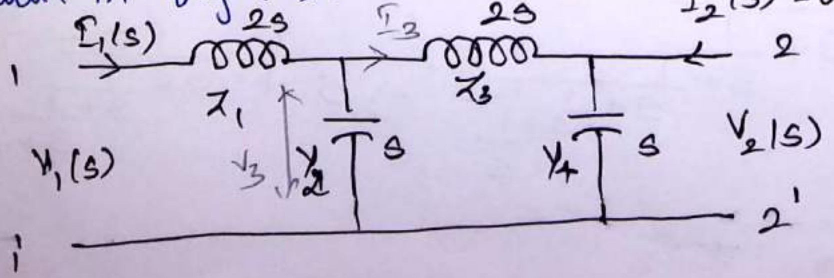
$$\therefore Z_{11} = \frac{s^6+5s^4+6s^2+1}{s^5+4s^3+3s}$$

2) The figure shown below is a ladder network having four elements. Find a) open circuit driving point impedance at port 1 (b) open circuit transfer impedance V_2/I_1 and (c) open circuit voltage ratio V_2/V_1.



Solution:

The given network can be converted into s-domain as shown in fig (a).



a) $Z_{11} = 2s + \cfrac{1}{s + \cfrac{1}{2s + \frac{1}{s}}} = 2s + \cfrac{1}{s + \cfrac{1}{\frac{2s^2+1}{s}}} = 2s + \cfrac{1}{s + \frac{s}{2s^2+1}}$

$= 2s + \cfrac{1}{\frac{2s^3+s+s}{2s^2+1}} = 2s + \frac{2s^2+1}{2s^3+s+s} = \frac{4s^4+4s^2+2s^2+1}{2s^3+2s}$

$$Z_{11} = \frac{4s^4+4s^2+2s^2+1}{2s^3+2s} = \frac{4s^4+6s^2+1}{2s^3+2s}$$

b) open circuit transfer impedance, V_2/I_1 (Z_{21}):

Let $V_2 = 1$

$\therefore I_1 = I_3 + V_3 Y_2$ (Apply KVL)

where $I_3 = Y_4 V_2 = s \times 1 = s$ (Apply ohm's law)

$V_3 = V_2 + I_3 Z_3 = 1 + s \times 2s = 1 + 2s^2$ (Apply KVL)

$I_1 = s + (1+2s^2)(s)$

$I_1 = 2s^3 + 2s$

$$Z_{21} = \frac{V_2}{I_1} = \frac{1}{2s^3+2s}$$

c) open circuit voltage ratio V_2/V_1 (G_{21}):

Let $V_2 = 1$

$\therefore V_1 = V_3 + I_1 Z_1$

$= (1+2s^2) + (2s^3+2s)(2s)$

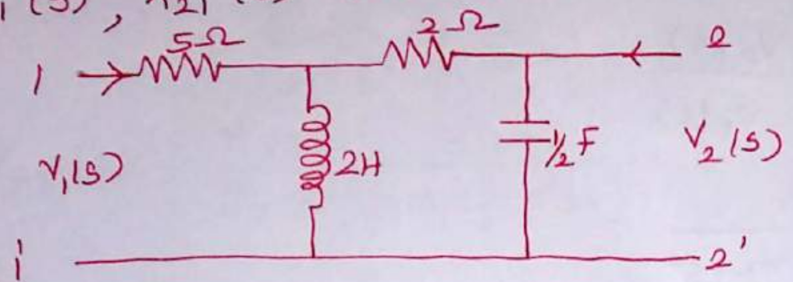
$= 1 + 2s^2 + 4s^4 + 4s^2$

$= 4s^4 + 6s^2 + 1$

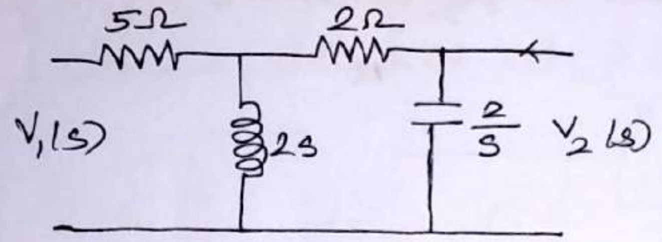
$$G_{21} = \frac{V_2}{V_1} = \frac{1}{4s^4+6s^2+1}$$

UNIT-1 (Additional problems)

1. For the two port network shown in fig, determine the $Z_{11}(s)$, $Z_{21}(s)$ and $G_{21}(s)$.



Solution:



i) To determine $Z_{11}(s)$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

Apply KVL at input, $V_1(s) = 5I_1 + 2sI_1 - 2sI_2$

$$V_1(s) = (5+2s)I_1(s) - 2sI_2(s) \quad \text{— (1)}$$

KCL ⇒ $$0 = \left(2 + \frac{2}{s} + 2s\right)I_2(s) - 2sI_1(s) \quad \text{— (2)}$$

Apply KVL at output, $V_2(s) = \frac{2}{s} I_2(s)$ — (3)

from (2), $$I_2(s) = \frac{2s^2 I_1(s)}{2s^2 + 2s + 2} \quad \text{— (4)}$$

Sub (4) in (1)

$$V_1(s) = (5+2s)I_1(s) - \frac{4s^2}{2s^2+2s+2} I_1(s)$$

$$\therefore Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = \frac{7s^2+7s+5}{s^2+s+1}$$

ii) $Z_{21}(s)$ at port 2,

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

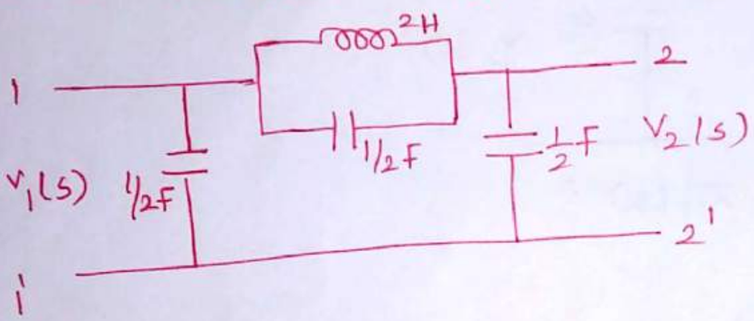
From ④, eq ③ becomes $V_2(s) = \dfrac{4s}{2s^2+2s+2} I_1(s)$

$\therefore Z_{21}(s) = \dfrac{V_2(s)}{I_1(s)} = \dfrac{2s}{s^2+s+1}$

iii) $G_{21}(s) = \dfrac{V_2(s)}{V_1(s)}$

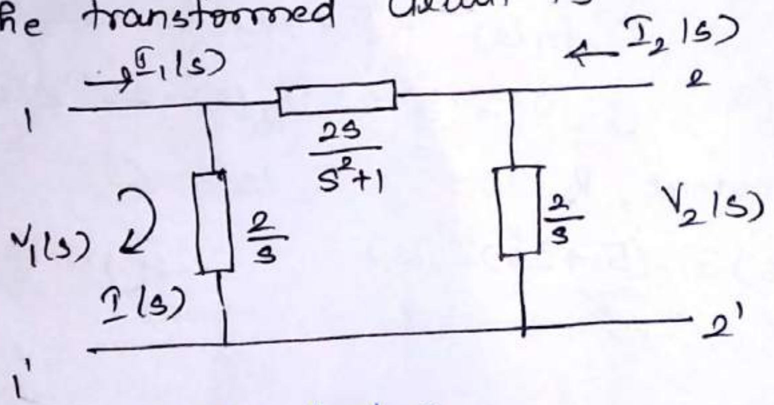
$= \dfrac{2s}{7s^2+7s+5}$

2. For the network shown in fig, determine the following transfer functions a) $G_{21}(s)$ b) $Z_{12}(s)$.



Solution:

The transformed circuit is



Voltage across port 2,

$$V_2(s) = V_1(s) \frac{2/s}{\frac{2}{s} + \frac{2s}{s^2+1}}$$

$$\therefore G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2/s}{\frac{4s^2+2}{s(s^2+1)}} = \frac{2(s^2+1)}{4s^2+2} = \frac{s^2+1}{2s^2+1}$$

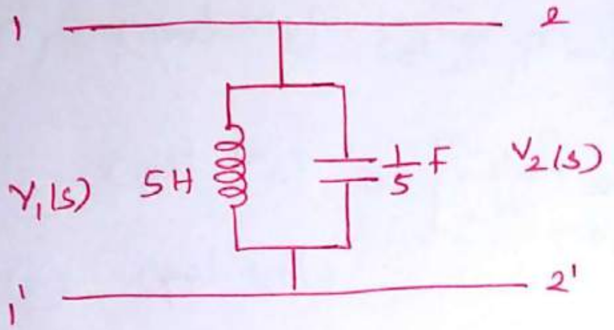
Voltage at port 1,

$$V_1(s) = I_1(s)\,\frac{2}{s}$$

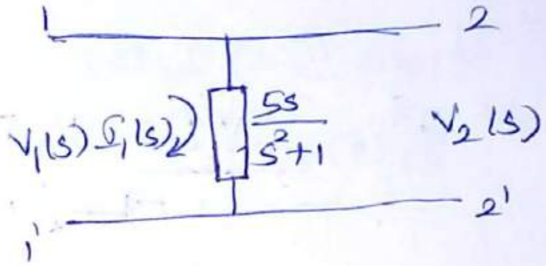
$$V_1(s) = \frac{I_2(s)\left(\frac{2}{s}\right)\times\left(\frac{2}{s}\right)}{\frac{2}{s}+\frac{2s}{s^2+1}+\frac{2}{s}} = I_2(s)\,\frac{4(s^2+1)}{s(6s^2+4)}$$

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)} = \frac{s^2+1}{s\left(\frac{3}{2}s^2+1\right)}$$

3) For the network shown in figure determine $Z_{21}(s)$ and $Y_{12}(s)$. Also find $g_{21}(s)$



solution:



Voltage at port 1,

$$V_1(s) = I_1(s)\,\frac{5s}{s^2+1}$$

Voltage at port 2,

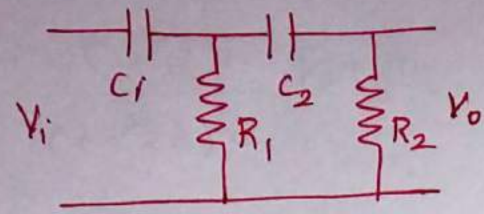
$$V_2(s) = I_1(s)\,\frac{5s}{s^2+1}$$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = 1$$

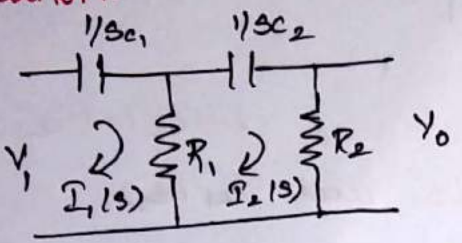
$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{5s}{s^2+1}$$

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)} = \frac{s^2+1}{5s}$$

4. find the expression of voltage transfer ratio for the network.



Solution:



$$V_1(s) = I_1(s)\left[R_1 + \frac{1}{sC_1}\right] - R_1 I_2(s) \quad (1^{st}\text{ Loop}) \quad - (1)$$

$$-R_1 I_1(s) + I_2(s)\left[R_1 + R_2 + \frac{1}{sC_2}\right] = 0 \quad (2^{nd}\text{ Loop}) \quad - (2)$$

(last loop) $-$ (3)

$$V_o(s) = I_2(s) R_2$$

from (1) $\Rightarrow I_1(s)$

$$I_1(s) = \frac{V_1(s) + R_1 I_2(s)}{\left(R_1 + \frac{1}{sC_1}\right)} \quad - (4)$$

sub (4) in (2)

$$-R_1\left[\frac{V_1(s) + R_1 I_2(s)}{R_1 + \frac{1}{sC_1}}\right] + I_2(s)\left[R_1 + R_2 + \frac{1}{sC_2}\right] = 0$$

~~$I_2(s)$~~ By simplification,

$$I_2(s)\left[\frac{-R_1^2 sC_1}{1 + R_1 sC_1} + \frac{R_1 sC_2 + R_2 sC_2 + 1}{sC_2}\right] = \frac{R_1 V_1(s) sC_1}{R_1 sC_1 + 1}$$

$$I_2(s)\left[\frac{-R_1^2 \cancel{s}C_1 sC_2 + R_1 sC_2 + R_1^2 \cancel{s}C_1 sC_2 + R_2 sC_2 + R_1 R_2 sC_1 sC_2 + 1 + R_1 + sC_1}{(1 + R_1 sC_1) sC_2}\right] =$$

$$\frac{R_1 V_1(s) sC_1}{R_1 sC_1 + 1}$$

$$I_2(s)\left[\frac{R_1 sC_2 + R_2 sC_2 + R_1 R_2 sC_1 sC_2 + R_1 sC_1 + 1}{(1+R_1 sC_1)sC_2}\right] = \frac{R_1 V_1(s) sC_1}{1+R_1 sC_1}$$

$$I_2(s) = \frac{R_1 V_1(s) sC_1\, sC_2}{sC_2(R_1+R_2) + 1 + R_1 sC_1(1+R_2 sC_2)} \quad \text{—} (5)$$

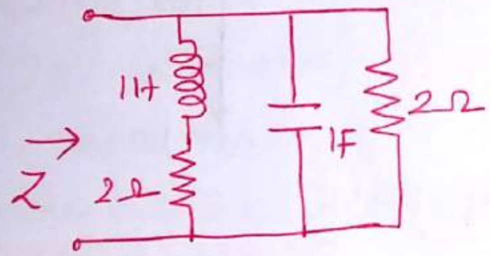
Sub (5) in (3)

$$V_o(s) = I_2(s) R_2$$

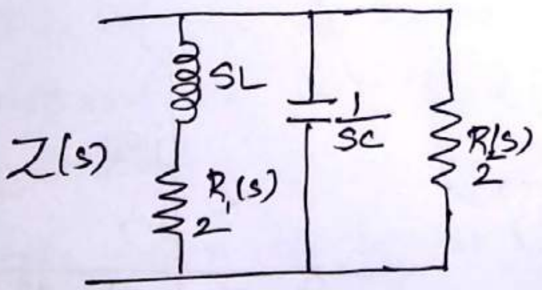
$$= \frac{R_1 R_2 V_1(s)\, sC_1\, sC_2}{(R_1+R_2)sC_2 + 1 + R_1 sC_1 (R_2 sC_2 + 1)}$$

$$\frac{V_o(s)}{V_1(s)} = \frac{R_1 R_2\, sC_1\, sC_2}{(R_1+R_2)sC_2 + 1 + R_1 sC_1 (R_2 sC_2 + 1)}$$

5. Find the driving point impedance.



Solution :



$$Z(s) = \left[R_2(s) \,\|\, \frac{1}{SC}\right] \,\|\, \left[SL + R_1(s)\right]$$

$$= \left[2 \,\|\, \tfrac{1}{s}\right] \,\|\, \left[s+2\right]$$

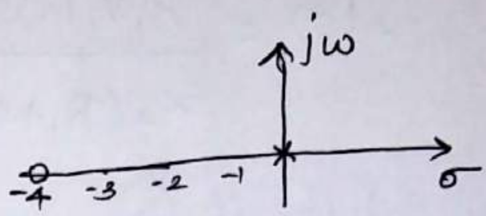
$$= \frac{2/s}{\frac{2s+1}{s}} \,\|\, (2+s)$$

$$= \left[\frac{2}{s} \times \frac{s}{2s+1}\right] \,\|\, (2+s)$$

$$= \frac{\left(\frac{2}{2s+1}\right) \times (2+s)}{\frac{2}{2s+1} + (2+s)} = \frac{4+2s}{2s+1} \times \frac{2s+1}{2s^2+5s+4} = \frac{2(s+2)}{2s^2+5s+4}$$

6. A function is given by $Z(s) = \frac{s+4}{s}$. find the pole zero plot.

Solution:

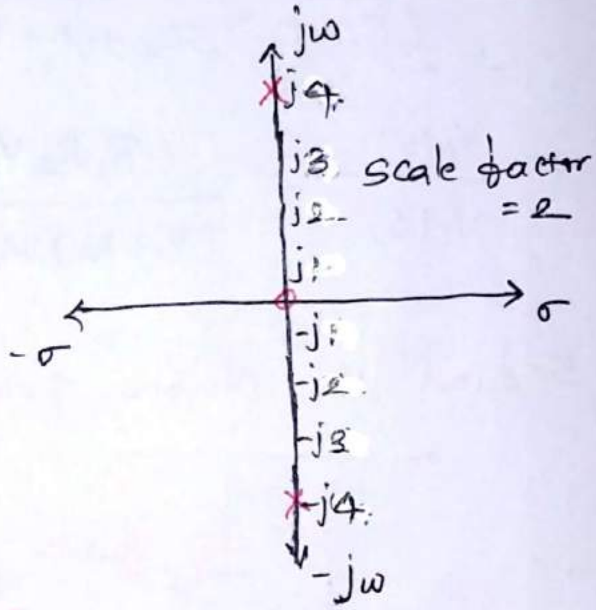


zero ⇒ $s+4=0$ ⇒ $s=-4$

pole ⇒ $s=0$

7. A function is given by $Z(s) = \frac{2s}{s^2+16}$. Draw its pole zero plot.

solution:



$s^2+16 = (s+j4)(s-j4)$

$\therefore Z(s) = \frac{2s}{(s+j4)(s-j4)}$

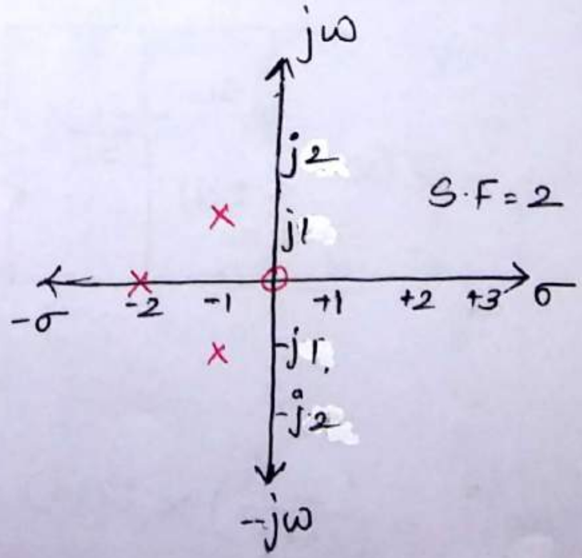
scale factor = 2

zero ⇒ $s=0$

pole ⇒ $s=-j4, +j4$

8. A Network function is given by $P(s) = \frac{2s}{(s+2)(s^2+2s+2)}$ obtain pole zero diagram.

Solution:



$s^2+2s+2 = (s+1-j)(s+1+j)$

$\therefore Z(s) = \frac{2s}{(s+2)(s+1-j)(s+1+j)}$

scale factor = 2

zero ⇒ $s=0$

pole ⇒ $s=-2, (-1+j)\ (-1-j)$

9. Check if the driving point impedance $Z(s)$,

$$Z(s) = \frac{s^4+s^2+1}{s^3+2s^2-2s+10}$$ can represent a passive one port network.

Solution:

The given function is not suitable to represent the impedance of a one port network due to following reasons

1. In numerator, one coefficient is Missing (s^3)
2. In Denominator, one coefficient is Negative $(-2s)$

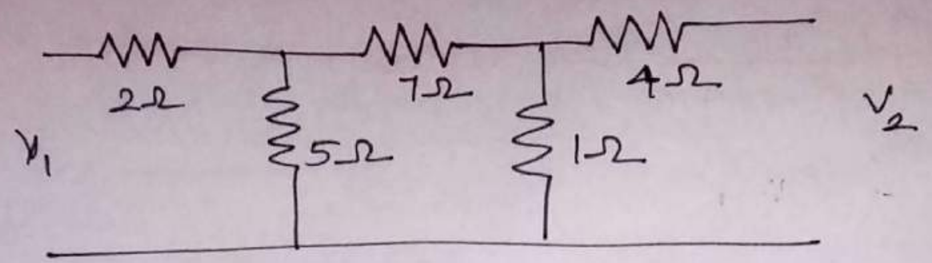
10. Explain with reasons why the following expression for $Z(s)$ is not suitable for representing a passive network. $$Z(s) = \frac{s^4-s^3+2s^2}{s+5}$$

Solution:

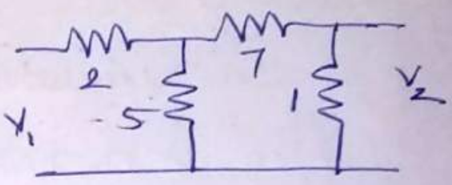
In numerator,

1. one coefficient is -ve $(-s^3)$
2. $s^4-s^3+2s^2 = s^2(s^2-s+2)$ (ie) double zero at $s=0$, $s=-0.5+j\sqrt{7/4}$ and $s=-0.5-j\sqrt{7/4}$. This is not permitted.
3. The degree of numerator is 4 while that of denominator is 1. Thus a difference of 3 is not permitted.
4. The term of lowest degree in numerator is 2 while that in the denominator is 0.

11. Determine open circuit impedance parameters for the network shown below.



i) port b-b' is open circuited



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$V_1 = I_1 Z_{eq_1}$$

$$Z_{eq_1} = [(7+1)||5]+2 = \frac{8\times5}{8+5}+2$$

$$Z_{eq_1} = \frac{66}{13}$$

$$V_1 = \frac{66}{13} \times I_1$$

$$Z_{11} = \frac{\frac{66}{13}\times I_1}{I_1}$$

$$\boxed{Z_{11} = \frac{66}{13}\ \Omega}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$V_2 = IR$$

Let us assume current through 1Ω be I_x

$$I_x = I_1 \times \frac{5}{(5+7+1)} = \frac{5I_1}{13}$$

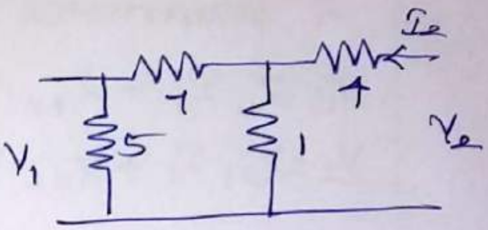
$$I_x = \frac{5I_1}{13}$$

$$V_2 = \frac{5I_1}{13}\times 1 = \frac{5I_1}{13}$$

$$Z_{21} = \frac{5I_1/13}{I_1}$$

$$\boxed{Z_{21} = \frac{5}{13}\ \Omega}$$

ii) port (a-a') is open circuited



$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$V_2 = I_2 Z_{eq_2}$$

$$Z_{eq_2} = ((7+5)\,||\,1) + 4 = \frac{12\times 1}{12+1} + 4$$

$$Z_{eq_2} = \frac{64}{13} \Rightarrow V_2 = \frac{64 I_2}{13}$$

$$\boxed{Z_{22} = \frac{64}{13}\ \Omega}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$V_1 = IR$$

Let us assume current through 5Ω is I_y

$$I_y = I_2 \times \frac{1}{(7+5+1)} = \frac{I_2}{13}$$

$$V_1 = \frac{5I_2}{13}$$

$$\boxed{Z_{12} = \frac{5}{13}\ \Omega}$$

12. In a two port network, $Z_{11} = 2\Omega$, $Z_{12} = Z_{21} = 5\Omega$, $Z_{22} = 1\Omega$. find i) y parameters ii) h- parameters iii) ABCD parameters.

Solution:

Z parameter equations are

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\therefore \; V_1 = 2I_1 + 5I_2 \quad - (1)$

$V_2 = 5I_1 + I_2 \quad - (2)$

To find y parameter:

Multiply eqn (1) with 2.5,

$2.5V_1 = 5I_1 + 12.5I_2 \quad - (3)$

sub (3) from (2),

$V_2 - 2.5V_1 = I_2 - 12.5I_2$

$V_2 - 2.5V_1 = -11.5I_2$

$I_2 = \frac{2.5}{11.5}V_1 - \frac{1}{11.5}V_2 \quad - (4)$

Multiply eqn (2) with 5,

$5V_2 = 25I_1 + 5I_2 \quad - (5)$

Sub (5) from (1)

$V_1 - 5V_2 = 2I_1 - 25I_1$

$V_1 - 5V_2 = -23I_1$

$I_1 = \frac{5}{23}V_2 - \frac{1}{23}V_1 \quad - (6)$

$\therefore$ from eq (4) & (6)

$Y_{12} = \frac{2.5}{11.5} \mho, \; Y_{21} = \frac{5}{23} \mho$

$Y_{11} = \frac{-1}{23} \mho, \; Y_{22} = \frac{-1}{11.5} \mho$

h- parameter:

Eq (6) & (2) is rearranged as

$V_1 = -23 I_1 + 5 V_2$ — (7)

$I_2 = -5 I_1 + V_2$ — (8)

from (7) & (8)

$h_{11} = -23, \ h_{12} = 5$

$h_{21} = -5, \ h_{22} = 1$

ABCD parameter:

Eq (4) & (2) is rearranged as

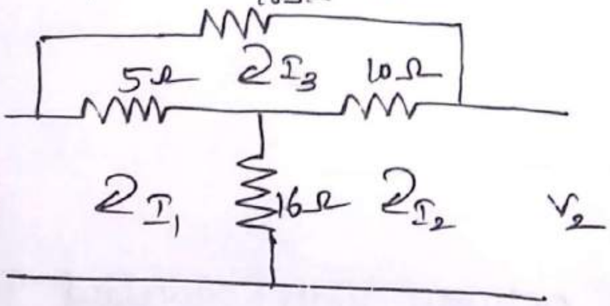
$V_1 = \frac{1}{2.5} V_2 + \frac{11.5}{2.5} I_2$

$I_1 = \frac{1}{5} V_2 - \frac{1}{5} I_2$

$\therefore A = \frac{1}{2.5} \quad B = -\frac{11.5}{2.5} \ \Omega$

$C = \frac{1}{5} \mho \quad D = \frac{1}{5}$

13. Find Z parameters of the network



Solution:

$21 I_1 - 16 I_2 - 5 I_3 = V_1$ — (1)

$-66 I_1 + 26 I_2 - 10 I_3 = V_2$ — (2)

$-5 I_1 - 10 I_2 + 25 I_3 = 0$ — (3)

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Cancel I_3 in 3 equations

$$(1) \times 5 \Rightarrow 105 I_1 - 80 I_2 - 25 \cancel{I_3} = 5V_1$$

$$(3) \Rightarrow -5I_1 - 10 I_2 + \cancel{25 I_3} = 0$$

$$100 I_1 - 90 I_2 = 5 V_1$$

$$20 I_1 - 18 I_2 = V_1 \quad — (4)$$

$$(2) \times 25 \Rightarrow -400 I_1 + 650 I_2 - \cancel{250 I_3} = 25 V_2$$

$$(3) \times 10 \Rightarrow -50 I_1 - 100 I_2 + \cancel{250 I_3} = 0$$

$$-450 I_1 + 550 I_2 = 25 V_2$$

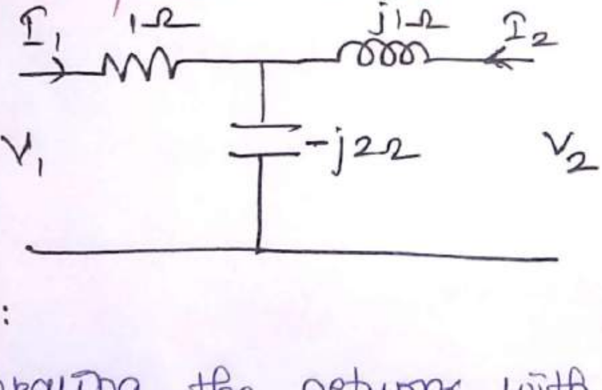
$$-18 I_1 + 22 I_2 = V_2 \quad - (5)$$

arrange (4) & (5) in Matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & -18 \\ -18 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore Z_{11} = 20\,\Omega, \; Z_{12} = Z_{21} = -18\,\Omega, \; Z_{22} = 22\,\Omega$$

14. obtain Z parameters of the network



Solution:

Comparing the network with standard T network,

$$Z_A = 1\Omega, \; Z_B = j1\Omega, \; Z_C = -j2\Omega$$

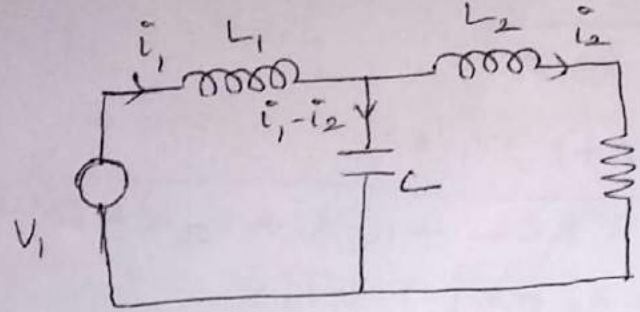
$$\therefore Z_{11} = Z_A + Z_C = 1 - j2\,\Omega$$

$$Z_{12} = Z_C = -j2\,\Omega$$

$$Z_{21} = Z_C = -j2\,\Omega$$

$$Z_{22} = Z_B + Z_C = j1 - j2 = -j1\,\Omega$$

15. find $Z_{11}(s)$ & $\frac{I_2(s)}{V_1(s)}$ for the following circuit



KVL equations for two loops are

$$I_1(s)\left(sL_1 + \frac{1}{sC}\right) - \frac{I_2(s)}{sC} = V_1(s) \quad \text{--- (1)}$$

$$\frac{-1}{sC} I_1(s) + \left(\frac{1}{sC} + sL_2 + R\right) I_2(s) = 0 \quad \text{--- (2)}$$

from eq (2),

$$I_2(s) = \frac{\frac{1}{sC} I_1(s)}{\left(\frac{1 + s^2L_2C + sCR}{sC}\right)}$$

$$I_2(s) = \frac{I_1(s)}{1 + s^2L_2C + sCR} \quad \text{--- (3)}$$

sub (3) in (1)

$$V_1(s) = I_1(s)\left[\frac{s^2L_1C + 1}{sC} - \frac{\cancel{I_1(s)}\ 1}{(1 + s^2L_2C + sCR)sC}\right]$$

$$= I_1(s)\left[\frac{s^3L_1C^2 + sC + s^4L_1L_2C^2 + s^2L_2C + s^3L_1RC^2 + sCR - 1}{sC\,(1 + s^2L_2C + sCR)}\right]$$

$$= I_1(s)\left[sC\,(s^2L_1C + 1 + s^3L_1L_2C + sL_2 + s^2L_1RC + R\right.$$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = \frac{s^3 L_1 L_2 C + s^2 RCL_1 + L_1 s + L_2 s + R}{s^2 L_2 C + sRC + 1}$$

$$I_2(s) = \frac{I_1(s)}{1 + s^2 CL_2 + sCR}$$

$$I_1(s) = \frac{(s^2 L_2 C + sCR + 1) V_1(s)}{s^3 L_1 L_2 C + s^2 RCL_1 + L_1 s + L_2 s + R}$$

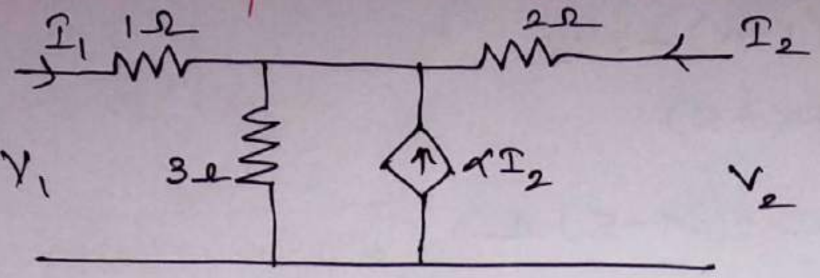
$$I_2(s) = \frac{(s^2 L_2 C + sCR + 1)\, V_1(s)}{(s^3 L_1 L_2 C + s^2 RCL_1 + sL_1 + sL_2 + R)(1 + s^2 CL_2 + sCR)}$$

$$\frac{I_2(s)}{V_1(s)} = \frac{(s^2 L_2 C + sCR + 1)}{[s^3 L_1 L_2 C + s^2 RCL_1 + sL_1 + sL_2 + R)(1 + s^2 CL_2 + sCR)}$$

$$\frac{I_2(s)}{V_1(s)} = \frac{1}{s^3 L_1 L_2 C + s^2 RCL_1 + sL_1 + sL_2 + R}$$

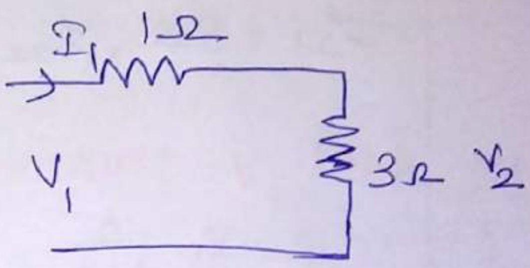
$$\frac{I_2(s)}{V_1(s)} = \frac{1}{s^3 L_1 L_2 C + s^2 RCL_1 + L_1 s + L_2 s + R}$$

16. find the Z parameters of the network shown in fig.



solution:

i) $I_2 = 0 \Rightarrow$ open circuit



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

To find Z_{11} & Z_{21} Apply ohm's law

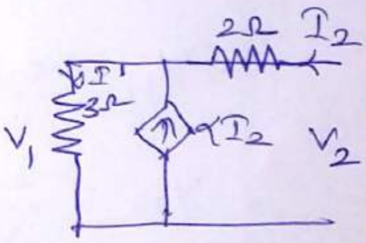
$$V_1 = I_1 Z_{eq} = I_1 4$$

$$\therefore Z_{11} = \frac{4I_1}{I_1} = 4\,\Omega$$

$$V_2 = 3I_1$$

$$\therefore Z_{21} = \frac{3I_1}{I_1} = 3\,\Omega$$

ii) $I_1 = 0 \Rightarrow$ open circuit



$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \& \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Let I' current flowing through 3Ω,

$$\therefore I' = I_2 + \alpha I_2$$

$$= I_2(1+\alpha)$$

$$V_1 = 3I' = 3I_2(1+\alpha)$$

$$Z_{12} = \frac{3I_2(1+\alpha)}{I_2} = 3(1+\alpha)\,\Omega$$

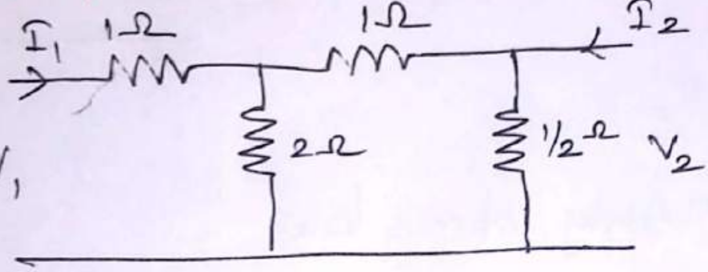
$$V_2 = V_1 + 2I_2$$
$$= 3I_2(1+\alpha) + 2I_2$$
$$= I_2(3\alpha + 5)$$

$$Z_{22} = \frac{V_2}{I_2} = (3\alpha+5)\,\Omega$$

$Z_{11} = 4\,\Omega$, $Z_{12} = 3(1+\alpha)\,\Omega$

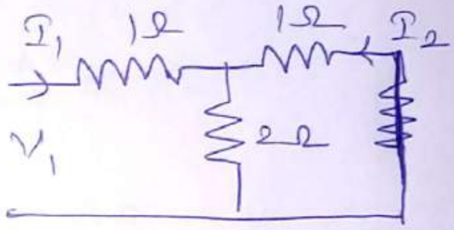
$Z_{21} = 3\,\Omega$, $Z_{22} = (3\alpha+5)\,\Omega$

17. Find Y-parameters for the network shown in fig.



Solution:

i) $V_2 = 0$



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$I_1 = \frac{V_1}{(1\|2)+1} = \frac{V_1}{1.67}$$

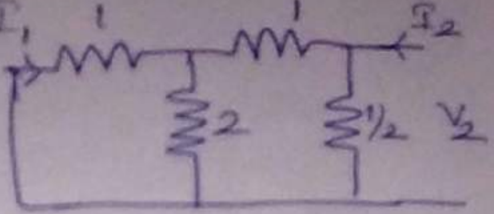
$$Y_{11} = 0.6\,\mho$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$-I_2 = I_1 \times \frac{2}{3} = \frac{V_1}{1.67}\left(2/3\right) = 0.4V_1$$

$$Y_{21} = \frac{-0.4V_1}{V_1} = -0.4\,\mho$$

ii) $V_1 = 0$



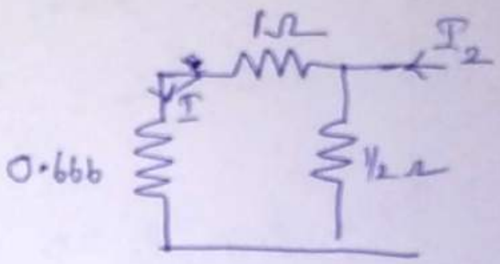
$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

$$I_2 = \frac{V_2}{([(1\|2)+1]\|1/2)}$$

$$= \frac{V_2}{1.67\|1/2} = \frac{V_2}{0.385}$$

$$I_2 = 2.6\,V_2$$

$$Y_{22} = \frac{2.6\,V_2}{V_2} = 2.6\,\mho$$



$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$-I_x = I_2 \times \frac{0.5}{2.1666} = 0.231\,I_2 \Rightarrow I_1 = I_x \times \frac{2}{3}$$

$$= 0.231\,I_2 \times \frac{2}{3}$$

$$V_2 = \frac{I_2}{2.6}$$

$$\therefore Y_{12} = \frac{-0.231\,I_2 \times 2.6 \times 2}{I_2 \times 3}$$

$$Y_{12} = -0.40\,\mho$$

18. obtain ABCD parameters and verify the condition for reciprocity.

i) When $I_2 = 0$

$$A = \frac{V_1}{V_2} \quad \& \quad C = \frac{I_1}{V_2}$$

$$V_1 = I_1 (8+2j)$$

$$V_2 = I_1 (3-4j)$$

$$\therefore A = \frac{8+2j}{3-4j} = 0.64 + 1.52j$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{I_1 (3-4j)} = \frac{1}{3-4j} = 0.12 + 0.16j \ \mho$$

ii) When $V_2 = 0$

$$B = \frac{-V_1}{I_2} \quad \& \quad D = \frac{-I_1}{I_2}$$

$$-I_2 = \frac{I_1 (3-4j)}{(6-4j)} = I_1 (0.65 - 0.23j)$$

$$V_1 = I_1 \{ (5+j6) + (3-4j) || 3 \}$$

$$= I_1 (6.96 + 5.3j)$$

$$B = \frac{I_1 (6.96 + 5.3j)}{I_1 (0.65 - 0.23j)}$$

$$B = 6.95 + 10.61j$$

$$D = \frac{-I_1}{I_2} = \frac{I_1}{I_1 (0.65 - 0.23j)}$$

$$D = 1.367 + 0.48j$$

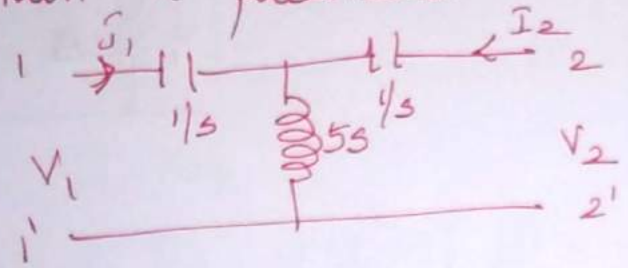
Reciprocity condition satisfy when $AD - BC = 1$

$$\left[(0.64+1.52j)(1.367+0.48j)\right] - \left[(6.95+10.61j)(0.12+0.16j)\right]$$

$$= 1.00 - 1.6\times10^{-4} = 1.008\angle 0.009^\circ \simeq 1$$

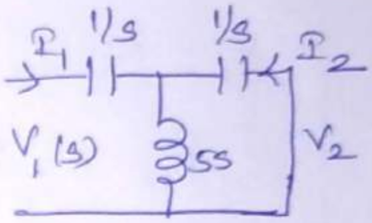
Hence condition is satisfied.

19. Obtain h parameters



Solution:

i) $V_2 = 0 \Rightarrow$ short circuit



$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ & $h_{21} =$

$$h_{11} = \frac{V_1}{I_1}$$

$$V_1 = I_1 Z_{eq}$$

$$Z_{eq} = \left(\tfrac{1}{s} \parallel 5s\right) + \frac{1}{s} = \frac{1/s \times 5s}{\frac{1}{s}+5s} + \frac{1}{s} = \frac{5s}{1+5s^2} + \frac{1}{s}$$

$$Z_{eq} = \frac{10s^2+1}{s(5s^2+1)}$$

$$V_1 = \frac{I_1(10s^2+1)}{s(5s^2+1)}$$

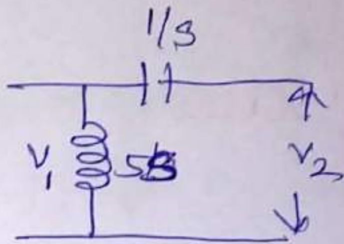
$$\therefore h_{11} = \frac{10s^2+1}{s(5s^2+1)}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$$

$$I_2 = -I_1 \times \frac{5s}{5s+\frac{1}{s}} = -I_1 \frac{5s^2}{5s^2+1}$$

$$h_{21} = \frac{-5s^2}{5s^2+1}$$

ii) $I_1 = 0 \Rightarrow$ open circuit



$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$

$$V_2 = I_2 Z_{eq}$$

$$V_2 = I_2 \left(5s + \frac{1}{s}\right) = I_2\left(\frac{5s^2+1}{s}\right)$$

$$V_1 = 5s\, I_2$$

$$\therefore h_{12} = \frac{5s \times s}{(5s^2+1)}$$

$$h_{12} = \frac{5s^2}{5s^2+1}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$

$$h_{22} = \frac{I_2}{I_2\left(\frac{5s^2+1}{s}\right)}$$

$$h_{22} = \frac{s}{5s^2+1}$$

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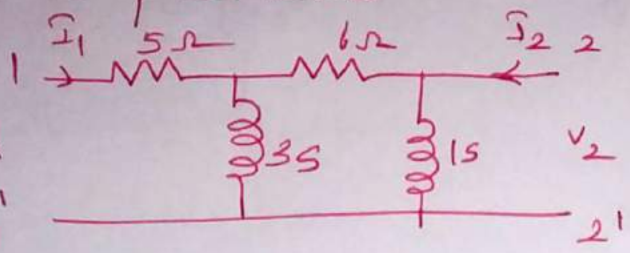
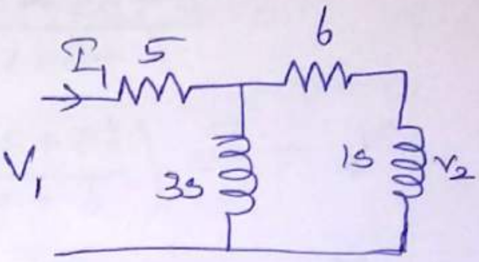
20. obtain z parameters



solution:

i) $I_2 = 0$



$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$V_1 = I_1 Z_{eq}$$

$$= I_1 \left(\left[(6+1s) \| 3s \right] + 5 \right)$$

$$= I_1 \left(\frac{6+1s \times 3s}{6+1s+3s} + 5 \right)$$

$$V_1 = I_1 \left(\frac{3s^2+48s+30}{4s+6} \right)$$

$$Z_{11} = \frac{3s^2+48s+30}{4s+6} \ \Omega$$

$$Z_{21} = \frac{V_2}{I_1}\bigg| I_2 = 0$$

$$-I_x = I_1 \times \frac{3s}{3s+6+5} = I_1 \frac{3s}{6+4s}$$

$$V_2 = s\left(-I_1 \frac{3s}{4s+6}\right)$$

$$V_2 = \frac{-3s^2}{4s+6} I_1$$

$$Z_{21} = \frac{-3s^2}{4s+6} \ \Omega$$

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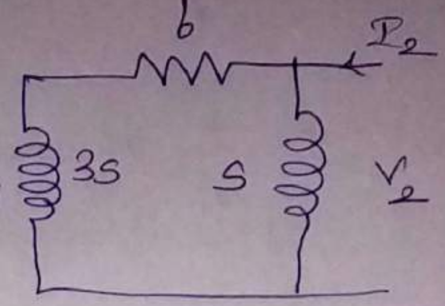
ii) $I_1 = 0$



$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$V_2 = I_2 Z_{eq}$$

$$Z_{eq} = (6+3s) \parallel s$$

$$= \frac{6+3s \times s}{6+3s+s} = \frac{6s+3s^2}{6+4s}$$

$$V_2 = I_2 \left(\frac{6s+3s^2}{6+4s}\right)$$

$$Z_{22} = \frac{6s+3s^2}{6+4s}\ \Omega$$

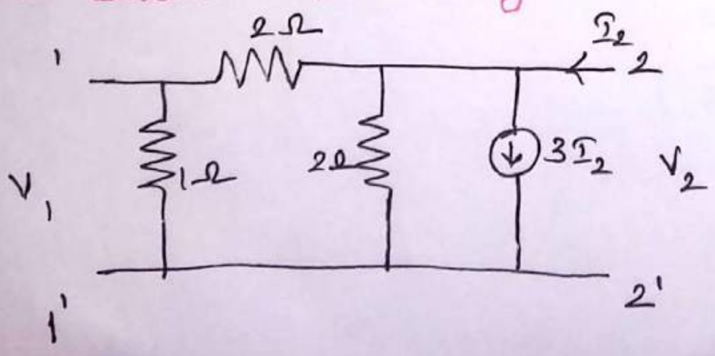
$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$I_y = -I_2 \times \frac{s}{3s+6+s} = -I_2 \frac{s}{6+4s}$$

$$V_1 = I_y \times 3s = -I_2 \frac{3s^2}{6+4s}$$

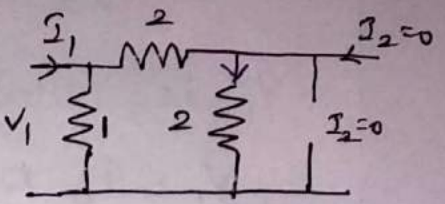
$$\therefore Z_{12} = \frac{-3s^2}{6+4s}\ \Omega$$

21. Determine the open circuit impedance parameters for the network shown in fig.



Solution:

i) Z_{11} & Z_{21} $(I_2 = 0)$



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

Apply ohm's law

$$V_1 = I_1 Z_{eq}$$

$$Z_{eq} = (2+2) \| 1 = \frac{4}{5}$$

$$V_1 = I_1 \frac{4}{5}$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{4}{5}\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

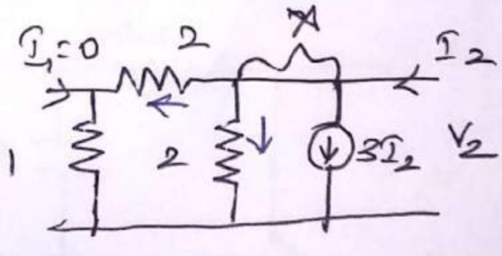
V_2 is across 2Ω resistor.

$$V_2 = I_x R_{2\Omega}$$

$$I_x = I_1 \times \frac{1}{5} = \frac{I_1}{5}$$

$$V_2 = \frac{2 I_1}{5}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{2}{5}\ \Omega$$



ii) Z_{22} & Z_{12} $(I_1 = 0)$

KCL at node x,

$$\frac{V_2}{2+1} + \frac{V_2}{2} + 3I_2 = I_2$$

$$\frac{5}{6} V_2 = -2 I_2$$

$$\frac{V_2}{I_2} = \frac{-12}{5}\ \Omega$$

$$\therefore Z_{22} = \frac{V_2}{I_2} = \frac{-12}{5}\ \Omega$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

V_1 is voltage across $1\,\Omega$ resistor.

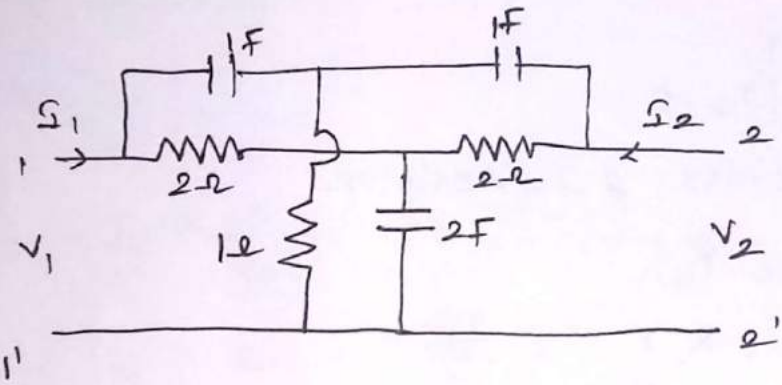
$$V_1 = I_y R_{1\Omega}$$

$$I_y = I_2 \times \frac{2}{5} = \frac{2I_2}{5}$$

$$V_1 = \frac{2I_2}{5}$$

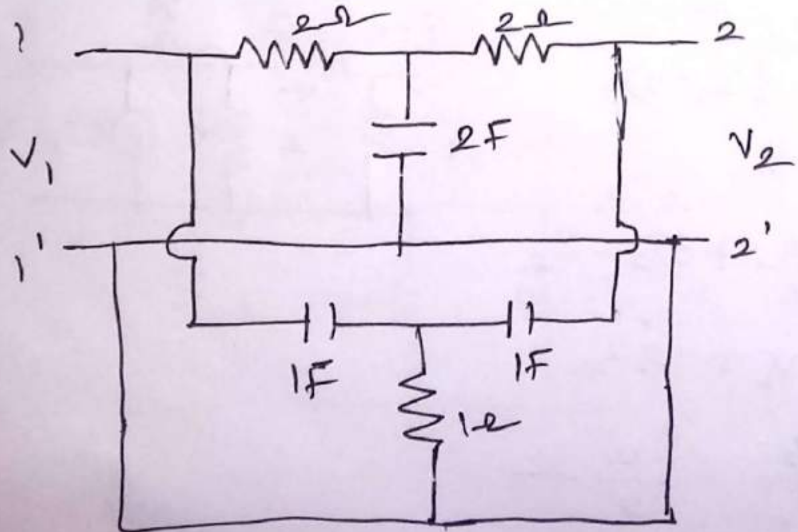
$$Z_{12} = \frac{V_1}{I_2} = \frac{2}{5}\,\Omega$$

22. find the short circuit parameters

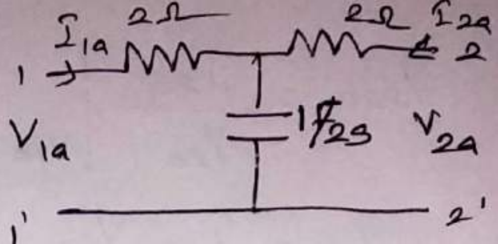


Solution:

Let us split the given network in parallel connection of two networks



a) for network a



$$Y_{11a} = \left.\frac{I_{1a}}{V_{1a}}\right|_{V_{2a}=0}$$

Apply ohms law

$$V_{1a} = I_{1a}\, Y_{eq}$$

$$Y_{eq} = 2 + \left(2 \,\|\, \frac{1}{2s}\right)$$

$$= \frac{8s+4}{4s+1}$$

$$Z_{11a} = \frac{V_{1a}}{I_{1a}} = \frac{8s+4}{4s+1}$$

$$Y_{11a} = \frac{I_{1a}}{V_{1a}} = \frac{4s+1}{8s+4}$$

$$Y_{12a} = \left.\frac{I_{2a}}{V_{1a}}\right|_{V_{2a}=0}$$

Apply current division Rule

$$-I_{2a} = I_{1a} \times \frac{1/2s}{2+\frac{1}{2s}}$$

$$-I_{2a} = \frac{1}{4s+1}\, I_{1a}$$

$$I_{1a} = \left(\frac{4s+1}{8s+4}\right) V_{1a}$$

$$\therefore \quad -I_{2a} = \frac{4s+1}{(4s+1)(8s+4)}\, V_{1a}$$

$$\therefore\ Y_{12a} = \frac{I_{2a}}{V_{1a}} = \frac{-1}{8s+4}$$

$$Y_{22a} = \frac{I_{2a}}{Y_{2a}} \Big|_{V_{1a}=0}$$

$$V_{2a} = I_{2a}\, Y_{eq}$$

$$Y_{eq} = \left(2 \,\|\, \frac{1}{2s}\right) + 2$$

$$= \frac{8s+4}{4s+1}$$

$$V_{2a} = I_{2a}\left(\frac{8s+4}{4s+1}\right)$$

$$Y_{22a} = \frac{I_{2a}}{V_{2a}} = \frac{4s+1}{8s+4}$$

$$Y_{21a} = \frac{I_{1a}}{V_{2a}} \Big|_{V_{1a}=0}$$

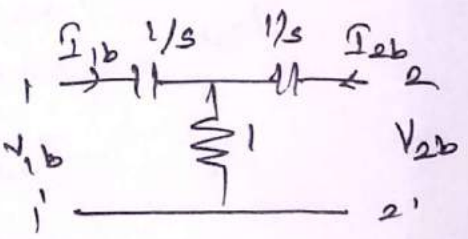
$$-I_{1a} = I_{2a} \times \frac{\frac{1}{2s}}{2+\frac{1}{2s}}$$

$$= \frac{1}{4s+1} I_{2a}$$

$$I_{2a} = \frac{4s+1}{8s+4} V_{2a}$$

$$Y_{21a} = \frac{I_{1a}}{Y_{2a}} = \frac{-1}{8s+4}$$

b) for network b:



$$Y_{11b} = \frac{I_{1b}}{V_{1b}} \Big|_{V_{2b}=0}$$

$$V_{1b} = I_{1b}\, Y_{eq}$$

$$Y_{eq} = \left(1 \,\|\, \frac{1}{s}\right) + \frac{1}{s}$$

$$= \frac{2s+1}{s(s+1)}$$

$$Y_{11b} = \frac{I_{1b}}{V_{1b}} = \frac{s(s+1)}{2s+1}$$

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Apply current division rule

$$-I_{2b} = I_{1b} \times \frac{1}{\frac{1}{s}+1}$$

$$= \frac{s}{s+1} I_{1b}$$

$$I_{1b} = \frac{s(s+1)}{2s+1} V_{1b}$$

$$-I_{2b} = \frac{s}{s+1} \times \frac{s(s+1)}{2s+1} V_{1b}$$

$$Y_{21b} = \frac{I_{2b}}{V_{1b}} = -\frac{s^2}{2s+1}$$

$$Y_{22b} = \frac{I_{2b}}{V_{2b}}\bigg|_{V_{1b}=0}$$

$$V_{2b} = I_{2b}\, Y_{eq}$$

$$Y_{eq} = \left[1 \,\|\, \frac{1}{s}\right] + \frac{1}{s}$$

$$= \frac{2s+1}{s(s+1)}$$

$$Y_{22b} = \frac{I_{2b}}{V_{2b}} = \frac{s(s+1)}{2s+1}$$

$$Y_{12b} = \frac{I_{1b}}{V_{2b}}\bigg|_{V_{1b}=0}$$

$$-I_{1b} = I_{2b} \times \frac{1}{\frac{1}{s}+1}$$

$$-I_{1b} = \frac{s}{s+1} V_{2b}$$

$$I_{2b} = \frac{s(s+1)}{2s+1} V_{2b}$$

$$Y_{12b} = \frac{I_{1b}}{V_{2b}} = \frac{-s^2}{2s+1}$$

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$$\begin{bmatrix} Y_{11a} & Y_{21a} \\ Y_{12a} & Y_{22a} \end{bmatrix} = \begin{bmatrix} \frac{4s+1}{8s+4} & \frac{-1}{8s+4} \\ \frac{-1}{8s+4} & \frac{4s+1}{8s+4} \end{bmatrix}$$

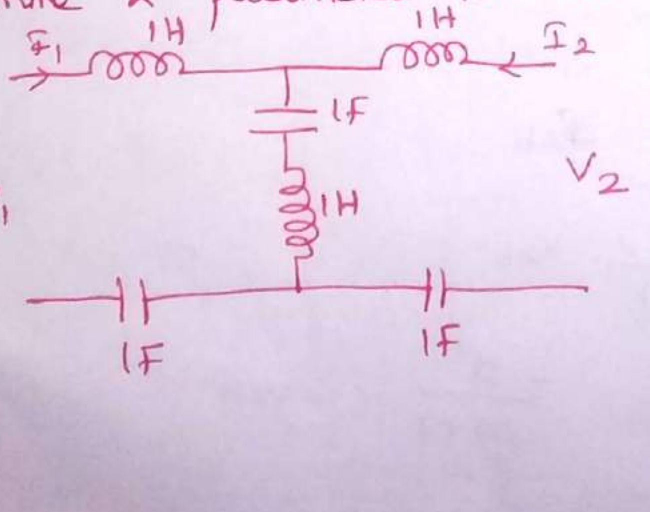
$$\begin{bmatrix} Y_{11b} & Y_{21b} \\ Y_{12b} & Y_{22b} \end{bmatrix} = \begin{bmatrix} \frac{s(s+1)}{2s+1} & \frac{-s^2}{2s+1} \\ \frac{-s^2}{2s+1} & \frac{s(s+1)}{2s+1} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{21a} + Y_{21b} \\ Y_{12a} + Y_{12b} & Y_{22a} + Y_{22b} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4s+1}{8s+4} + \frac{s(s+1)}{2s+1} & \frac{-1}{8s+4} - \frac{s^2}{2s+1} \\ \frac{-1}{8s+4} - \frac{s^2}{2s+1} & \frac{4s+1}{8s+4} + \frac{s(s+1)}{2s+1} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{21} \\ Y_{12} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{4s^2+8s+1}{4(2s+1)} & -\frac{4s^2+1}{4(2s+1)} \\ -\frac{4s^2+1}{4(2s+1)} & \frac{4s^2+8s+1}{4(2s+1)} \end{bmatrix}$$

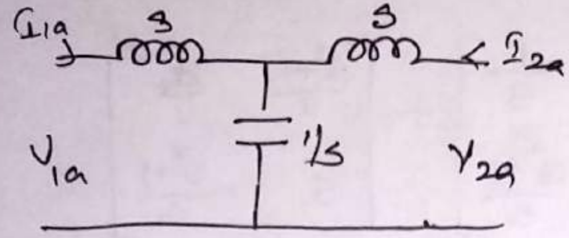
23. Determine Z parameters for the network



Solution:

The given network can be splitted into two individual networks.

network a



The network is in the form of T,

$Z_{11} = Z_a + Z_c$

$Z_{12} = Z_c$

$Z_{21} = Z_c$

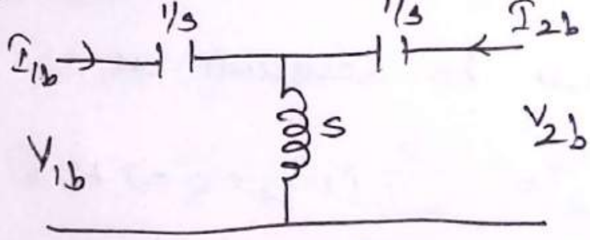
$Z_{22} = Z_b + Z_c$

$Z_{11a} = s + \frac{1}{s} = \frac{s^2+1}{s}$

$Z_{12a} = Z_{21a} = \frac{1}{s}$

$Z_{22a} = s + \frac{1}{s} = \frac{s^2+1}{s}$

network b :



$Z_{11b} = \frac{1}{s} + s = \frac{s^2+1}{s}$

$Z_{12b} = Z_{21b} = s$

$Z_{22b} = s + \frac{1}{s} = \frac{s^2+1}{s}$

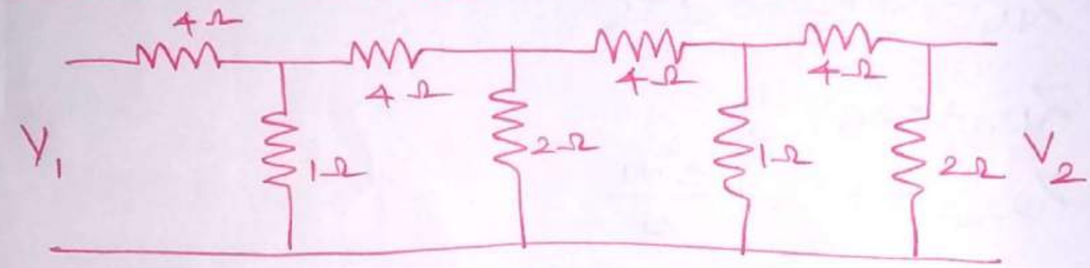
Z parameters for overall network is

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} + \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s^2+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{s^2+1}{s} \end{bmatrix} + \begin{bmatrix} \frac{s^2+1}{s} & s \\ s & \frac{s^2+1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2(s^2+1)}{s} & s+\frac{1}{s} \\ s+\frac{1}{s} & \frac{2(s^2+1)}{s} \end{bmatrix}$$

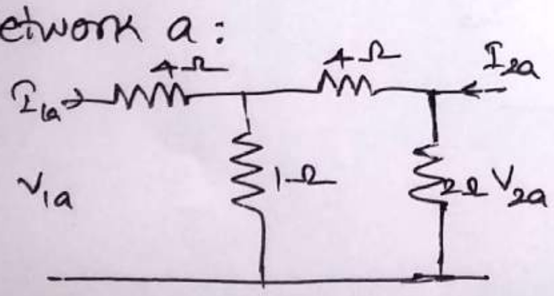
24. Determine ABCD parameters of 2 networks connected in cascade as shown.



solution:

The network can be redrawn as

network a:



i) $I_2 = 0 \Rightarrow A$ & C

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

Apply ohm's law

$$V_{1a} = I_{1a} Z_{eqa}$$

$$Z_{eqa} = ((4+2)\|1)+4 = \frac{6}{7}+4 = \frac{34}{7}$$

$$V_{1a} = I_{1a}\frac{34}{7}$$

Scanned by CamScanner

Apply current division rule

$V_{2a} = I_{2a} R_{2\Omega}$

$I_{2a} = I_{1a} \times \frac{1}{7} = \frac{I_1}{7}$

$V_2 = \frac{2 I_{1a}}{7}$

$A = \frac{V_{1a}}{V_{2a}} = \frac{34 I_{1a}}{7} \times \frac{7}{2 I_{1a}}$

$A = 17$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_{1a}}{2 I_{1a}} \times 7$

$C = \frac{7}{2}$

ii) $V_2 = 0 \Rightarrow B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$

Current division rule

$-I_{2a} = I_{1a} \times \frac{1}{5}$

Apply ohm's law

$V_{1a} = I_{1a} Z_{eqa}$

$Z_{eqa} = (4 \| 1) + 4 = \frac{4}{5} + 4 = \frac{24}{5}$

$V_{1a} = \frac{24}{5} I_{1a}$

$B = -\frac{24 I_{1a}}{5} \times \frac{5}{-I_{1a}} = 24$

$B = 24$

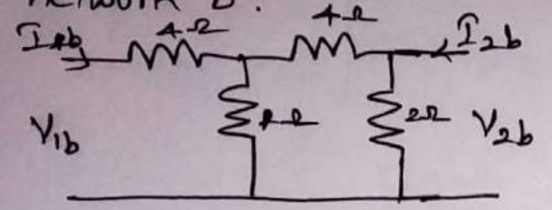
$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$

from previous step,

$D = \frac{-I_{1a}}{-I_{1a} \times 1/5} = 5$

$$\therefore \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} = \begin{bmatrix} 17 & 24 \\ 7/2 & 5 \end{bmatrix}$$

network b :



network b is similar to network a.

So the network b takes the same value of network a.

i) $I_2 = 0$

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$

ohm's law

$V_{1b} = I_{1b} Z_{eqb}$

$Z_{eqb} = ((4+2)||1) + 4$

$= \frac{6}{7} + 4 = \frac{34}{7}$

$A = \frac{34}{7}$

$$\therefore \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} = \begin{bmatrix} 17 & 24 \\ 7/2 & 5 \end{bmatrix}$$

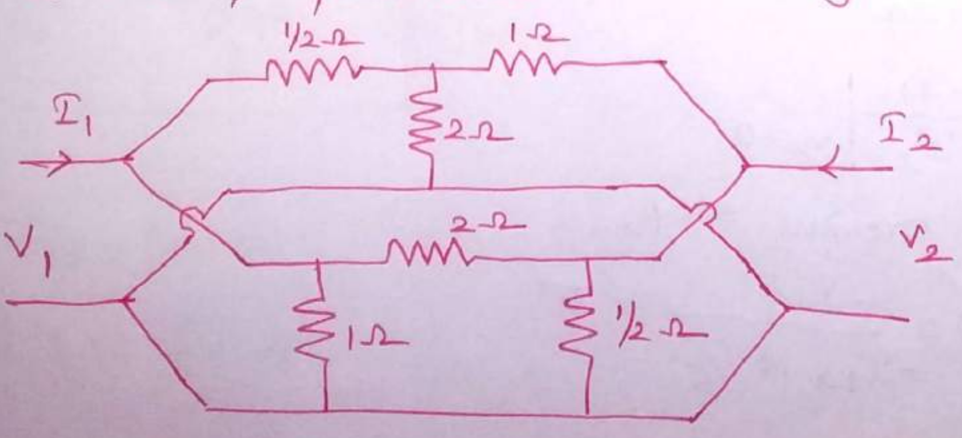
The overall network parameter is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 24 \\ 7/2 & 5 \end{bmatrix} * \begin{bmatrix} 17 & 24 \\ 7/2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 373 & 528 \\ 77 & 109 \end{bmatrix}$$

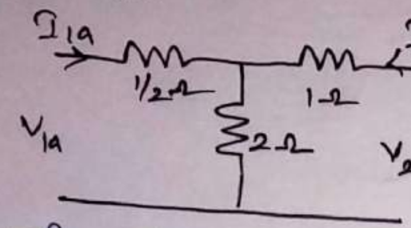
25. find the y parameters for the given network.



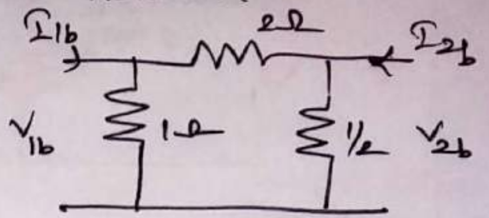
Solution :

The given network is redrawn as

network a



network b



The given is a T network

$$Z_{11a} = \frac{1}{2} + 2 = \frac{5}{2}\,\Omega$$

$$Z_{12a} = Z_{21a} = 2\,\Omega$$

$$Z_{22a} = 2 + 1 = 3\,\Omega$$

$$\Delta Z = \begin{bmatrix} \frac{5}{2} & 2 \\ 2 & 3 \end{bmatrix} = \frac{7}{2}\,\Omega$$

using interrelationship,

$$Y_{11a} = \frac{Z_{22a}}{\Delta Z} = \frac{3}{7/2} = \frac{6}{7}\,\mho$$

$$Y_{12a} = Y_{2a} = \frac{-Z_{21a}}{\Delta Z} = \frac{-2}{7/2} = \frac{-4}{7}\,\mho$$

$$Y_{22a} = \frac{Z_{11a}}{\Delta Z} = \frac{5/2}{7/2} = \frac{5}{7}\,\mho$$

The given is a π- network

$$Y_{11b} = Y_1 + Y_2$$

$$Y_{21b} = Y_{12b} = -Y_2$$

$$Y_{22b} = Y_2 + Y_3$$

$$Y_{11b} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}\,\mho$$

$$Y_{12b} = Y_{21b} = -Y_2 = \frac{-1}{2}\,\mho$$

$$Y_{22b} = Y_2 + Y_3$$

$$= \frac{1}{2} + 2 = \frac{5}{2}\,\mho$$

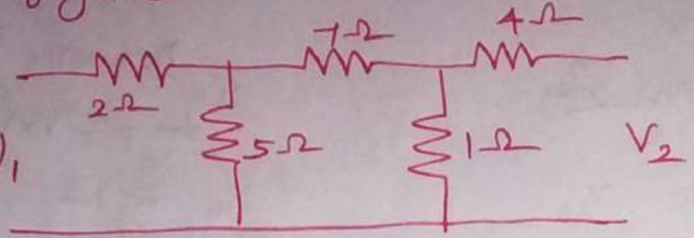
∴ Y parameters for overall network is

$$Y = Y_a + Y_b$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 6/7 & -4/7 \\ -4/7 & 5/7 \end{bmatrix} + \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

$$= \begin{bmatrix} 2.35 & -1.07 \\ -1.07 & 3.21 \end{bmatrix}$$

26. Obtain T network and ABCD parameters for the figure.



Solution:

i) Obtain Z parameters for the given network by $I_1 = 0$ & $I_2 = 0$

The answer is

$$Z_{11} = 66/13\ \Omega,\quad Z_{12} = Z_{21} = \frac{5}{13}\ \Omega,$$

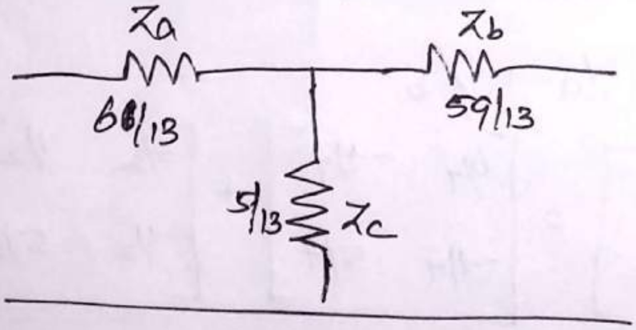
$$Z_{22} = \frac{64}{13}\ \Omega$$

ii) T network parameters

$$Z_a = Z_{11} - Z_{21} = \frac{66}{13} - \frac{5}{13} = \frac{61}{13} = 4.7\ \Omega$$

$$Z_b = Z_{22} - Z_{12} = \frac{64}{13} - \frac{5}{13} = \frac{59}{13} = 4.5\ \Omega$$

$$Z_c = Z_{12} = Z_{21} = \frac{5}{13} = 0.38\ \Omega$$



iii) ABCD parameters (It is from T parameters)

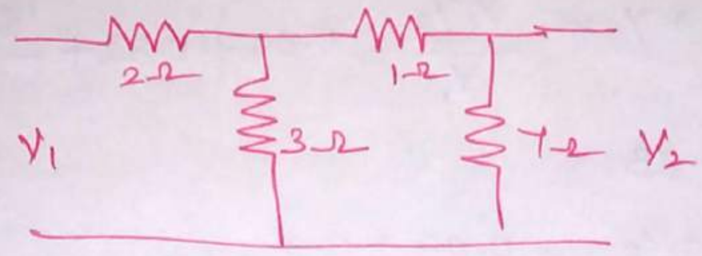
$$A = \frac{Z_a + Z_c}{Z_c} = \frac{61/13 + 5/13}{5/13} = \frac{66}{5}$$

$$B = \frac{Z_b Z_a}{Z_c} + Z_a + Z_b = \frac{\left(\frac{59}{13}\right)(61/13)}{5/13} + \frac{61}{13} + \frac{59}{13} = 65.3$$

$$C = \frac{1}{Z_c} = \frac{13}{5}$$

$$D = \frac{Z_b + Z_c}{Z_c} = \frac{59/13 + 5/13}{5/13} = \frac{64}{5}$$

27. find the π Equivalent and ABCD parameters



Solution:

i) obtain y parameters for the given network by making $V_1 = 0$ & $V_2 = 0$

$$Y_{11} = \frac{4}{11} \mho, \quad Y_{12} = \cancel{Y_{21}} = \frac{-23}{77} \mho$$

$$Y_{21} = \frac{-3}{11} \mho, \quad Y_{22} = \frac{46}{77} \mho$$

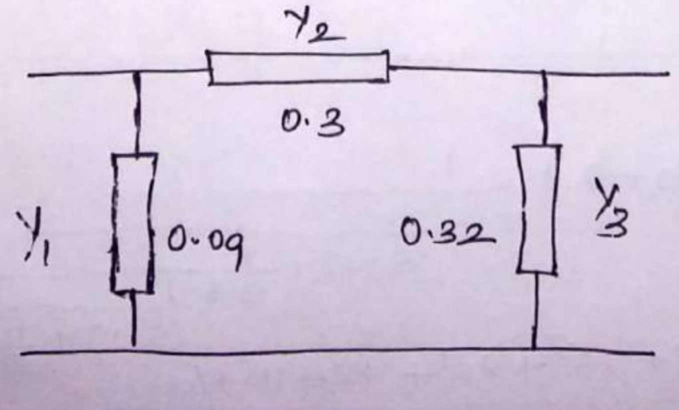
ii) obtain π network parameters:

It is from y parameters

$$Y_1 = Y_{11} + Y_{21} = \frac{4}{11} - \frac{3}{11} = \frac{1}{11} = 0.09 \mho$$

$$Y_2 = -Y_{12} = \frac{23}{77} = 0.3 \mho$$

$$Y_3 = Y_{22} + Y_{21} = \frac{46}{77} - \frac{3}{11} = \frac{25}{77} = 0.32 \mho$$



iii) ABCD parameters

It is obtain from π Network parameters

$$A = \frac{Y_3 + Y_2}{Y_2} = \frac{0.32 + 0.3}{0.3} = 2.06$$

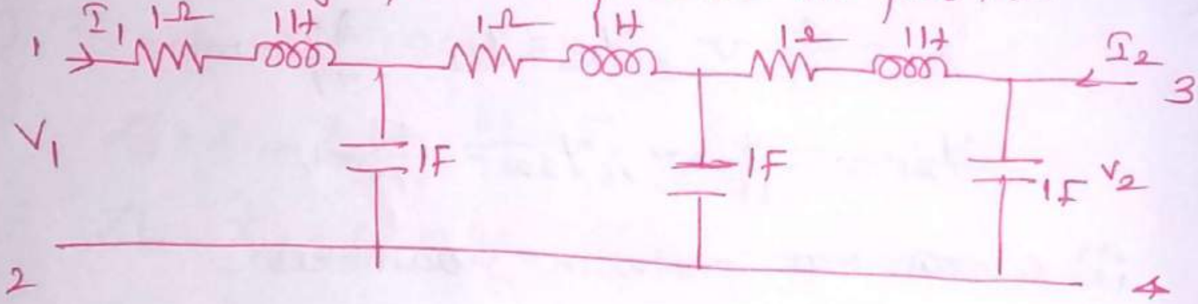
$$B = \frac{1}{Y_2} = \frac{1}{0.3} = 3.33$$

$$C = Y_1 + Y_3 + \frac{Y_1 Y_3}{Y_1} = 0.09 + 0.32 + \frac{(0.09 \times 0.32)}{0.09}$$

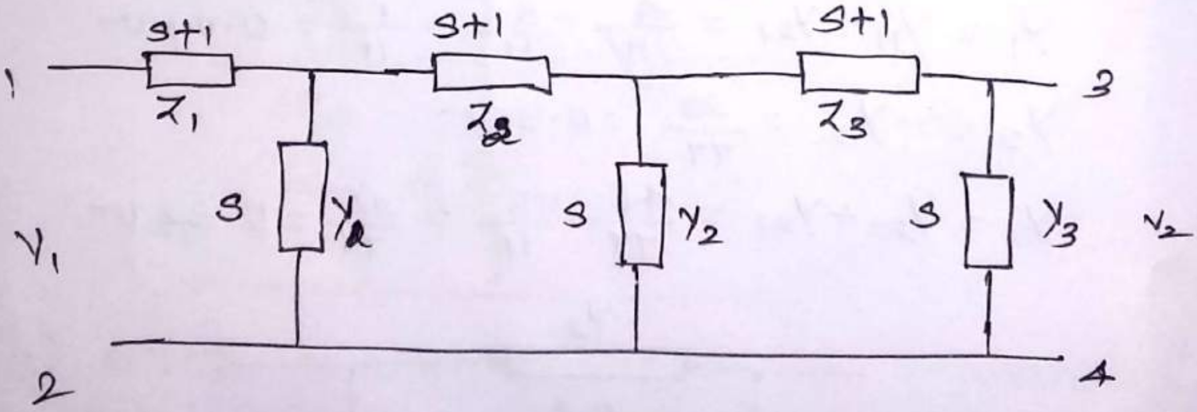
$$C = 0.73$$

$$D = \frac{Y_1 + Y_2}{Y_2} = \frac{0.09 + 0.3}{0.3} = 1.3$$

28. for the ladder shown in figure. obtain open circuit driving point impedance at port 1-2.



solution:



$$Z_{11} = (s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \frac{1}{s}}}}}$$

$$Z_{11} = \frac{s^6 + 3s^5 + 8s^4 + 11s^3 + 11s^2 + 6s + 1}{s^5 + 2s^4 + 5s^3 + 4s^2 + 3s}$$