4

Electromagnetic Induction

4.1 Introduction

Uptill now we have discussed the basic properties, concepts of magnetism and magnetic circuits. Similarly we have studied, the magnetic effects of an electric current. But we have not seen the generation of e.m.f. with the help of magnetism. The e.m.f. can be generated by different ways, by chemical action, by heating thermocouples etc. But the most popular and extensively used method of generating an e.m.f. is based on electromagnetism.

After the magnetic effects of an electric current, attempts were made to produce electric current with the help of magnetism rather than getting magnetism due to current carrying conductor. In 1831, an English Physicist, **Michael Faraday** succeeded in getting e.m.f. from magnetic flux. The phenomenon by which e.m.f. is obtained from flux is called **electromagnetic induction**. Let us discuss, what is electromagnetic induction and its effect on the electrical engineering branch, in brief.

4.2 Faraday's Experiment

Let us study first the experiment conducted by **Faraday** to get understanding of electromagnetic induction.

Consider a coil having 'N' turns connected to a galvanometer as shown in the Fig. 4.1. Galvanometer indicates flow of current in the circuit, if any. A permanent magnet is moved relative to coil, such that magnetic lines of force associated with coil get changed. Whenever, there is motion of permanent magnet, galvanometer deflects indicating flow of current through the circuit.

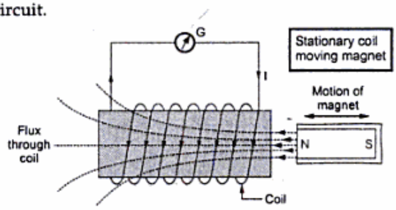


Fig. 4.1 Faraday's experiment

Key Point : *The galvanometer deflects in one direction, when magnet is moved towards a coil. It deflects in other direction, when moved away from the coil.*

The deflection continues as long as motion of magnet exists. More quickly the magnet is moved, the greater is the deflection. Now deflection of galvanometer indicates flow of current. But to exist flow of current there must be presence of e.m.f. Hence such movement of flux lines with respect to coil generates an e.m.f. which drives current through the coil. This is the situation where coil in which e.m.f. is generated is fixed and magnet is moved to create relative motion of flux with respect to coil.

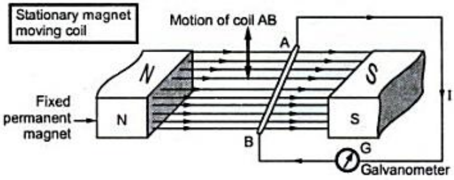


Fig. 4.2 Another form of Faraday's experiment

Similar observations can be made by moving a coil in the magnetic field of fixed permanent magnet, creating relative motion between flux and coil. This arrangement is shown in the Fig. 4.2. The coil AB is moved by some external means in the magnetic field of fixed permanent magnet. Coil is connected to galvanometer.

Whenever conductor AB is moved in the direction shown in the Fig. 4.2 the galvanometer deflects indicating flow of current through coil AB.

Key Point: *The deflection is on one side when conductor is moved up. While it is in other direction, when it is moved down.*

Similarly greater is the deflection if conductor is moved quickly in magnetic field.

In both cases, basically there is **change of flux lines with respect to conductor** i.e there is cutting of the flux lines by the conductor in which e.m.f. induced.

With this experiment Faraday stated laws called **Faraday's Laws of Electromagnetic Induction.**

This phenomenon of cutting of flux lines by the conductor to get the induced e.m.f. in the conductor or coil is called ***electromagnetic induction.***

Thus, to have induced e.m.f. there must exist,

1) A coil or conductor.
2) A magnetic field (permanent magnet or electromagnet).
3) Relative motion between conductor and magnetic flux (achieved by moving conductor with respect to flux or moving with respect to conductor.)

Key Point: *The e.m.f. exists as long as relative motion persists.*

4.3 Faraday's Laws of Electromagnetic Induction

From the experiment discussed above, Michael Faraday a British scientist stated two laws of electromagnetic induction.

4.3.1 First Law

Whenever the number of magnetic lines of force (flux) linking with a coil or circuit changes, an e.m.f. gets induced in that coil or circuit.

4.3.2 Second Law

The magnitude of the induced e.m.f. is directly proportional to the rate of change of flux linkages (flux × turns of coil).

Flux linkages = Flux × Number of turns of coil

The law can be explained as below.

Consider a coil having N turns. The initial flux linking with a coil is ϕ_1

$$\therefore \quad \text{Initial flux linkages} = N\phi_1$$

In time interval t, the flux linking with the coil changes from ϕ_1 to ϕ_2.

$$\therefore \quad \text{Final flux linkages} = N\phi_2$$

$$\therefore \quad \text{Rate of change of flux linkages} = \frac{N\phi_2 - N\phi_1}{t}$$

Now as per the first law, e.m.f. will get induced in the coil and as per second law the magnitude of e.m.f. is proportional to the rate of change of flux linkages.

$$\therefore \quad e \propto \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \quad e = K \times \frac{N\phi_2 - N\phi_1}{t}$$

$$\therefore \quad e = N\frac{d\phi}{dt}$$

With K as unity to get units of e as volts, $d\phi$ is change in flux, dt is change in time hence $(d\phi / dt)$ is rate of change of flux.

Now as per Lenz's law (discussed later), the induced e.m.f. sets up a current in such a direction so as to oppose the very cause producing it. Mathematically this opposition is expressed by a negative sign.

Thus such an induced e.m.f. is mathematically expressed alongwith its sign as,

$$\therefore \quad \boxed{e = -N\frac{d\phi}{dt} \quad \text{volts}}$$

4.4 Nature of the Induced E.M.F.

E.M.F. gets induced in a conductor, whenever there exists change in flux with that conductor, according to Faraday's Law. Such change in flux can be brought about by different methods.

Depending upon the nature of methods, the induced e.m.f. is classified as,

1) Dynamically induced e.m.f. and
2) Statically induced e.m.f.

4.5 Dynamically Induced E.M.F.

The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux. Both these methods are discussed earlier in discussion of Faraday's experiment .

Key Point : *Such an induced e.m.f. which is due to physical movement of coil, conductor with respect to flux or movement of magnet with respect to stationary coil, conductor is called **dynamically induced e.m.f. or motional induced e.m.f.***

4.5.1 Magnitude of Dynamically Induced E.M.F.

Consider a conductor of length l metres moving in the air gap between the poles of the magnet.

If plane of the motion of the conductor is parallel to the plane of the magnetic field then there is no cutting of flux lines and there can not be any induced e.m.f. in the conductor such condition is shown in the Fig. 4.3(a).

Key Point : *When plane of the flux is parallel to the plane of the motion of conductors then there is no cutting of flux, hence no induced e.m.f.*

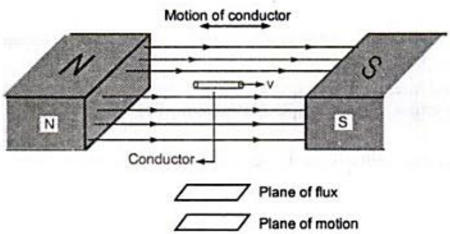


Fig. 4.3 (a) No cutting of flux

In second case as shown in the Fig. 4.3(b), the velocity direction i.e. motion of conductor is perpendicular to the flux. Hence whole length of conductor is cutting the flux line hence there is maximum possible induced e.m.f. in the conductor. **Under such condition plane of flux and plane of motion are perpendicular to each other.**

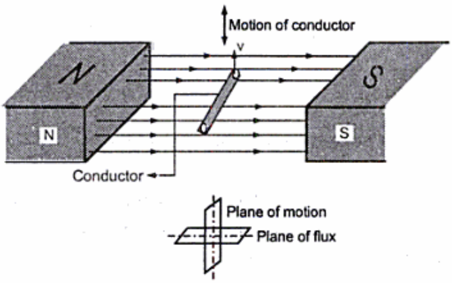
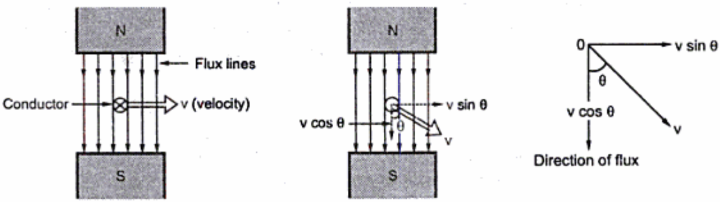


Fig. 4.3 (b) Maximum cutting of flux

Key Point : *When plane of the flux is perpendicular to the plane of the motion of the conductors then the cutting of flux is maximum and hence induced e.m.f. is also maximum.*

Consider a conductor moving with velocity v m/s such that its plane of motion or direction of velocity is perpendicular to the direction of flux lines as shown in Fig. 4.4 (a).



(a) **(b)** **(c)**

Fig. 4.4

B = Flux density in Wb/m^2

l = Active length of conductor in metres.

(This is the length of conductor which is actually responsible for cutting of flux lines.)

v = Velocity in m/sec.

Let this conductor is moved through distance dx in a small time interval dt, then

$$\text{Area swept by conductor} = l \times dx \quad m^2$$

$$\therefore \quad \text{Flux cut by conductor} = \text{Flux density} \times \text{Area swept}$$

$$d\phi = B \times l \times dx \quad Wb$$

According to Faraday's law, magnitude of induced e.m.f. is proportional to the rate of change of flux.

$$\therefore \quad e = \frac{\text{Flux cut}}{\text{Time}}$$

$$= \frac{d\phi}{dt} \quad \text{[Here N = 1 as single conductor]}$$

$$= \frac{B\, l\, dx}{dt}$$

But $\frac{dx}{dt}$ = Rate of change of displacement

= Velocity of the conductor

= v

$$\therefore \quad e = B\, l\, v \quad \text{volts}$$

This is the induced e.m.f. when plane of motion is exactly perpendicular to the plane of flux. This is maximum possible e.m.f. as plane of motion is at right angles to plane of the flux.

But if conductor is moving with a velocity v but at a certain angle θ measured with respect to direction of the field (plane of the flux) as shown in the Fig. 4.4 (b) then component of velocity which is $v \sin \theta$ is perpendicular to the direction of flux and hence responsible for the induced e.m.f.. The other component $v \cos \theta$ is parallel to the plane of the flux and hence will not contribute to the dynamically induced e.m.f.

Under this condition magnitude of induced e.m.f. is given by,

$$e = B\, l\, v \sin\theta \quad \text{volts}$$

where θ is measured with respect to plane of the flux.

Example 4.1 : *A conductor of 2 m length moves with a uniform velocity of 1.27 m/sec under a magnetic field having a flux density of 1.2 Wb/m² (tesla). Calculate the magnitude of induced e.m.f. if conductor moves,*

i) at right angles to axis of field.

ii) at an angle of 60° to the direction of field.

Solution : i) The magnitude of induced e.m.f.

$$e = B\, l\, v \quad \text{for } \theta = 90°$$

$$\therefore \quad e = 1.2 \times 2 \times 1.27 = \mathbf{3.048} \text{ volts}$$

ii) $e = B\, l\, v \sin\theta$ where $\theta = 60°$

$$e = 1.2 \times 2 \times 1.27 \times \sin 60 = \mathbf{2.6397} \text{ volts.}$$

Example 4.2 : *A coil carries 200 turns gives rise a flux of 500 μWb when carrying a certain current. If this current is reversed in $\frac{1}{10}$ th of a second. Find the average e.m.f. induced in the coil.*

Solution : The magnitude of induced e.m.f. is,

$$= N\frac{d\phi}{dt}$$

where $d\phi$ is change in flux linkages i.e. change in $N\phi$. Now in this problem flux is 500×10^{-6} for given current. After reversing this current, flux will reverse its direction. So flux becomes (-500×10^{-6}).

$$\therefore \quad d\phi = \phi_2 - \phi_1 = -500\times10^{-6} - (+500\times10^{-6}).$$

This happens in time dt = 0.1 sec.

$$\therefore \quad \text{Average e.m.f.} = -N\frac{d\phi}{dt} = -200\times\frac{(-1\times10^{3})}{0.1} = \textbf{2 volts}$$

4.5.2 Direction of Dynamically Induced E.M.F.

The direction of induced e.m.f. can be decided by using two rules.

1) Fleming's Right Hand Rule

As discussed earlier, the Fleming's Left Hand Rule is used to get direction of force experienced by conductor carrying current, placed in a magnetic field while Fleming's Right Hand Rule is to be used to get direction of induced e.m.f. when conductor is moving in a magnetic field.

According to Fleming's right hand rule, outstretch the three fingers of right hand namely the thumb, fore finger and the middle finger, perpendicular to each other. Arrange the right hand so that first finger point in the direction of flux lines (from N to S) and thumb in the direction of motion of conductor with respect to the flux then the middle finger will point in the direction of the induced e.m.f. (or current).

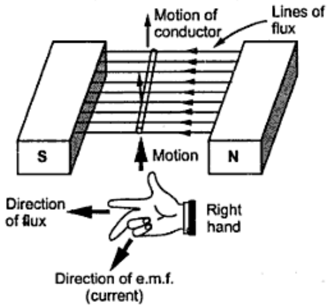


Fig. 4.5 (a)

Consider the conductor moving in a magnetic field as shown in the Fig. 4.5 (a). It can be verified using Fleming's right hand rule that the direction of the current due to the induced e.m.f. is coming out. Symbolically this is shown in the Fig. 4.5 (b).

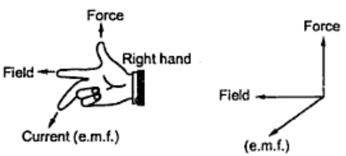


Fig. 4.5 (b)

> **Key Point** : *In practice though magnet is moved keeping the conductor stationary, while application of rule, thumb should point in the direction of relative motion of conductor with respect to flux, assuming the flux stationary.*

This rule mainly gives direction of current which induced e.m.f. in conductor will set up when closed path is provided to it.

Verify the direction of the current through conductor in the four cases shown in the Fig. 4.6 by the use of Fleming's right hand rule.

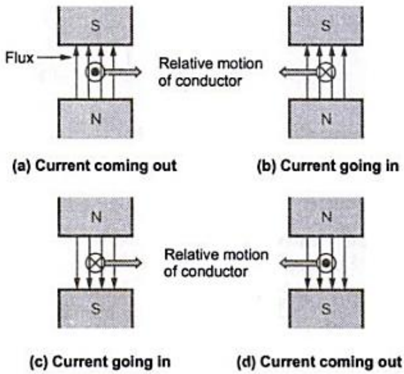


Fig. 4.6 Verifying Fleming's right hand rule

2) Lenz's Law

This rule is based on the principles derived by German Physicist Heinrich Lenz.

The Lenz's law states that, '**The direction of an induced e.m.f. produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the e.m.f.**'

In short the induced e.m.f. always opposes the cause producing it, which is represented by a negative sign, mathematically in its expression.

$$\therefore \quad e = -N\frac{d\phi}{dt}$$

The explanation can be given as below :

Consider a solenoid as shown in the Fig. 4.7. Let a bar magnet is moved towards coil such that N-pole of magnet is facing a coil which will circulate the current through the coil.

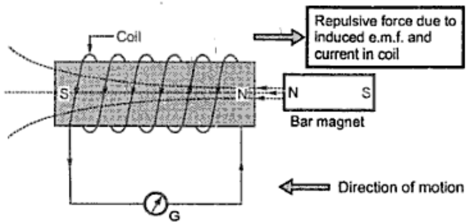


Fig. 4.7 Lenz's law

According to Lenz's Law, the direction of current due to induced e.m.f. is so as to oppose the cause. The cause is motion of bar magnet towards coil. So e.m.f. will set up a current through coil in such a way that the end of solenoid facing bar magnet will become N-pole. Hence two like poles will face each other experiencing force of repulsion which is opposite to the motion of bar magnet as shown in the Fig. 4.7.

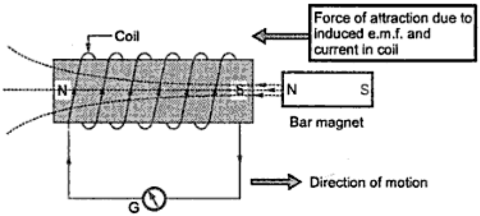


Fig. 4.8 Lenz's law

If the same bar magnet is moved away from the coil, then induced e.m.f. will set up a current in the direction which will cause, the end of solenoid facing bar magnet to behave as S-pole. Because of this two unlike poles face each other and there will be force of attraction which is direction of magnet, away from the coil. The galvanometer shows deflection in other direction as shown in the Fig. 4.8.

The Lenz's law can be summarized as,

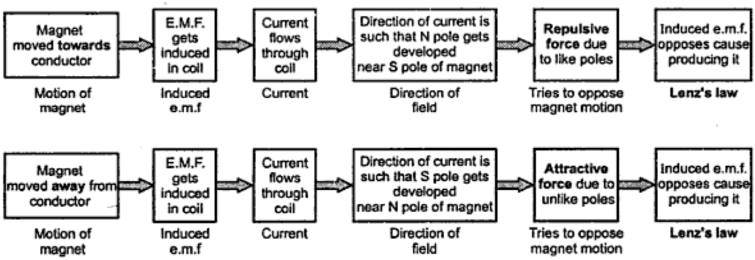


Fig. 4.9 Concept of Lenz's law

4.6 Statically Induced E.M.F.

Key Point : *The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f. in a coil which is without physical movement of coil or a magnet is called* ***statically induced e.m.f.***

Explanation : To have an induced e.m.f. there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.

Consider an electromagnet which is producing the necessary flux for producing e.m.f. Now let current through the coil of an electromagnet be an alternating one. Such alternating current means **it changes its magnitude periodically with time.** This produces the flux which is also alternating i.e. changing with time. Thus there exists $d\phi/dt$ associated with coil placed in the viscinity of an electromagnet. This is responsible for producing an e.m.f. in the coil. This is called statically induced e.m.f.

Key Point : *It can be noted that there is no physical movement of magnet or conductor, it is the alternating supply which is responsible for such an induced e.m.f.*

The concept of statically induced e.m.f. is shown in the Fig. 4.10.

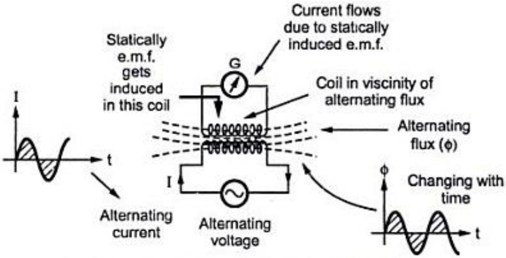


Fig. 4.10 Concept of statically induced e.m.f.

Such an induced e.m.f. can be observed in case of a device known as transformer.

Note : Due to alternating flux linking with the coil itself, the e.m.f. gets induced in that coil itself which carries an alternating current.

The statically induced e.m.f. is further classified as,

1) Self induced e.m.f. and 2) Mutually induced e.m.f.

We shall study now these two types of statically induced e.m.f.s.

4.7 Self Induced E.M.F.

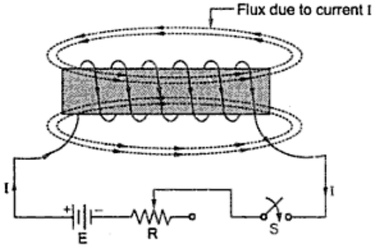


Fig. 4.11

Consider a coil having 'N' turns and carrying current 'I' when switch 'S' is in closed position. The current magnitude can be varied with the help of variable resistance connected in series with battery, coil and switch as shown in the Fig. 4.11.

The flux produced by the coil links with the coil itself. The total flux linkages of coil will be N ϕ Wb-turns. Now if the current 'I' is changed with the help of variable resistance, then flux produced will also change, due to which flux linkages will also change.

Hence according to Faraday's law, due to rate of change of flux linkages there will be induced e.m.f. in the coil. So without physically moving coil or flux there is induced e.m.f. in the coil. The phenomenon is called **self induction.**

The e.m.f. induced in a coil due to the change of its own flux linked with it is called **self induced e.m.f.**

Key Point: *The self induced e.m.f. lasts till the current in the coil is changing. The direction of such induced e.m.f. can be obtained by Lenz's law.*

4.7.1 Self Inductance

According to Lenz's law the direction of this induced e.m.f. will be so as to oppose the cause producing it. The cause is the current I hence the self induced e.m.f. will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced e.m.f. reduces the current tries to keep it to its original value. If current is decreased, self induced e.m.f. increases the current and tries to maintain it back to its original value. **So any change in current through coil is opposed by the coil.**

This property of the coil which opposes any change in the current passing through it is called **Self Inductance or Only Inductance.**

It is analogous to electrical inertia or electromagnetic inertia.

4.7.2 Magnitude of Self Induced E.M.F.

From the Faraday's law of electromagnetic induction, self induced e.m.f. can be expressed as

$$e = -N\frac{d\phi}{dt}$$

Negative sign indicates that direction of this e.m.f. is opposing change in current due to which it exists.

The flux can be expressed as,

$$\phi = (\text{Flux / Ampere}) \times \text{Ampere} = \frac{\phi}{I} \times I$$

Now for a circuit, as long as permeability 'μ' is constant, ratio of flux to current (i.e. B/H) remains constant.

$\therefore$ Rate of change of flux $= \frac{\phi}{I} \times$ rate of change of current

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -N \cdot \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$$

The constant $\frac{N\phi}{I}$ in this expression is nothing but the quantitative measure of the property due to which coil opposes any change in current.

So this constant $\frac{N\phi}{I}$ is called **coefficient of self inductance and denoted by 'L'.**

$$\therefore \quad \boxed{L = \frac{N\phi}{I}}$$

It can be defined as flux linkages per ampere current in it. Its unit is **henry (H).**

A circuit possesses **a self inductance of 1 H** when a current of 1 A through it produces flux linkages of 1 Wb-turn in it.

$$\therefore \quad \boxed{e = -L\frac{dI}{dt} \quad \text{volts}}$$

From this equation , the unit henry of self inductance can be defined as below.

Key Point: *A circuit possesses an inductance of 1 H when a current through coil is changing uniformly at the rate of one ampere per second inducing an opposing e.m.f. 1 volt in it.*

The coefficient of self inductance is also defined as the e.m.f. induced in volts when the current in the circuit changes uniformly at the rate of one ampere per second.

4.7.3 Expressions for Coefficient of Self Inductance (L)

$$L = \frac{N\phi}{I} \quad \dots (1)$$

But $$\phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S}$$

$\therefore$ $$L = \frac{N.NI}{I.S}$$

$\therefore$ $$\boxed{L = \frac{N^2}{S} \quad \text{henries}} \quad \dots (2)$$

Now $$S = \frac{l}{\mu a}$$

$$L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$\therefore$ $$\boxed{L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l} \quad \text{henries}} \quad \dots (3)$$

Where l = length of magnetic circuit

a = area of cross-section of magnetic circuit through which flux is passing.

4.7.4 Factors Affecting Self Inductance of a Coil

Now as defined in last section,

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

We can define factors on which self inductance of a coil depends as,

1) It is directly proportional to the square of number of turns of a coil. This means for same length, if number of turns are more then self inductance of coil will be more.
2) It is directly proportional to the cross-sectional area of the magnetic circuit.
3) It is inversely proportional to the length of the magnetic circuit.
4) It is directly proportional to the relative permeability of the core. So for iron and other magnetic materials inductance is high as their relative permeabilities are high.

5) For air cored or non magnetic cored magnetic circuits, $\mu_r=1$ and constant, hence self inductance coefficient is also small and always constant.

As against this for magnetic materials, as current i.e. magnetic field strength H (NI/*l*) is changed, μ_r also changes. Due to this change in current, cause change in value of self inductance. So for magnetic materials it is not constant but varies with current.

Key Point: *For magnetic materials, L changes as the current I.*

6) Since the relative permeability of iron varies with respect to flux density, the coefficient of self inductance varies with respect to flux density.

7) If the conductor is bent back on itself, then magnetic fields produced by current through it will be opposite to each other and hence will neutralize each other. Hence **inductance will be zero** under such condition.

Example 4.3 : *If a coil has 500 turns is linked with a flux of 50 mWb, when carrying a current of 125 A. Calculate the inductance of the coil. If this current is reduced to zero uniformly in 0.1 sec, calculate the self induced e.m.f. in the coil.*

Solution : The inductance is given by,

$$L = \frac{N\phi}{I}$$

Where $N = 500, \quad \phi = 50 \text{ mWb} = 50\times10^{-3} \text{ Wb}, \quad I = 25 \text{ A}$

$$\therefore \quad L = \frac{500\times50\times10^{-3}}{125} = 0.2 \text{ H}$$

$$e = -L\frac{dI}{dt} = -L\left[\frac{\text{Final value of I} - \text{Initial value of I}}{\text{Time}}\right]$$

$$= -0.2\times\left(\frac{0-125}{0.1}\right) = \mathbf{250 \ volts}$$

This is positive because current is decreased. So this 'e' will try to oppose this decrease, means will try to increase current and will help the growth of the current.

Example 4.4 : *A coil is wound uniformly on an iron core. The relative permeability of the iron is 1400. The length of the magnetic circuit is 70 cm. The cross-sectional area of the core is 5 cm². The coil has 1000 turns. Calculate,*

i) Reluctance of magnetic circuit ii) Inductance of coil in henries.

iii) E.M.F. induced in coil if a current of 10 A is uniformly reversed in 0.2 seconds.

Solution : $\mu_r = 1400, \; L = 70 \text{ cm} = 0.7 \text{ m}, \; N = 1000$

$$A = 5 \text{ cm}^2 = 5\times10^{-4} \text{ m}^2, \; \mu_0 = 4\pi\times10^{-7}$$

i) $$S = \frac{l}{\mu_0\mu_r a} = \frac{0.7}{4\pi\times10^{-7}\times1400\times5\times10^{-4}} = 7.957\times10^5 \text{ AT/Wb}$$

ii) $$L = \frac{N^2}{S} = \frac{(1000)^2}{7.957\times10^5} = \mathbf{1.2566} \quad H$$

iii) A current of + 10 A is made – 10 A in 0.2 sec.

$$\therefore \quad \frac{dI}{dt} = \frac{-10-10}{0.2} = -100$$

$$e = -L\frac{dI}{dt} = -1.2566\times(-100) = \mathbf{125.66 \ volts}$$

Again it is positive indicating that this e.m.f. opposes the reversal i.e. decrease of current from +10 towards –10 A.

4.8 Mutually Induced E.M.F.

If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced e.m.f. in the second coil, then such an e.m.f. is called **mutually induced e.m.f.**

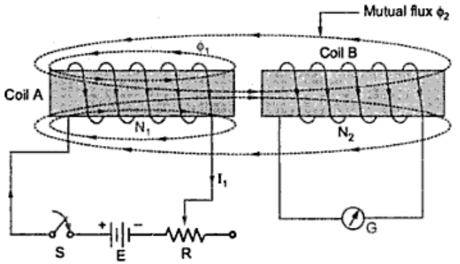


Fig. 4.12 Mutually induced e.m.f.

Consider two coils which are placed adjacent to each other as shown in the Fig. 4.12. The coil A has N_1 turns while coil B has N_2 number of turns. The coil A has switch S, variable resistance R and battery of 'E' volts in series with it. A galvanometer is connected across coil B to sense induced e.m.f. and current because of it.

Current through coil A is I_1 producing flux ϕ_1. Part of this flux will link with coil B i.e. will complete its path through coil B as shown in the Fig. 4.12. This is the mutual flux ϕ_2.

Now if current through coil A is changed by means of variable resistance R, then flux ϕ_1 changes. Due to this, flux associated with coil B, which is mutual flux ϕ_2 also changes. Due to Faraday's law there will be induced e.m.f. in coil B which will set up a current through coil B, which will be detected by galvanometer G.

Key Point : *Any change in current through coil A produces e.m.f. in coil B, this phenomenon is called mutual induction and e.m.f. is called mutually induced e.m.f.*

4.8.1 Magnitude of Mutually Induced E.M.F.

Let N_1 = Number of turns of coil A

N_2 = Number of turns of coil B

I_1 = Current flowing through coil A

ϕ_1 = Flux produced due to current I_1 in webers.

ϕ_2 = Flux linking with coil B

According to Faraday's law, the induced e.m.f. in coil B is,

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

Negative sign indicates that this e.m.f. will set up a current which will oppose the change of flux linking with it.

Now $$\phi_2 = \frac{\phi_2}{I_1} \times I_1$$

If permeability of the surroundings is assumed constant then $\phi_2 \propto I_1$ and hence ϕ_2 / I_1 is constant.

$\therefore$ Rate of change of ϕ_2 = $\frac{\phi_2}{I_1} \times$ Rate of change of current I_1

$$\therefore \quad \frac{d\phi_2}{dt} = \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -N_2 \cdot \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -\left(\frac{N_2 \phi_2}{I_1}\right) \frac{dI_1}{dt}$$

Here $\left(\frac{N_2 \phi_2}{I_1}\right)$ is called coefficient of mutual inductance denoted by M.

$$\therefore \quad \boxed{e_2 = -M \frac{dI_1}{dt} \quad \text{volts}}$$

Coefficient of mutual inductance is defined as the property by which e.m.f. gets induced in the second coil because of change in current through first coil.

Coefficient of mutual inductance is also called mutual inductance. It is measured in henries.

4.8.2 Definitions of Mutual Inductance and its Unit

1) The coefficient of mutual inductance is defined as the flux linkages of the coil per ampere current in other coil.
2) It can also be defined as equal to e.m.f. induced in volts in one coil when current in other coil changes uniformly at a rate of one ampere per second.

Similarly its unit can be defined as follows :

1. Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current of one ampere flowing through one coil produces a flux linkage of one weber turn in the other coil.
2. Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current changing uniformly at the rate of one ampere per second in one coil, induces as e.m.f. of one volt in the other coil.

4.8.3 Expressions of the Mutual Inductance (M)

1) $$\boxed{M = \frac{N_2 \phi_2}{I_1}}$$

2) ϕ_2 is the part of the flux ϕ_1 produced due to I_1. Let K_1 be the fraction of ϕ_1 which is linking with coil B.

$$\therefore \quad \phi_2 = K_1 \phi_1$$

$$\therefore \quad \boxed{M = \frac{N_2 K_1 \phi_1}{I_1}}$$

3) The flux ϕ_1 can be expressed as,

$$\phi_1 = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{N_1 I_1}{S}$$

$$\therefore \quad M = \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)$$

$$\boxed{M = \frac{K_1 N_1 N_2}{S}}$$

If all the flux produced by coil A links with coil B then $K_1 = 1$.

$$M = \frac{N_1 N_2}{S}$$

4) Now $\quad S = \frac{l}{\mu a} \quad$ and $\quad K_1 = 1$

Then $$M = \frac{N_1 N_2}{\left(\frac{l}{\mu a}\right)} = \frac{N_1 N_2 a \mu}{l}$$

$$\therefore \quad \boxed{M = \frac{N_1 N_2 \, a \, \mu_0 \mu_r}{l}}$$

5) If second coil carries current I_2, producing flux ϕ_2, the part of which links with coil A i.e. ϕ_1 then,

$$\phi_1 = K_2 \phi_2 \quad \text{and} \quad M = \frac{N_1 \phi_1}{I_2}$$

$$M = \frac{N_1 K_2 \phi_2}{I_2}$$

Now $$\phi_2 = \frac{N_2 I_2}{S}$$

$$\therefore \quad M = \frac{N_1 K_2 N_2 I_2}{I_2 S}$$

$$\therefore \quad \boxed{M = \frac{K_2 N_1 N_2}{S}}$$

If entire flux produced by coil B_2 links with coil 1, $K_2 = 1$ hence,

$$M = \frac{N_1 N_2}{S}$$

4.8.4 Coefficient of Coupling or Magnetic Coupling Coefficient

We know that, $M = \frac{N_2 K_1 \phi_1}{I_1}$ and $M = \frac{N_1 K_2 \phi_2}{I_2}$

Multiplying the two expressions of M,

$$M \times M = \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2}$$

$$\therefore \quad M^2 = K_1 K_2 \left(\frac{N_1 \phi_1}{I_1}\right)\left(\frac{N_2 \phi_2}{I_2}\right)$$

But $$\frac{N_1 \phi_1}{I_1} = \text{Self inductance of coil 1} = L_1$$

$$\frac{N_2 \phi_2}{I_2} = \text{Self inductance of coil 2} = L_2$$

$$\therefore \quad M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \cdot \sqrt{L_1 L_2} = K\sqrt{L_1 L_2}$$

where $$K = \sqrt{K_1 K_2}$$

The K is called **coefficient of coupling.**

If entire flux produced by one coil links with other then $K = K_1 = K_2 = 1$ and maximum mutual inductance existing between the coil is $M = K\sqrt{L_1 L_2}$.

This gives an idea about magnetic coupling between the two coils. When entire flux produced by one coil links with other, this coefficient is maximum i.e. Unity.

It can be defined as the ratio of the actual mutual inductance present between the two coils to the maximum possible value of the mutual inductance.

The expression for K is,

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Key Point: *When K = 1 coils are said to be* ***tightly coupled*** *and if K is a fraction the coils are said to be* ***loosely coupled.***

4.9 Effective Inductance of Series Connection

Similar to the resistances, the two inductances can be coupled in series. The inductances can be connected in series either in series aiding mode called **cumulatively coupled connection** or series opposition mode called **differentially coupled connection.**

4.9.1 Series Aiding or Cumulatively Coupled Connection

Two coils are said to be cumulatively coupled if their fluxes are always in the same direction at any instant.

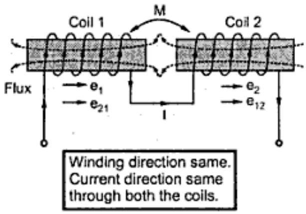


Fig. 4.13 Series aiding

For this, winding direction of the two coils on the core must be the same so that both will carry current in same direction. The Fig. 4.13 shows cumulatively coupled connection.

Coil 1 has self inductance L_1 and Coil 2 has self inductance L_2.

While both have a mutual inductance of M.

4.9.2 Equivalent Inductance of Series Aiding Connection

Refer to Fig. 4.13 which shows two coil of self inductances L_1 and L_2 connected in series aiding mode. The mutual inductance between the two is M.

If current flow through the circuit is changing at the rate of $\frac{di}{dt}$ then total e.m.f. induced will be due to self induced e.m.f.s and due to mutually induced e.m.f.s.

Due to flux linking with coil 1 itself, there is self induced e.m.f.,

$$e_1 = -L_1 \frac{di}{dt}$$

Due to flux produced by coil 2 linking with coil 1 there is mutually induced e.m.f.,

$$e_{21} = -M\frac{di}{dt}$$

Due to flux produced by coil 1 linking with coil 2 there is mutually induced e.m.f.,

$$e_{12} = -M\frac{di}{dt}$$

Due to flux produced by coil 2 linking with itself there is self induced e.m.f.

$$e_2 = -L_2 \frac{di}{dt}$$

The total induced e.m.f. is addition of these e.m.f.s as all are in the same direction,

$$e = e_1 + e_{21} + e_{12} + e_2 = -L_1 \frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} - L_2 \frac{di}{dt}$$

$$= -[L_1 + L_2 + 2M]\frac{di}{dt} = -L_{eq}\frac{di}{dt}$$

Where L_{eq} = Equivalent inductance

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2 + 2M}$$

4.9.3 Series Opposition or Differentially Coupled Connection

Two coils are said to be differently coupled if their fluxes are always in the opposite direction at any instant.

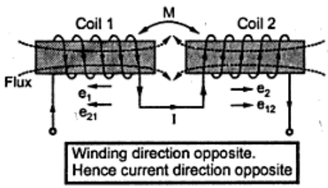


Fig. 4.14 Series opposition

Such a connection is shown in the Fig. 4.14.

Coil 1 has self inductance L_1

Coil 2 has self inductance L_2

and the mutual inductance between the two is M.

4.9.4 Equivalent Inductance of Series Opposition Connection

In series opposition, flux produced by coil 2 is in opposite direction to the flux produced by coil 1.

If current in the circuit is changed at a rate $\frac{di}{dt}$ then their self induced e.m.f.s will oppose the applied voltage but mutually induced e.m.f. will assist the applied voltage.

Similar to the cumulative connection there will exist four e.m.f.s which are,

$$e_1 = -L_1 \frac{di}{dt}, \qquad e_{21} = +M\frac{di}{dt}$$

$$e_{12} = +M\frac{di}{dt} \quad \text{and} \quad e_2 = -L_2 \frac{di}{dt}$$

Hence the total e.m.f. is the addition of these four e.m.f.s,

$$\therefore \quad e = e_1 + e_{21} + e_{12} + e_2$$

$$= -L_1 \frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} - L_2 \frac{di}{dt}$$

$$= -[L_1 + L_2 - 2M]\frac{di}{dt} = -L_{eq} \frac{di}{dt}$$

Where L_{eq} = Equivalent inductance of the differentially coupled connection.

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2 - 2M}$$

Example 4.5 : *Two coils A and B are kept in parallel planes, such that 70 % of the flux produced by coil A links with coil B . Coil A has 10,000 turns. Coil B has 12,000 turns. A current of 4 A in coil A produces a flux of 0.04 mWb while a current of 4A in coil B produces a flux of 0.08 mWb. Calculate,*
i) Self inductances L_A and L_B ii) Mutual inductance M iii) Coupling coefficient.

Solution : The given values are,

$$N_A = 10{,}000, \quad N_B = 12{,}000, \quad \phi_B = 0.7\,\phi_A$$

$$\therefore \quad K_A = \frac{\phi_B}{\phi_A} = 0.7$$

$$\phi_A = 0.04\times10^{-3} \text{ Wb} \quad \text{for } I_A = 4 \text{ A.}$$

$$\phi_B = 0.08\times10^{-3} \text{ Wb} \quad \text{for } I_B = 4 \text{ A}$$

i) Self Inductance $L_A = \frac{N_A \phi_A}{I_A} = \frac{10{,}000 \times 0.04 \times 10^{-3}}{4} = \mathbf{0.1\ H}$

and $L_B = \frac{N_B \phi_B}{I_B} = \frac{12000 \times 0.08 \times 10^{-3}}{4} = \mathbf{0.24\ H}$

ii) Mutual Inductance $M = \frac{N_B \phi_B}{I_A} = \frac{N_B K_A \phi_A}{I_A}$

$$= \frac{12000 \times 0.7 \times 0.04 \times 10^{-3}}{4} = \mathbf{0.084\ H}$$

iii) Coupling Coefficient $K = \frac{M}{\sqrt{L_A L_B}} = \frac{0.084}{\sqrt{0.1 \times 0.24}} = \mathbf{0.5422}$

4.10 Energy Stored in the Magnetic Field

We know that energy is required to establish flux i.e. magnetic field but it is not required to maintain it. This is similar to the fact that the energy is required to raise the water through a certain height (h) which is 'mgh' joules. But energy is not required to maintain the water at height 'h'. This energy 'mgh' gets stored in it as its potential energy and can be utilized for many purposes.

Key Point : *The energy required to establish magnetic field then gets stored into it as a potential energy. This energy can be recovered when magnetic field established, collapses.*

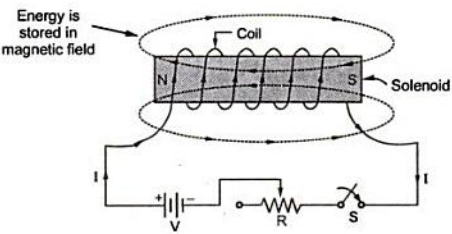


Fig. 4.15 Energy stored in magnetic field

This can be explained as below.

Consider a solenoid, the current through which can be controlled with the help of switch S, resistance R shown in the Fig. 4.15.

Initially switch 'S' is open, so current through coil, I is zero. When switch is closed, current will try to built its value equal to I. Neglect the resistance of coil.

It will take some time to increase the current from 'zero' to 'I' say 'dt' seconds.

In the mean time, flux linkages associated with the coil will change, due to which there will be self induced e.m.f. in the coil whose value is given by,

$$e = -L\frac{dI}{dt}$$

So at every instant, coil will try to oppose the increase in the current. To overcome this opposition, supply has to provide the energy to the circuit. This is nothing but the energy required to establish the current i.e. magnetic field or flux around the coil.

Once current achieves its maximum value 'I' then change in current stops. Hence there can not be any induced e.m.f. in the coil and no energy will be drawn from the supply. So no energy is required to maintain the established flux. This is because, induced e.m.f. lasts as long as there is change in flux lines associated with the coil, according to Faraday's law.

Key Point : *Now the energy which supply has provided, gets stored in the coil which is energy stored in the magnetic field, as its potential energy.*

When current is again reduced to zero by opening the switch then current through the coil starts decreasing and flux starts decreasing. So there is induced e.m.f. in the coil according to Faraday's law. But as per Lenz's law it will try to oppose cause producing it which is decrease in current. So this induced e.m.f. now will try to maintain current to its original value. So instantaneously this induced e.m.f. acts as a source and supplies the energy to the source. This is nothing but the same energy which is stored in the magnetic field which gets recovered while field collapses. So energy stored while increase in the current is returned back to the supply when current decreases i.e. when field collapses.

Key Point : *The energy which is stored in the coil earlier is returned back to the supply. No additional energy can exist as coil can not generate any energy.*

The expression for this energy stored is derived below.

4.10.1 Expression for Energy Stored in the Magnetic Field

Let the induced e.m.f. in a coil be,

$$e = -L\frac{dI}{dt}$$

This opposes a supply voltage. So supply voltage 'V' supplies energy to overcome this, which ultimately gets stored in the magnetic field.

$$\therefore \quad V = -e = -\left[-L\frac{dI}{dt}\right] = L\frac{dI}{dt}$$

$$\therefore \quad \text{Power supplied} = V \times I = L\frac{dI}{dt} \times I$$

$\therefore$ Energy supplied in time dt is,

$$E = \text{Power} \times \text{Time} = L\frac{dI}{dt} \times I \times dt$$

$$= L\,di \times I \quad \text{joules.}$$

This is energy supplied for change in current of dI but actually current changes from zero to I.

∴ Integrating above total energy stored is,

$$E = \int_0^I L\,dI\,I = L\int_0^I dI\,I$$

$$= L\left[\frac{I^2}{2}\right]_0^I = L\left[\frac{I^2}{2}-0\right]$$

$$\therefore \quad \boxed{E = \frac{1}{2}LI^2 \quad \text{joules}}$$

4.10.2 Energy Stored Per Unit Volume

The above expression for the energy stored can be expressed in the different form as,

$$E = \frac{1}{2}LI^2 \text{ joules}$$

Now $L = \dfrac{N\phi}{I}$

$$\therefore \quad E = \frac{1}{2}\frac{N\phi}{I}I^2 \text{ joules} = \frac{1}{2}N\phi I \text{ joules}$$

Now $NI = Hl$ ampere-turns

$$\phi = Ba$$

$$\therefore \quad E = \frac{1}{2}BaHl$$

But $a \times l = \text{Area} \times \text{Length} = \text{Volume of magnetic circuit}$

∴ Energy stored per unit volume is,

$$= \frac{1}{2}BH$$

But $B = \mu H$

∴ Energy per unit volume,

$$= \frac{1}{2}\mu H^2 \quad \text{joules / m}^3$$

$$\boxed{E/\text{unit volume} = \frac{1}{2}\frac{B^2}{\mu} \quad \text{joules/m}^3}$$

Where $\mu = \mu_0\mu_r$

In case of inductive circuit when circuit is opened with the help of switch then current decays and finally becomes zero. In such case energy stored is recovered and if there is resistance in the circuit, appears in the form of heat across the resistance.

While if the resistance is not present then this energy appears in the form of an arc across the switch, when switch is opened.

If the medium is air, $\mu_r = 1$ hence $\mu = \mu_0$ must be used in the above expressions of energy.

➠ **Example 4.6 :** *A coil is wound on an iron core to form a solenoid. A certain current is passed through the coil which is producing a flux of 40 μWb. The length of magnetic circuit is 75 cm while its cross-sectional area is 3 cm^2. Calculate the energy stored in the circuit. Assume relative permeability of iron as 1500.*

Solution :

$$l = 75 \text{ cm} = 0.75 \text{ m}, \quad a = 3 \text{ cm}^2 = 3\times10^{-4} \text{ m}^2$$

$$\phi = 40 \ \mu\text{Wb} = 40\times10^{-6} \text{ Wb}, \quad \mu_r = 1500$$

$$\therefore \quad B = \frac{\phi}{a} = \frac{40\times10^{-6}}{3\times10^{-4}} = 0.133 \text{ Wb/m}^2$$

∴ Energy stored per unit volume,

$$\frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r} = \frac{1}{2}\frac{(0.133)^2}{4\pi\times10^{-7}\times1500} = 4.7157 \quad \text{J/m}^3$$

$$\therefore \text{Total energy stored} = \text{Energy per unit volume}\times\text{Volume} = E\times(a\times l)$$

$$= 4.7157\times(3\times10^{-4}\times0.75) = \mathbf{0.00106 \ joules}$$

4.11 Lifting Power of Electromagnets

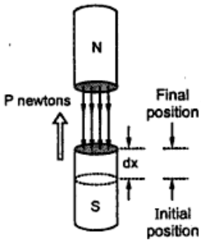


Fig. 4.16

Force of attraction between the two magnetized surfaces forms the basis of operation of devices like lifting magnets, solenoid valves, magnetically operated contactors, clutches etc.

Consider two poles of two magnetized surface N and S having an air gap of length 'l ' m between them and a cross-sectional area of 'a' sq.m. Let P newtons be the force of attraction between them. This is shown in the Fig. 4.16.

The energy stored in a magnetic field in air per unit volume is,

$$E = \frac{1}{2}\frac{B^2}{\mu_0} \text{ J/m}^3 \qquad \dots \mu_r = 1$$

$\therefore \quad \text{Energy stored} = \frac{1}{2}\frac{B^2}{\mu_0} a \times l \quad J$

If south pole is moved further by distance dx then energy stored will further increase by,

$$= \frac{B^2}{2\mu_0} a \times dx \quad \text{joules}$$

This increased energy must be equal to the mechanical work done to move pole by distance dx which is,

$$P \times dx = (\text{Force} \times \text{displacement})$$

$$\therefore \quad P\,dx = \frac{B^2}{2\mu_0} a \times dx$$

$$\boxed{P = \frac{B^2 a}{2\mu_0} \quad \text{newtons}}$$

This is the force in newtons existing between two magnetized surfaces.

Examples with Solutions

Example 4.7 : *A square coil of 20 cm side is rotated about its axis at a speed of 200 revolutions per minute in a magnetic field of density 0.8 Wb/m². If the number of turns of coil is 25, determine maximum e.m.f. induced in the coil.*

Solution : The arrangement is shown in the Fig. 4.17.

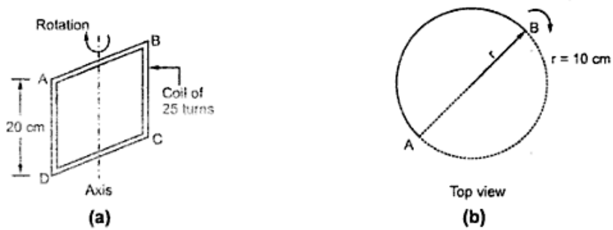


Fig. 4.17

As shown in the Fig. 4.17 (b) the active length responsible for cutting flux lines becomes $l = 20$ cm = 0.2 m.

Now $\quad N = 200$ r.p.m.

We want the velocity of m/sec

$$v = r\,\omega$$

$$= r \times \frac{2\pi N}{60} \text{ [where N is in r.p.m. and r = 10 cm = 0.1 m]}$$

$$\therefore \quad v = 0.1 \times \frac{2\pi \times 200}{60} = 2.094 \text{ m/sec.}$$

$$B = 0.8 \text{ Wb/m}^2 \quad \text{and} \quad \text{Active length} = 0.2 \text{ m}$$

The maximum e.m.f. induced in conductor AB, shown in Fig. 4.17 (b) will be,

$$e = B\,l\,v \sin\theta$$

For e_{max}, $\theta = 90°$

$$\therefore \quad e = 0.8 \times 0.2 \times 2.094 = 0.335 \text{ volts}$$

The e.m.f. induced in sides BC and AD is almost zero as their plane of rotation becomes parallel to plane of field.

And maximum e.m.f. induced in conductor CD will be same as AB = 0.335 volts.

∴ e.m.f. induced in one turn of the coil [AB + CD]

$$= 2 \times 0.335 = 0.67 \text{ volts}$$

In all, there are 25 turns in that coil,

∴ Total e.m.f. induced in a coil is

$$= 25 \times 0.67 = \mathbf{16.75 \text{ volts}}$$

➡ **Example 4.8 :** *A conductor has 50 cm length is mounted on the periphery of a rotating part of d.c. machine. The diameter of a rotating drum is 75 cm. The drum is rotated at a speed of 1500 r.p.m. The flux density through which conductor passes at right angles is 1.1 T. Calculate the induced e.m.f. in the conductor.*

Solution : The active length l = 50 cm = 0.5 m., N = 1500 r.p.m., B = 1.1 T, $\theta = 90°$.

The rotating drum on which conductor is mounted is called **armature** of a d.c. machine.

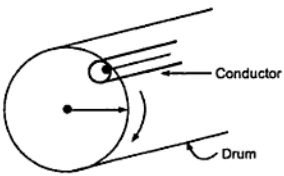


Fig. 4.18

The arrangement is as shown in Fig. 4.18.

The linear velocity

$$v = r\,\omega = r \times \frac{2\pi N}{60}$$

$$= 0.375 \times \frac{2\pi \times 1500}{60} = 58.9 \text{ m/sec.}$$

$$r = \frac{75}{2} = 37.5 \text{ cm}$$

$$\therefore \quad \text{Induced e.m.f. in a conductor} = B\,l\,v = 1.1 \times 0.5 \times 58.9$$

$$= \mathbf{32.397 \text{ volts}}$$

Example 4.9 : *Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius.*

Solution : Given values are, N = 200, l = 25 cm = 0.25 m, r = 5 cm = 0.05 m

$$\therefore \quad \text{c/s area} = \frac{\pi}{4} d^2 \quad \text{where d = Diameter}$$

$$a = \frac{\pi}{4}(2r)^2 = \frac{\pi}{4} \times (2 \times 0.05)^2$$

$$a = 7.853 \times 10^{-3} \text{ m}^2$$

For paper, $\mu_r = 1$

$$\therefore \quad S = \frac{l}{\mu_0 a} = \frac{0.25}{4\pi \times 10^{-7} \times 7.853 \times 10^{-3}} = 2.533 \times 10^7 \text{ AT/Wb}$$

$$L = \frac{N^2}{S} = \frac{(200)^2}{2.533 \times 10^7} = 1.579 \times 10^{-3} \text{H}$$

$$= \mathbf{1.579 \text{ mH}}$$

Example 4.10 : *An electromagnet is wound with 800 turns. Find the value of average e.m.f. induced and current through coil, if it is moved to that magnetic field is changed from 1 mWb to 0.25 mWb in 0.2 sec. The resistance of the coil is 500 Ω.*

Solution : Given values are, N = 800, ϕ_2 = 0.25 mWb, ϕ_1 = 1 mWb, t = 0.2 sec., R = 500Ω

$$\text{Induced e.m.f} \quad e = -N\frac{d\phi}{dt} = -800\left[\frac{\phi_2 - \phi_1}{dt}\right] = -\left[\frac{0.25 \times 10^{-3} - 1 \times 10^{-3}}{0.2}\right]$$

$$= 3 \text{ volts}$$

$$\therefore \quad \text{Current I} = \frac{\text{e.m.f.}}{R} = \frac{3}{500} = 6 \times 10^{-3} \text{ A} = \mathbf{6 \text{ mA}}$$

Example 4.11 : *A solenoid is wound with 1000 turns having area of cross-section 25 cm^2. When 2.5 A current flows through the coil, the flux density is 0.8 Wb/m^2 and when current is increased to 5 A, the flux density becomes 1.2 Wb/m^2. Find the average value of self inductance within given current limits. If this change in current is achieved within 0.04 sec., calculate the self induced e.m.f.*

Solution : Given values are, $N = 1000$, $a = 25\ cm^2 = 25 \times 10^{-4}\ m^2$, $I_1 = 2.5$ A,

$B_1 = 0.8\ Wb/m^2$, $I_2 = 5$ A, $B_2 = 1.2\ Wb/m^2$, $t = 0.04$ sec.

$$L = \frac{N\phi}{I} \quad \text{i.e.} \quad L = N\frac{d\phi}{dI}$$

$$L = N\left[\frac{\phi_2 - \phi_1}{I_2 - I_1}\right] \quad \text{as } B = \frac{\phi}{a}$$

$$= Na\left[\frac{\frac{\phi_2}{a} - \frac{\phi_1}{a}}{I_2 - I_1}\right] = Na\left[\frac{B_2 - B_1}{I_2 - I_1}\right]$$

$$\therefore \quad L = 1000 \times 25 \times 10^{-4} \times \left[\frac{1.2 - 0.8}{5 - 2.5}\right] = \mathbf{0.4\ H}$$

Now $$e = -L\frac{dI}{dt} = -0.4\left[\frac{I_2 - I_1}{dt}\right] = -0.4\left[\frac{5 - 2.5}{0.004}\right] = \mathbf{-25\ volts}$$

Negative sign indicates that it opposes change in current.

Example 4.12 : *An iron ring of mean length of 100 cm and cross-sectional area of 10 cm^2 has an air gap of 1 mm cut in it. It is wound with a coil of 100 turns. Assuming relative permeability of iron as 500, calculate the inductance of a coil.*

Solution : Given values are, $N = 100$, $a = 10\ cm^2 = 10 \times 10^{-4}\ m^2$, $\mu_r = 500$

Length of iron is l_i = Mean length – Air gap length

$= 100\ cm - 1 \times 10^{-1}\ cm = 99.9\ cm = 0.999\ m$

Length of air gap is $l_g = 1\ mm = 1 \times 10^{-3}\ m$

$$\therefore \quad S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.999}{4\pi \times 10^{-7} \times 500 \times 10 \times 10^{-4}}$$

$$= 1.5899 \times 10^6\ AT/Wb$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 7.9577 \times 10^5\ AT/Wb.$$

$$\therefore \quad \text{Total } S = S_i + S_g = 2.38567 \times 10^6\ AT/Wb$$

$$\therefore \quad L = \frac{N^2}{S} = \frac{(100)^2}{2.38597 \times 10^6} = 4.191 \times 10^{-3}\ H$$

$$= \mathbf{4.191\ mH}$$

Example 4.13 : *An iron cored toroid of relative permeability 980 has a mean length of 120 cm and core area of 100 mm². A current of 0.3 A establishes a flux of 40 μWb, calculate*

i) the number of turns of coil ii) self inductance iii) energy stored in magnetic field.

Solution : Given values are $\mu_r = 980$, $l = 120$ cm = 1.2 m, $a = 100\ \text{mm}^2 = 100 \times 10^{-6}$ m

$$I = 0.3\ \text{A},\ \phi = 40\ \mu\ \text{Wb} = 40 \times 10^{-6}\ \text{Wb}$$

i) Flux density $$B = \frac{\phi}{a} = \frac{40\times10^{-6}}{100\times10^{-6}} = 0.4\ \text{Wb}$$

Field strength $$H = \frac{B}{\mu_0\mu_r} = \frac{0.4}{4\pi\times10^{-7}\times980} = 324.8\ \text{AT/m}$$

$$H = \frac{NI}{l}$$

$$\therefore \quad 324.8 = \frac{N\times0.3}{1.2}$$

$$\therefore \quad N = 1299.2 \approx \mathbf{1300\ turns}$$

ii) $$L = \frac{N\phi}{I} = \frac{1300\times40\times10^{-6}}{0.3} = \mathbf{0.1733\ H}$$

iii) Energy stored $E = \frac{1}{2} L I^2 = \frac{1}{2}\times0.1733\times(0.3)^2 = \mathbf{7.8\times10^{-3}\ joules}$

Otherwise alternatively energy stored can be calculated as,

$$E = \frac{1}{2}\frac{B^2}{\mu_0\mu_r}\times \text{Volume} = \frac{1}{2}\times\frac{(0.4)^2}{4\pi\times10^{-7}\times980}\times(a\times l)$$

$$= \frac{1}{2}\times129.9224\times\frac{1}{2}\times(100\times10^{-6}\times1.2)$$

$$= \mathbf{7.8\times10^{-3}\ J}$$

Example 4.14 : *A coil of 200 turns having a mean diameter of 6 cm is placed coaxially at the centre of a solenoid of 50 cm long with 1500 turns and carrying current of 2.5 A. Calculate the mutual inductance between the two coils.*

Solution : Given values are,

$N_1 = 1500$ (solenoid), $N_2 = 200$, $l_1 = 50$ cm = 0.5 m, $I_1 = 2.5$ A

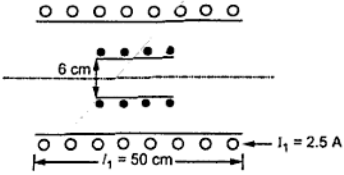


Fig. 4.19

Now magnetic field strength H at the centre of coil due to solenoid current is

$$H = \frac{N_1 I_1}{l_1}$$

$$= \frac{1500 \times 2.5}{0.5} = 7500 \text{ AT/m}$$

$\therefore$ Flux density at centre is $\quad B = \mu_0 H \quad (\mu_r = 1)$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 7500 = 9.424 \times 10^{-3} \text{ Wb/m}^2$$

$\therefore$ Flux linking with second coil is,

$$\phi_2 = B \times a_2 = 9.424 \times 10^{-3} \times \frac{\pi}{4} \times (d_2)^2$$

$$= 9.424 \times 10^{-3} \times \frac{\pi}{4} \times (6 \times 10^{-2})^2 = 2.664 \times 10^{-5} \text{ Wb}$$

$\therefore$ Mutual inductance between the coils is,

$$M = \frac{N_2 \phi_2}{I_1} = \frac{200 \times 2.664 \times 10^{-5}}{2.5} = \mathbf{2.1318 \times 10^{-3} \text{ H}}$$

Example 4.15 : *Two coils with a coefficient of coupling of 0.5 between them are connected in series so as to magnetize a) in the same direction (series aiding), b) in the opposite direction (series opposition). The corresponding values of equivalent inductance for a) is 1.9 H and b) 0.7 H. Find the self inductance of each coil, mutual inductance between the coil.*

(May-2006)

Solution : Given values are, $K = 0.5$

Now for series aiding, $\quad L_{eq} = L_1 + L_2 - 2M = 1.9 \text{ H} \quad$ ...(1)

For series opposition, $\quad L_{eq} = L_1 + L_2 - 2M = 0.7 \text{ H} \quad$ ...(2)

and $\quad M = K\sqrt{L_1 L_2} = 0.5\sqrt{L_1 L_2} \quad$ ... (3)

Subtracting (2) from (1), $\quad 4M = 1.2 \quad$ i.e. $M = \mathbf{0.3 \text{ H}}$

Substituting in (3), $\quad 0.3 = 0.5\sqrt{L_1 L_2} \quad$ i.e. $L_1 L_2 = 0.36$

$$L_2 = \frac{0.36}{L_1}$$

Substituting in (1), $L_1 + \frac{0.36}{L_1} + 2 \times 0.3 = 1.9$

$$\therefore L_1^2 + 0.36 - 1.3\, L_1 = 0$$

$$\therefore \quad L_1 = \frac{1.3 \pm \sqrt{(1.3)^2 - 4 \times 0.36}}{2}$$

$$L_1 = 0.9\text{ H or } L_1 = 0.4\text{ H}$$

$$L_2 = 0.4\text{ H or } L_2 = 0.9\text{ H}$$

Example 4.16 : *A coil of 800 turns of copper wire those diameter is of 0.375 mm. The length of the core is 90 cm. The diameter of core is 2.5 cm. Find the resistance and inductance of the coil. Assume specific resistance of copper as 1.73×10^{-6} Ω-cm.*

Solution :

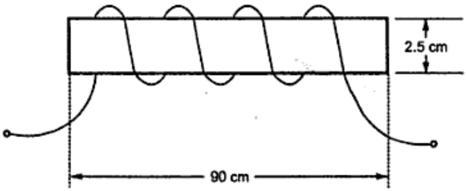


Fig. 4.20

Length of the coil $= (\pi \times d) \times$ Number of turns

As $\pi \times d =$ Circumference of 1 turn

And $d =$ Diameter of the core

$\therefore$ Length of the coil $= (\pi \times 2.5 \times 10^{-2}) \times 800 = 62.83$ m

$$\rho = 1.73 \times 10^{-6}\ \Omega\text{-cm} = 1.73 \times 10^{-8}\ \Omega\text{-m}$$

$$\therefore \quad R = \frac{\rho l}{a}$$

Where $a = \frac{\pi}{4} d^2$ where d = Diameter of coil

$$\therefore \quad d = 0.375\text{ mm} = 0.375 \times 10^{-3}\text{ m}$$

$$\therefore \quad a = \frac{\pi}{4} \times (0.375 \times 10^{-3})^2 = 1.104 \times 10^{-7}\text{ m}^2$$

$\therefore \qquad R = \frac{\rho l}{a} = \frac{1.73\times10^{-8}\times 62.68}{1.1044\times10^{-7}} = 9.84\ \Omega$

While $\qquad L = \frac{N^2}{S}$

Reluctance $\qquad S = \frac{l}{\mu_0\mu_r a}$ where l = Length of core = 0.9 m

a = c/s area of core $= \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(2.5\times10^{-2})^2$

$= 4.908\times10^{-4}\,m^2$

Assume $\qquad \mu_r = 1$

$\therefore \qquad S = \frac{0.9}{4\pi\times10^{-7}\times1\times4.908\times10^{-4}} = 1.45902\times10^9$ AT/Wb

$L = \frac{N^2}{S} = \frac{(800)^2}{1.45902\times10^9} = 4.386\times10^{-4}$ H

$=$ **0.4386 mH**

Example 4.17 : *A length of an air cored solenoid is 1.7 m and area of cross-section is 12 cm². The number of turns of coil is 1000.*
Calculate :
i) The self inductance. ii) The energy stored in magnetic field when a current of 10 A flows through the coil. **(Dec.-97)**

Solution : l = 1.7 m, $a = 12\ cm^2 = 12\times10^{-4}\,m^2$, $\mu_0 = 4\pi\times10^{-7}$, N = 1000, I = 10 A

$S = \frac{l}{\mu_0 a} = \frac{1.7}{4\pi\times10^{-7}\times12\times10^{-4}}$

$= 1.1273\times10^9$ AT/Wb ... $\mu_r = 1$ as air cored

$L = \frac{N^2}{S} = \frac{(1000)^2}{1.1273\times10^9} = 8.87\times10^{-4}$ H = **88.7 mH**

Now if $\qquad E = \frac{1}{2}LI^2 = \frac{1}{2}\times(88.7\times10^{-3})\times(10)^2 =$ **0.0443 J**

Example 4.18 : *Two coils having 3000 and 2000 turns are wound on a magnetic ring. 60% of flux produced in first coil links with the second coil. A current of 3 A produces flux of 0.5 mWb in the first coil and 0.3 mWb in the second coil. Determine the mutual inductance and coefficient of coupling.* **(Dec.-98)**

Solution : $N_1 = 3000, N_2 = 2000, \phi_1 = 0.5$ mWb, $\phi_2 = 0.3$ mWb

$$I_1 = I_2 = 3\text{ A} \quad \text{and} \quad \phi_2 = 0.6\,\phi_1$$

$$M = \frac{N_2\,\phi_2}{I_1} = \frac{2000\times 0.3\times 10^{-3}}{3} = \mathbf{0.2\ H}$$

$$L_1 = \frac{N_1\phi_1}{I_1} = \frac{3000\times 0.5\times 10^{-3}}{3} = 0.5\text{ H}$$

$$L_2 = \frac{N_2\phi_2}{I_2} = \frac{2000\times 0.3\times 10^{-3}}{3} = 0.2\text{ H}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.2}{\sqrt{0.5\times 0.2}} = \mathbf{0.6324}$$

Example 4.19 : *Two coils having 1000 and 300 turns are wound on a common magnetic path with perfect magnetic coupling. The reluctance of the path is 3×10^6 AT/Wb. Find the mutual inductance between them. If the current in 1000 turns coil changes uniformly from 5 A to zero in 10 milliseconds, find the induced e.m.f. in the other coil.*

(Dec. -99, Dec.-2000)

Solution : $N_1 = 1000, N_2 = 300, K = 1, S = 3\times10^6$ AT/Wb

Now,
$$M = \frac{N_1 N_2}{S} = \frac{1000\times 300}{3\times10^6} = \mathbf{0.1\ H}$$

$$e_2 = -M\frac{dI_1}{dt} = -0.1\times\left(\frac{0-5}{10\times10^{-3}}\right) = \mathbf{50\ V}$$

This the induced e.m.f. in other coil.

Example 4.20 : *Two coils A and B are placed such that 40 % of flux produced by coil A links with coil B coils A and B have 2000 and 1000 turns respectively. A current of 2.5 A in coil A produces a flux of 0.035 mWb in coil B. For the above coil combination, find out (i) M, the mutual inductance and (ii) the coefficient of coupling K_A, K_B and K (iii) Self inductances L_A and L_B.*

(May-2000)

Solution : $N_A = 2000,\ N_B = 1000,\ K_A = 0.4,\ \phi_B = 0.4\,\phi_A$

$I_A = 2.5$ A and $\phi_B = 0.035$ mWb

(i) Mutual Inductance,
$$M = \frac{N_B\,\phi_B}{I_A} = \frac{1000\times 0.035\times 10^{-3}}{2.5} = \mathbf{0.014\ H}$$

(ii)
$$\phi_B = 0.035\text{ mWb} \quad \text{and} \quad \phi_B = 0.4\,\phi_A$$

$$\therefore \quad \phi_A = \frac{\phi_B}{0.4} = \frac{0.035}{0.4} = 0.0875 \text{ mWb}$$

$$\therefore \quad L_A = \frac{N_A \phi_A}{I_A} = \frac{2000 \times 0.0875 \times 10^{-3}}{2.5}$$

$$\therefore \quad L_A = \mathbf{0.07\ H}$$

Assuming that same current in coil B produces 0.035 mWb in coil B.

$$\therefore \quad L_B = \frac{N_B \phi_B}{I_B} = \frac{1000 \times 0.035 \times 10^{-3}}{2.5} = \mathbf{0.014\ H}$$

(iii)

$$M = \frac{N_A \phi_A}{I_B} \qquad M = \frac{N_A K_B \phi_B}{I_B}$$

$$\therefore \quad 0.014 = \frac{2000 \times K_B \times 0.035 \times 10^{-3}}{2.5}$$

$$\therefore \quad K_B = \mathbf{0.5}$$

$\phi_B = K_A \phi_A$ and it is given that 40% of ϕ_A links with coil B,

$$\therefore \quad K_A = \mathbf{0.4}$$

$$K = \sqrt{K_A K_B} = \sqrt{0.4 \times 0.5} = \mathbf{0.4472}$$

Example 4.21 : *Two windings connected in series are wound on a ferromagnetic ring having cross-sectional area of 750 mm² and a mean diameter of 175 mm. The two windings have 250 and 750 turns, while the relative permeability of material is 1500. Assuming no leakage of flux, calculate the self inductances of each winding and the mutual inductance as well. Calculate e.m.f. induced in coil 2 if current is coil 1 in increased uniformly from zero to 5 A in 0.01 sec.* **(Dec.-2001)**

Solution :

$$l = \text{Length of magnetic circuit} = \pi \times d_{mean}$$

$$l = \pi \times 175 \times 10^{-3} = 0.5497 \text{ m}$$

$$a = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2 = 7.5 \times 10^{-4} \text{ m}^2$$

$$N_1 = 250, \quad N_2 = 750, \mu_r = 1500$$

Self inductance, $L = \dfrac{N\phi}{I}$ but $\phi = \dfrac{NI}{S}$

$$\therefore \quad L = \frac{N\ NI}{I S} = \frac{N^2}{S}$$

We have, $S = \frac{l}{\mu a}$

$$\therefore \quad S = \frac{l}{\mu_0 \mu_r a} = \frac{0.5497}{(4\pi\times10^{-7})(1500)(7.5\times10^{-4})}$$

$$= 388833.2 \text{ AT/Wb}$$

$$\therefore \quad L_1 = \frac{N_1^2}{S} = \frac{(250)^2}{388833.2} = \mathbf{0.1607\ H}$$

$$L_2 = \frac{N_2^2}{S} = \frac{(750)^2}{388833.2} = \mathbf{1.4466\ H}$$

The mutual inductance between the two windings is given by,

$$M = \frac{N_1 N_2}{S} = \frac{(250)(750)}{388833.20} = \mathbf{0.4822\ H}$$

$$\therefore \quad M = \mathbf{0.4822\ H}$$

E.M.F. induced in coil 2 is,

$$e_2 = -M\frac{dI_1}{dt} = -0.4822\times\frac{(5-0)}{0.01} = \mathbf{-241.1\ V}$$

Example 4.22 : *If a current of 5 A flowing in coil with 1000 turns wound on a ring of ferromagnetic material produces a flux of 0.5 mWb in the ring. Calculate (i) self inductance of coil (ii) e.m.f. induced in the coil when current is switched off and reaches zero value in 2 millisec. (iii) mutual inductance between the coils, if a second coil with 750 turns is wound uniformly over the first one.* **(May-2003)**

Solution : $\phi = 0.5$ mWb, N = 1000, I = 5 A

i) $$L = \frac{N\phi}{I} = \frac{1000\times0.5\times10^{-3}}{5} = \mathbf{0.1\ H}$$

ii) $$e = -L\frac{dI}{dt} = -0.1\left[\frac{0-5}{2\times10^{-3}}\right] = \mathbf{250\ V}$$

iii) Let N_2 = 750 of other coil

As other coil is wound on first, all the flux produced by coil 1 links with the second coil.

$$\therefore \quad \phi_2 = K_1 \phi_1 = \phi_1 \quad \text{as} \quad K_1 = 1$$

$$\therefore \quad M = \frac{N_2\phi_2}{I_1} = \frac{N_2[K_1\phi_1]}{I_1}$$

$$= \frac{750\times 05\times 10^{-3}}{5} = 0.075 \text{ H}$$

Example 4.23 : *An electric conductor of effective length of 0.3 metre is made to move with a constant velocity of 5 metre per second perpendicular to a magnetic field of uniform flux density 0.5 tesla. Find the e.m.f. induced in it. If this e.m.f. is used to supply a current of 25 A, find the force on the conductor, and state its direction w.r.t. motion of conductor, ignoring friction. Find the power required to keep the conductor moving across the field.* **(Dec.-2003)**

Solution : $l = 0.3$ m, $v = 5$ m/s, $B = 0.5$ T

$\therefore$ $$e = Blv = 0.3\times 5\times 0.5 = \mathbf{0.75\ V}$$

Now $$I = 25 \text{ A}$$

$\therefore$ $$F = B\,I\,l = 0.5\times 25\times 0.3 = \mathbf{3.75\ N}$$

The direction of this force is so as to oppose the motion of conductor, as per Lenz's law.

The power required to keep the conductor moving is,

$$P = e\times I = 0.75\times 3.75 = \mathbf{2.8125\ W}$$

Example 4.24 : *Two identical coils P and Q, each with 1500 turns, are placed in parallel planes near to each other, so that 70% of the flux produced by current in coil P links with coil Q. If a current of 4 A is passed through any one coil, it produces a flux of 0.04 mWb linking with itself. Find the self inductances of the two coils, the mutual inductance and coefficient of coupling between them.* **(Dec.-2003)**

Solution : $N_P = N_Q = 1500$, $\phi_Q = 0.7\ \phi_P$ ... 70% linking

Let $I_P = 4$ A and $\phi_P = 0.04$ mWb

$\therefore$ $$L_P = \frac{N_P\,\phi_P}{I_P} = \frac{1500\times 0.04\times 10^{-3}}{4} = \mathbf{15\ mH}$$

Let $I_Q = 4$ A then $\phi_Q = 0.04$ mWb

$\therefore$ $$L_Q = \frac{N_Q\,\phi_Q}{I_Q} = \frac{1500\times 0.04\times 10^{-3}}{4} = \mathbf{15\ mH}$$

$$M = \frac{N_Q\,\phi_Q}{I_P} = \frac{N_Q\ 0.7\,\phi_P}{I_P} = \frac{1500\times 0.7\times 0.04\times 10^{-3}}{4} = \mathbf{10.5\ mH}$$

And $$K = \frac{M}{\sqrt{L_P\,L_Q}} = \frac{10.5\times 10^{-3}}{\sqrt{(15\times 10^{-3})^2}} = \mathbf{0.7}$$

➡ **Example 4.25 :** *A coil of 450 turns is uniformly wound around a ring of an iron alloy of mean circumference of 100 cm and cross-sectional area 1.125 sq. cm. When a current of 0.5 ampere is linearly reduced to zero in 0.01 second, the e.m.f. induced in the coil is 2 V. Find the relative permeability of the iron alloy.* **(May-2004)**

Solution : $N = 450,\ l_i = 100\ \text{cm},\ a = 1.125\ \text{cm}^2$

$$e = -L\frac{dI}{dt}$$

$$dI = +0.5 \text{ to zero i.e. } 0 - 0.5 = -0.5$$

$$dt = 0.01 \text{ sec},\ e = 2\ \text{V}$$

$$\therefore \quad 2 = -L\frac{(-0.5)}{0.01} \quad \text{i.e. } \mathbf{L = 0.04\ H}$$

Now $$L = \frac{N^2}{S} \text{ and } S = \frac{l_i}{\mu_0 \mu_r a}$$

$$\therefore \quad 0.04 = \frac{(450)^2}{\dfrac{l_i}{\mu_0 \mu_r a}}$$

$$\therefore \quad \frac{1}{\mu_r} = \frac{(450)^2 \times (1.125\times10^{-4}) \times (4\pi\times10^{-7})}{100\times10^{-2}\times0.04}$$

$$\therefore \quad \mu_r = 1397.245 \approx \mathbf{1398}$$

➡ **Example 4.26 :** *A straight conductor 1.5 m long lies in a plane perpendicular to a uniform magnetic field of flux density 1.2 tesla. When a current of 'I' ampere is passed through it, it makes the conductor move across the magnetic field with a velocity of 1 m/s. Ignoring resistance of the conductor and friction, find the current 'I', if the power of the moving conductor is 90 watt. Find the e.m.f. induced in the conductor and the force on it. State the sense of the force w.r.t. the velocity, and sense of the e.m.f. induced w.r.t. current.* **(May-2004)**

Solution : $l = 1.5\ \text{m},\ B = 1.2\ \text{T},\ v = 1\ \text{m/s},\ P = 90\ \text{W}$

$$e = B\,l\,v = 1.2\times1.5\times1 = 1.8\ \text{V}$$

$$P = e\times I$$

$$\therefore \quad 90 = 1.8\times I$$

$$\therefore \quad I = \mathbf{50\ A}$$

$$\therefore \quad F = B\,I\,l = 1.2\times50\times1.5 = \mathbf{90\ N}$$

The force is so as to oppose the velocity while the sense of e.m.f. is so as to oppose the current.

Example 4.27 : *Two coils A and B, have self inductances of 120 μH and 300 μH respectively. A current of 1 A through coil 'A' produces flux linkage of 100 μWb turns in coil 'B'. Calculate*

i) mutual inductance between the coil.

ii) average e.m.f. induced in coil 'B' if current of 1 A in coil 'A' is reversed at a uniform rate in 0.1 sec. Also find coefficient of coupling. **(Dec.-2004)**

Solution : $L_A = 120\ \mu H,\ L_B = 300\ \mu H$

$$I_A = 1\text{ A produces } N_B\phi_B = 100\ \mu Wb$$

i) $$M = \frac{N_B\phi_B}{I_A} = \frac{100\times10^{-6}}{1} = \mathbf{100\ \mu H} \quad \text{...Mutual inductance}$$

ii) $$e_B = -M\frac{dI_A}{dt}$$

The current in coil A is reversed i.e. it is –1 A in 0.1 sec.

$\therefore$ $$\Delta I = (\text{New value} - \text{Original value}) = (-1-1) = -2\text{ A}$$

and $$\Delta t = 0.1\text{ sec}$$

$\therefore$ $$\frac{dI_A}{dt} = \frac{\Delta I}{\Delta t} = \frac{-2}{0.1} = -20\text{ A/sec}$$

$\therefore$ $$e_B = -100\times10^{6}\times(-20) = \mathbf{2\ mV} \quad \text{... Induced e.m.f. in B}$$

$$K = \frac{M}{\sqrt{L_A L_B}} = \frac{100\times10^{-6}}{\sqrt{120\times10^{-6}\times300\times10^{-6}}}$$

$$= \mathbf{0.527} \quad \text{... Coefficient of coupling}$$

Example 4.28 : *A magnetic core is in the form of a closed ring of mean length 20 cm and cross-sectional area 1 cms². Its relative permeability is 2400. A coil of 2000 turns is uniformly wound around it. Find the flux density set up in the core if a current of 66 mA is passed through the coil. Find the energy stored in the magnetic field set up.*

Find the inductance of the coil, if an air gap of 1 mm is cut in the ring perpendicular to the direction of the flux. **(May-2005)**

Solution : Given $l = 20$ cm, $a = 1\text{ cm}^2$, $\mu_r = 2400$, $N = 2000$, $I = 66$ mA

Case 1 : $$S = \frac{l}{\mu_0\mu_r a} = \frac{20\times10^{-2}}{4\pi\times10^{-7}\times2400\times1\times10^{-4}} = 663.1455\times10^{3}\ \text{AT/Wb}$$

$$\text{m.m.f} = NI = 2000\times66\times10^{-3} = 132\text{ AT}$$

$\therefore$ $$\phi = \frac{NI}{S} = \frac{132}{663.1455\times10^{3}} = 1.9905\times10^{-4}\ \text{Wb}$$

$\therefore \quad B = \frac{\phi}{a} = \frac{1.9905\times10^{-4}}{1\times10^{-4}} = \mathbf{1.9905\ Wb/m^2}$ i.e. T ...Flux density

$$L = \frac{N^2}{S} = \frac{(2000)^2}{663.1455\times10^3} = 6.03185 \text{ H or } L = \frac{N\phi}{I}$$

$\therefore \quad E = \frac{1}{2}LI^2 = \frac{1}{2}\times6.03185\times(66\times10^{-3})^2 = \mathbf{13.1373\ mJ}$ ...Energy stored

Case 2 : New air gap is cut of length l_g = 1 mm in the ring.

$\therefore \quad l_i$ = Iron length = $l - l_g = 20\times10^{-2} - 1\times10^{-3} = 0.199$ m

$\therefore \quad S = S_i + S_g = \frac{l_i}{\mu_0\mu_r a} + \frac{l_g}{\mu_0 a}$ ... $\mu_r = 1$ for air gap

$$= \frac{1}{\mu_0 a}\left[\frac{li}{\mu_r} + l_g\right] = \frac{1}{4\pi\times10^{-7}\times1\times10^{-4}}\left[\frac{0.199}{2400} + 1\times10^{-3}\right]$$

$= 8.6175\times10^6$ AT/Wb ...Total reluctance

$\therefore \quad L = \frac{N^2}{S} = \frac{(2000)^2}{8.6175\times10^6} = \mathbf{0.4641\ H}$ ...New inductance.

Example 4.29 : *Two long, single-layered solenoids 'x' and 'y' have the same length and the same number of turns. The cross-sectional areas of the two are 'a_x' and 'a_y' respectively, with 'a_y' < 'a_x'. They are placed coaxially, with solenoid 'y' placed within the solenoid 'x'. Show that the coefficient of coupling between them is equal to $\sqrt{a_y / a_x}$.* **(May-2005)**

Solution : The arrangement is shown in the Fig. 4.21.

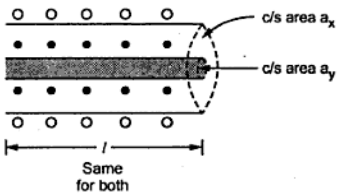


Fig. 4.21

It is known that

$$L = \frac{N^2\mu_0\mu_r a}{l}$$

For coil x,

$$L_x = \frac{N^2\mu_0\mu_r a_x}{l}$$

and $$L_y = \frac{N^2\mu_0\mu_r a_y}{l}$$

The number of turns N and μ_r is same for both.

Considering coil y,

$$M = \frac{N_1 N_2 a_y \mu_0 \mu_r}{l} \quad \text{where } N_1 = N_2 = N$$

$$\therefore \quad M = \frac{N^2 a_y \mu_0 \mu_r}{l}$$

The coefficient of coupling is given by,

$$K = \frac{M}{\sqrt{L_x L_y}} = \frac{\frac{N^2 a_y \mu_0 \mu_r}{l}}{\sqrt{\frac{N^2 \mu_0 \mu_r a_x}{l} \times \frac{N^2 \mu_0 \mu_r a_y}{l}}}$$

$$= \frac{\left(\frac{N^2 \mu_0 \mu_r}{l}\right) a_y}{\left(\frac{N^2 \mu_0 \mu_r}{l}\right) \sqrt{a_x a_y}} = \sqrt{\frac{a_y}{a_x}}$$

$$\therefore \quad K = \sqrt{\frac{a_y}{a_x}} \quad \text{...Proved}$$

Example 4.30 : *An iron ring wound with 500 turns solenoid produces a flux density of 0.94 tesla in the ring carrying a current of 2.4 Amp. The mean length of iron path is 80 cm and that of air gap is 1 mm. Determine i) the relative permeability of iron, ii) the self inductance and iii) energy stored in the above arrangement, if the area of cross-section of ring is 20* cm^2. **(Dec.-2005)**

Solution : $N = 500, \; I = 2.4 \text{ A}, \; B = 0.94 \text{ T}, \; a = 20 \text{ cm}^2$

l_i = Length of iron path = 80 cm

l_g = Length of air gap = 1 mm

i) $$\phi = B \times a = 0.94 \times 20 \times 10^{-4} = 1.88 \times 10^{-3} \text{ Wb}$$

$$\phi = \frac{\text{m.m.f}}{\text{reluctance}} = \frac{NI}{S}$$

$$\therefore \quad S = \frac{500 \times 2.4}{1.88 \times 10^{-3}} = 638297.8723 \text{ AT/Wb}$$

But $$S = S_i + S_g = \frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a} \quad \text{... } \mu_r = 1 \text{ for air gap}$$

$$\therefore \quad 638297.8723 = \left[\frac{80 \times 10^{-2}}{\mu_r} + \frac{1 \times 10^{-3}}{1}\right] \frac{1}{4\pi \times 10^{-7} \times 20 \times 10^{-4}}$$

$\therefore \quad \mu_r = \mathbf{1324.02}$

ii) $\quad L = \frac{N^2}{S} = \frac{(500)^2}{638297.8723} = \mathbf{0.3916\ H}$

iii) $\quad E = \frac{1}{2} LI^2 = \frac{1}{2} \times 0.3916 \times (2.4)^2 = \mathbf{1.1278\ J}$

Example 4.31 : *An air cored solenoid 1 m in length and 10 cm in diameter has 5,000 turns. Calculate : (i) the self inductance and (ii) the energy stored in the magnetic field when current of 2 A flows in solenoid.* **(Dec.-2006)**

Solution : $l = 1$ m, d = 10 cm, N = 5000, $\mu = \mu_0$ as air cored

$$a = \frac{\pi}{4} d^2 = \frac{100\pi}{4} \text{cm}^2 = 7.854 \times 10^{-3} \text{m}^2$$

$$\therefore \quad S = \frac{l}{\mu_0 a} = \frac{1}{4\pi \times 10^{-7} \times 7.854 \times 10^{-3}} = 101.3209 \times 10^6 \text{ AT/Wb}$$

i) $\quad L = \frac{N^2}{S} = \frac{(5000)^2}{101.3209 \times 10^6} = \mathbf{0.2467\ H}$

ii) $\quad I = 2$ A

$\therefore \quad E = \frac{1}{2} LI^2 = \frac{1}{2} \times 0.2467 \times 2^2 = \mathbf{0.4934\ J}$

Example 4.32 : *An iron ring of 10 cm in diameter and 8 cm² in cross-section is wound with 300 turns of wire. For a flux density of 1.2 Wb/m² and relative permeability of 500, find the exciting current, the inductance and the energy stored.* **(May-2007)**

Solution : d = 10 cm, a = 8 cm², N = 300, B = 1.2 Wb/m², $\mu_r = 500$

$$l = \pi \times d = \pi \times 10 \text{ cm} = 0.3141 \text{ m}$$

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{0.3141}{4\pi \times 10^{-7} \times 500 \times 8 \times 10^{-4}} = 624.882 \times 10^3 \text{ AT/Wb}$$

$$\phi = B \times a = 1.2 \times 8 \times 10^{-4} = 9.6 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad \phi = \frac{NI}{S}$$

$$\therefore \quad 9.6 \times 10^{-4} = \frac{300 \times I}{624.882 \times 10^3}$$

$\therefore \quad I = \mathbf{2\ A}$

$$L = \frac{N^2}{S} = \frac{(300)^2}{624.882 \times 10^3} = \mathbf{0.14402\ H}$$

$\therefore \quad E = \frac{1}{2} LI^2 = \frac{1}{2} \times 0.14402 \times (2)^2 = \mathbf{0.288\ J}$

Review Questions

1. *State the Faraday's laws of electromagnetism.*
2. *What is the difference between dynamically induced e.m.f. and statically induced e.m.f. ?*
3. *Derive the expression for the magnitude of the dynamically induced e.m.f.*
4. *Explain clearly the difference between self inductance and mutual inductance. State their units.*
5. *Derive the various expressions for the self inductance.*
6. *Explain the factors on which self inductances depends.*
7. *Derive the various expressions for the mutual inductance.*
8. *Derive the expression for coefficient of coupling.*
9. *Derive the expression for the equivalent inductance when two inductances are connected in*
   i) Series aiding (cumulatively coupled) ii) Series opposition (differentially coupled).
10. *How energy gets stored in the magnetic field ?*
11. *Derive the expression for energy stored in the magnetic field.*
12. *Write a note on lifting power of an electromagnet.*
13. *Two identical 1000 turn coils X and Y lie in parallel planes such that 60% of the flux produced by one coil links with the other. A current of 5 A in X produces a flux of 5×10^{-6} Wb in itself. If the current in X changes from + 6 A to – 6 A in 0.01 sec, what will be the magnitude of the e.m.f. induced in Y ? Calculate the self inductance of each coil.* **(Ans. : 0.72 V, 0.001 H)**
14. *Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius. Also calculate energy stored in it if current rises from zero to 5 A.* **(Ans. : 1.579 mH, 0.01973 J)**
15. *Two 200 turns, air cored solenoids, 25 cm long have a cross-sectional area of 3 cm^2 each. The mutual inductance between them is 0.5 μH. Find the self inductance of the coils and the coefficient of coupling.* **(Ans. : 60.31 μH, 0.00828)**
16. *Two coils A and B having 5000 and 2500 turns respectively are wound on a magnetic ring. 60 % of the flux produced by coil A links with coil B. A current of 1 A produces a flux of 0.25 mWb in coil A while same current produces a flux of 0.15 mWb in coil B. Find the mutual inductance and coefficient of coupling.* **(Ans. : 1.25 H, 0.375 H, 0.5477)**
17. *A conductor has 1.9 m length. It moves at right angles to a uniform magnetic field. The flux density of the magnetic field is 0.9 tesla. The velocity of the conductor is 65 m/sec. Calculate the e.m.f. induced in the conductor.* **(Ans. : 111.15 volts)**
18. *An air cored coil has 800 turns. Length of the coil is 6 cm while its diameter is 4 cm. Find the current required to establish flux density of 0.01 T in core and self inductance of the coil.* **(Ans. : 0.5968 A, 16.844 mH)**

19. *A flux of 0.25 mWb is produced when a current of 2.5 a passes through a coil of 1000 turns. Calculate*

    i) Self inductance

    ii) E.M.F. induced in the coil if the current of 2.5 A is reduced to zero in 1 milliseconds.

    iii) If second coil of 100 turns is placed near to the first on the same iron ring, calculate the mutual inductance between the coils. **(Ans. : 0.1 H, 250 V, 0.01 H)**

20. *Two coils A and B having 180 and 275 number of turns respectively are closely wound on an iron magnetic circuit, which has a mean length of 1.5 m and cross-sectional area of 150 cm. The relative permeability of iron is 1500. Determine mutual inductance between the coils. When will be the e.m.f. induced in a coil B if the current in coil A changes uniformly from 0 to 2.5 a in 0.03 seconds?* **(Ans. : 0.933 H, – 77.75 Volts)**

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