ANSWER KEY

CIA-II - 14E304 - ELECTROMAGNETIC THEORY

PART - A:

A1: Point form: $(\nabla \cdot \vec{J}) = \dfrac{-\partial \rho_v}{\partial t}$ (1)

Integral form: $\oint_S \vec{J} \cdot d\vec{s} = \dfrac{-dQ_i}{dt}$ (1)

A2: Point form of ohms law:

The relationship between $\vec{J}$ and $\vec{E}$ can also be expressed interms of conductivity of the material

$$\vec{J} = \sigma \vec{E}$$

A3: Properties of dielectric material: (Any two)

1. The dielectric donot contain any free charges but contain bound charges.
2. Bound charges are under the internal molecular and atomic forces and cannot contribute to the conduction.
3. Due to the polarization, the dielectrics can store the energy.
4. Due to the polarization, the flux density of the dielectric increases by amount equal to the polarization.
5. The induced dipoles produce their own electric field and align in the direction of the applied electric field.
6. The electric field outside and inside the dielectric gets modified due to the induced electric dipoles.
7. Due to electric field $\vec{E}$, small electric dipoles gets induced inside the dielectric. This is called polarization.

A4:

$E_{tan} = D_{tan} = 0$

$D_N = \rho_s$

$E_N = \dfrac{\rho_s}{\varepsilon} = \dfrac{\rho_s}{\varepsilon_0 \varepsilon_r}$

A5: Uniqueness Theorem:

If the solution of Laplace's equation satisfies the boundry condition then that solution is unique, by whatever method it is obtained.

A6: curl: (1)

The curl of a vector in the direction of the unit vector is the ratio of the line Integral of a vector around a closed contour, to the enclosed area bounded by the contour, as the enclosed area tends to zero.

$$\lim_{\Delta S_N \to 0} \frac{\oint \vec{H} \cdot \vec{dL}}{\Delta S_N} = J_N$$

Properties of curl: (Any two) (1)

1. The curl of a vector is a vector quantity
2. $\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$
3. $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$
4. $\nabla \cdot (\nabla \times \vec{A}) = 0$
5. $\nabla \times \nabla V = 0$

A7: Amperes circuital law:

The line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path,

$$\oint \vec{H} \cdot \vec{dL} = I \qquad (1)$$

Point form of Amp. circuital law:

$$\nabla \times \vec{H} = \vec{J} \qquad (1)$$

A8: Self Inductance: (1)

The ratio of total flux linkage to the current through the circuit is called inductance and it is given by

$$L = \frac{N\phi}{I}.$$

The inductance is known as Self Inductance

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Mutual Inductance: (1)

The Mutual Inductance between the two circuit is defined as the flux linkage of one circuit to the current in the other circuit.

$$M = \frac{\text{Flux linkage of circuit 1}}{\text{current in circuit 2}}$$

A9: Solution:

$$M = K\sqrt{L_1 L_2} = 0.05\sqrt{800 \times 200} = 20\ \mu H \quad (1)$$

$$L = L_1 + L_2 + 2M = 800\times10^{-6} + 200\times10^{-6} + 2\times20\times10^{-6}$$

$$\boxed{L = 1040\ \mu H} \quad (1)$$

A10: Magnetic Torque: (1)

The magnetic torque on the loop is defined as the vector product of the Force (F) and the moment arm (r)

$$T = F \times r \quad N.m$$

Magnetic Dipole: (1)

The Magnetic field which is having two poles and forms a closed loop is called Magnetic Dipole.

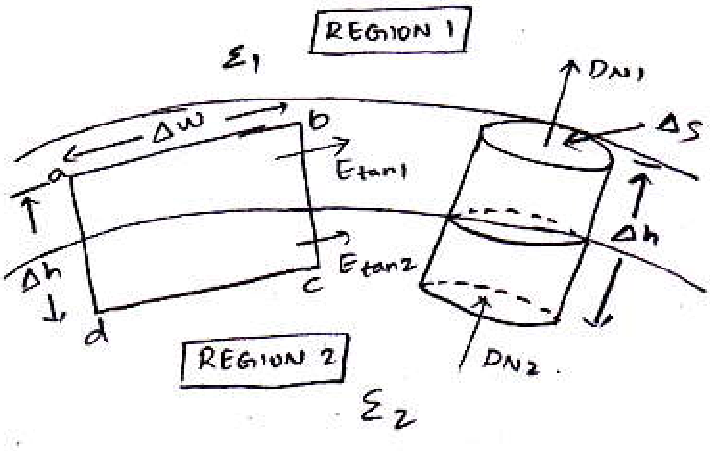
PART-B

B1) a) i) Boundry Conditions between Dielectric to Dielectric:

Tangential Component:

$$\oint \vec{E}\cdot\vec{dl} = 0 \quad \text{—— (1)}$$

Evaluating the integral $\vec{E}\cdot\vec{dl}$ along the path a, b, c, d, a in clockwise direction.



$$\int_a^b \vec{E}\cdot\vec{dl} + \int_b^c \vec{E}\cdot\vec{dl} + \int_c^d \vec{E}\cdot\vec{dl} + \int_d^a \vec{E}\cdot\vec{dl} = 0 \quad \text{— (2)}$$

Now, $\vec{E_1} = \vec{E_{1t}} + \vec{E_{1N}}$ $\quad |\vec{E_{1t}}| = E_{tan1}$ $\quad |\vec{E_{1N}}| = E_{1N}$

$\vec{E_2} = \vec{E_{2t}} + \vec{E_{2N}}$ $\quad |\vec{E_{2t}}| = E_{tan2}$ $\quad |\vec{E_{2N}}| = E_{2N}$

To analyse boundry condition, $\Delta h \to 0$

$\therefore$ In eq (2) $\int_b^c \vec{E}\cdot\vec{dl} + \int_d^a \vec{E}\cdot\vec{dl} = 0$ as $\Delta h \to 0$

$$\therefore \int_a^b \vec{E}\cdot\vec{dl} + \int_c^d \vec{E}\cdot\vec{dl} = 0 \quad \text{— (3)}$$

for $a-b \Rightarrow E = E_{tan1}$ $\quad dL = \Delta w$

for $c-d \Rightarrow E = E_{tan2}$ $\quad dL = \Delta w$

$$E_{tan1}(\Delta w) - E_{tan2}\,\Delta w = 0$$

$$\boxed{E_{tan1} = E_{tan2}} \qquad (1)$$

w.r.t $\vec{D} = \varepsilon\vec{E}$ $\quad D_{tan1} = \varepsilon_1 E_{tan1}$

$D_{tan2} = \varepsilon_2 E_{tan2}$

$$\frac{D_{tan1}}{\varepsilon_1} = \frac{D_{tan2}}{\varepsilon_2}$$

$$\boxed{\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}} \qquad (1)$$

<u>Normal component:</u> consider Gaussian Surface

$$\oint \vec{D}\cdot\vec{ds} = Q$$

$$\left[\int_{top} + \int_{bottom} + \int_{lateral}\right] \vec{D}\cdot\vec{ds} = Q$$

As $\Delta h \to 0$ $\quad \int_{lateral} \vec{D}\cdot\vec{ds} = 0$

$$\left[\int_{top} + \int_{bottom}\right] \vec{D}\cdot\vec{ds} = Q$$

$$D_{N1}\,\Delta S - D_{N2}\,\Delta S = Q$$

But $\quad Q = \rho_s\,\Delta s$

$$\boxed{D_{N1} - D_{N2} = \rho_s} \qquad (1)$$

$\rho_s = 0$ for ideal dielectric, then

$$\boxed{D_{N1} = D_{N2}}$$

$$D_{N1} = \varepsilon_1 E_{N1} \qquad D_{N2} = \varepsilon_2 E_{N2}$$

$$\boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}} \qquad (1)$$

Conductor to Dielectric Medium (2)

$$E_{tan} = D_{tan} = 0$$

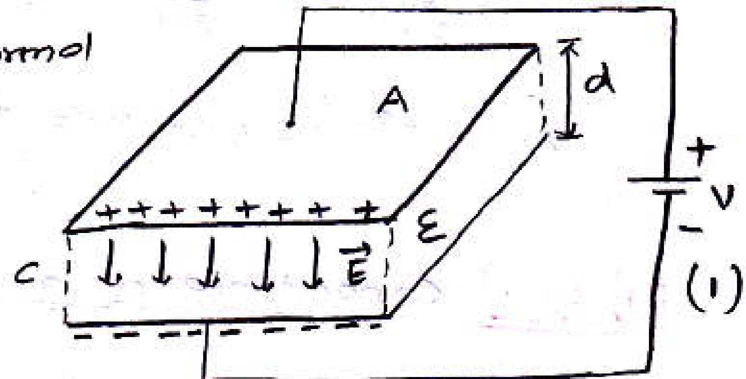
$$D_N = \rho_s$$

$$E_N = \frac{\rho_s}{\varepsilon}$$

B1) a) ii) Energy Stored in a capacitor:

Let $\vec{a_N}$ is the direction normal to the plates

$$\therefore\ \vec{E} = \frac{V}{d}\,\vec{a_N}$$



(1)

The energy stored is given by,

$$W_E = \frac{1}{2}\int_{Vol} \vec{D}\cdot\vec{E}\,dv \qquad \vec{D} = \varepsilon\vec{E} \qquad (1)$$

$$= \frac{1}{2}\int_{Vol} \varepsilon\,\vec{E}\cdot\vec{E}\,dv \qquad \vec{E}\cdot\vec{E} = |\vec{E}|^2$$

$$= \frac{1}{2}\int_{Vol} \varepsilon\,|\vec{E}|^2\,dv$$

$$= \frac{1}{2}\int_{Vol} \varepsilon\left(\frac{V^2}{d^2}\right)dv \qquad |\vec{E}| = \frac{V}{d} \qquad (1)$$

but $\int_{vol} dv = \text{volume} = A \times d$

$$W_E = \frac{1}{2} \varepsilon \frac{V^2 A d}{d^2}$$

$$= \frac{1}{2} \frac{\varepsilon A}{d} V^2 = \frac{1}{2} C V^2 \quad \left[\because C = \frac{\varepsilon A}{d}\right]$$

$$\boxed{W_E = \frac{1}{2} C V^2} \qquad (2)$$

B1) a) iii) Given:

$V = 2x^2 - 3y^2 + z^2$

To find:

Statisfies the laplace equation or not

Solution:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad (1)$$

$$= \frac{\partial^2}{\partial x^2}\left[2x^2 - 3y^2 + z^2\right] + \frac{\partial^2}{\partial y^2}\left[2x^2 - 3y^2 + z^2\right] + \frac{\partial^2}{\partial z^2}\left[2x^2 - 3y^2 + z^2\right] \quad (1)$$

$$= \frac{\partial}{\partial x}[4x] + \frac{\partial}{\partial y}[-6y] + \frac{\partial}{\partial z}[2z] = 4 - 6 + 2 = \underline{\underline{0}} \quad (2)$$

As, $\nabla^2 V = 0$, the field statisfies the laplace equation.

B1) b) i) Capacitance of a parallel plate capacitor:

Let,

Q = charge on each plate

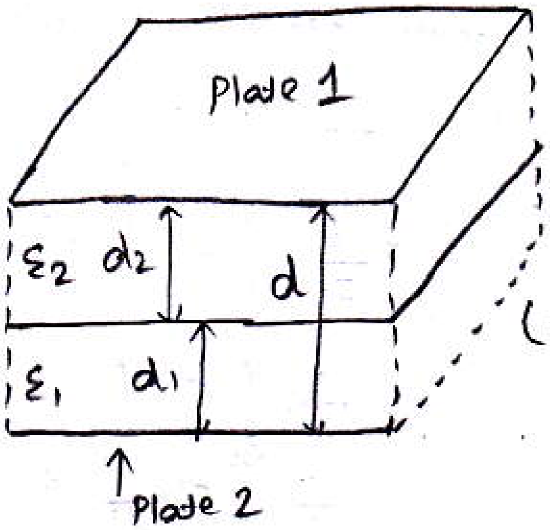
$\vec{E_1}$ = field intensity in region d_1

$\vec{E_2}$ = field intensity in region d_2

Both the intensities are uniform

$V_1 = E_1 d_1$

$V_2 = E_2 d_2$



(1)

$$V = V_1 + V_2 = E_1 d_1 + E_2 d_2 \quad \text{--- (1)}$$

At a dielectric - dielectric interface,

$$D_{N1} = D_{N2}$$

Now $D_1 = \varepsilon_1 E_1$ & $D_2 = \varepsilon_2 E_2$

$$E_1 = \frac{D_1}{\varepsilon_1} \quad \& \quad E_2 = \frac{D_2}{\varepsilon_2} \quad \text{--- (2)}$$

Sub (2) in (1) we get

$$V = \frac{D_1}{\varepsilon_1} d_1 + \frac{D_2}{\varepsilon_2} d_2 \quad \text{--- (3)}$$

The magnitude of surface charge is same on each plate,

$$\therefore \rho_s = D_1 = D_2 \quad \text{--- (4)}$$

Sub eq (4) in eq (3)

$$V = \frac{\rho_s}{\varepsilon_1} d_1 + \frac{\rho_s}{\varepsilon_2} d_2 = \rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right] \quad \text{--- (5)} \qquad (2)$$

Now, $C = \dfrac{Q}{V} = \dfrac{Q}{\rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right]}$ (1)

But $Q = \rho_s A$

$$C = \frac{\rho_s A}{\rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right]} = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} = \frac{1}{\frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}} \quad \boxed{C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}} \ (2)$$

where, $C_1 = \dfrac{\varepsilon_1 A}{d_1}$ & $C_2 = \dfrac{\varepsilon_2 A}{d_2}$

B1) b) ii)

Given: Diameter = 2 cm

a = radius of Sphere $= \frac{d}{2} = 1 \text{ cm} = 1 \times 10^{-2}$ m

$\varepsilon_r = 2.26$

thickness = 3 cm

To Find:

capacitance

Solution: $$C = \frac{4\pi}{\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{r_1}\right) + \frac{1}{\varepsilon_0 r_1}} \text{ F} \qquad (2)$$

$a = 1 \times 10^{-2}$ m

$r_1 = a + \text{thickness} = 1 + 3 = 4 \text{ cm} = 4 \times 10^{-2}$ m (1)

$$c = \frac{4\pi}{\frac{1}{2.26}\left[\frac{1}{1\times10^{-2}} - \frac{1}{4\times10^{-2}}\right] + \frac{1}{8.854\times10^{-12}\times4\times10^{-2}}} \qquad (1)$$

$$\boxed{c = 1.9121 \text{ pF}} \qquad (1)$$

B1) b) iii) Poisson's and Laplace equation:

From the Gauss's law in the point form, poissons equation can be derived.

Consider the Gauss's Law in the point form as

$$\nabla \cdot \vec{D} = \rho_v \quad \text{—} (1)$$

w.k.t $\quad \vec{D} = \varepsilon \vec{E} \quad$ — (2)

$$\nabla \cdot \varepsilon \vec{E} = \rho_v \quad \text{—} (3)$$

from gradient $\quad \vec{E} = -\nabla V \quad$ — (4)

Sub (4) in (3) we get

$$\nabla \cdot \varepsilon(-\nabla V) = \rho_v$$

$$-\varepsilon[\nabla \cdot \nabla V] = \rho_v$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon} \quad \text{—} (5)$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}} \quad \text{—} (6) \qquad (2)$$

Equation (6) is called poisson's equation.

In certain region, $\rho_v = 0$

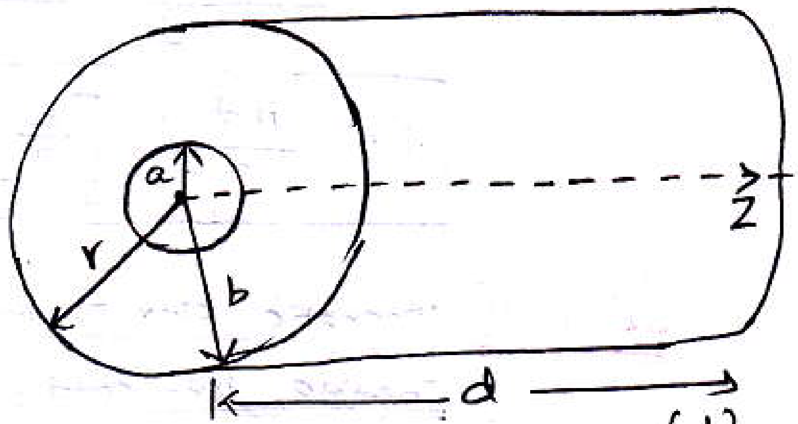
$$\boxed{\nabla^2 V = 0} \qquad (2)$$

This is a special case of poisson's equation called as Laplace's equation.

B2) a) i) Inductance of a co-axial cable:

Magnetic field intensity at any point between inner and outer conductor is given by

$$\vec{H} = \frac{I}{2\pi r} \quad a < r < b$$



(1)

w.k.t $\vec{B} = \mu \vec{H}$

$$\vec{B} = \frac{\mu I}{2\pi r} \qquad (1)$$

Axis of cable along z-axis ∴ $\vec{B}$ will be in radial plane extending from $r=a$ to $r=b$ and $z=0$ to $z=d$.

The total magnetic flux is given by

$$\phi = \int_S \vec{B} \cdot \vec{ds}$$

$$\vec{ds} = dr\, dz\, a_\phi \qquad (1)$$

$$\phi = \int_{z=0}^{z=d} \int_{r=a}^{r=b} \left(\frac{\mu I}{2\pi r}\right) a_\phi \,(dr\, dz\, a_\phi) \qquad (1)$$

$$\phi = \int_{z=0}^{d} \int_{r=a}^{b} \frac{\mu I}{2\pi r}\, dr\, dz$$

$$\phi = \frac{\mu I}{2\pi} \int_{z=0}^{d} \int_{r=a}^{b} \frac{1}{r}\, dr\, dz$$

$$\phi = \frac{\mu I}{2\pi} [z]_0^d \, [\ln r]_a^b$$

$$\phi = \frac{\mu I d}{2\pi} \ln\left[\frac{b}{a}\right]$$

Inductance of a co-axial cable is given by

$$L = \frac{\text{Total Flux Linkage}}{\text{Total current}} \qquad (1)$$

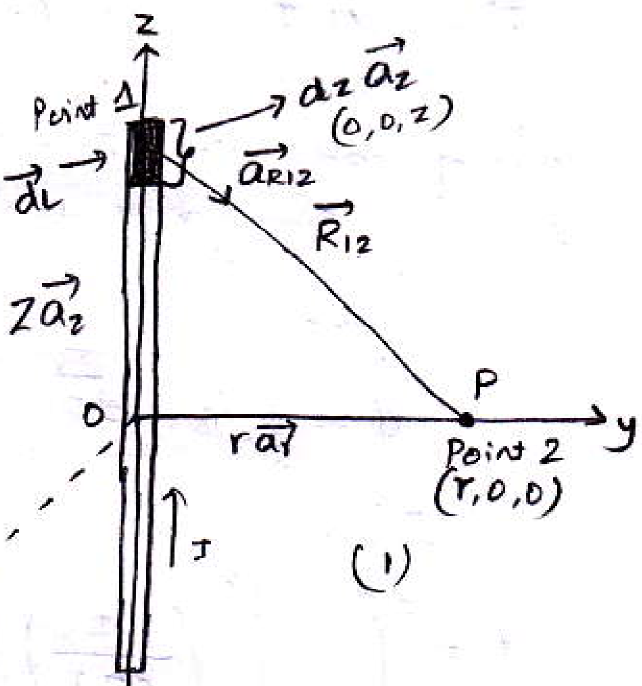
$$L = \frac{\frac{\mu I d}{2\pi} \ln\left[\frac{b}{a}\right]}{I}$$

$$\boxed{L = \frac{\mu d}{2\pi} \ln\left[\frac{b}{a}\right] \text{ Henry}} \qquad (1)$$

B2) a) ii) Magnetic Flux Intensity and density due to an Infinite long conductor using Biot - Savarts law:

Consider small differential element at Point 1, along the z-axis at a distance of z from orgin,

$$\therefore \boxed{I\,\vec{dL} = I\,dz\,\vec{a_z}} \quad \text{—— ①}$$



(1)

The distance vector joining Point 1 to point 2 is $\vec{R_{12}}$

$$\vec{R_{12}} = r\vec{a_r} + 0\,\vec{a_\phi} - z\,\vec{a_z}$$

$$\boxed{R_{12} = r\vec{a_r} - z\,\vec{a_z}} \quad \text{—— ②} \qquad (1)$$

$$\vec{a_{R12}} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{-z\vec{a_z} + r\vec{a_r}}{\sqrt{r^2+z^2}} = \frac{r\vec{a_r} - z\vec{a_z}}{\sqrt{r^2+z^2}}$$

$$I\,\vec{dL} \times a_{R12} = \frac{I r\,dz\,\vec{a_\phi}}{\sqrt{r^2+z^2}}$$

According to Biot Savarts law, $\vec{dH}$ at point 2 is

$$\vec{dH} = \frac{I\,\vec{dL} \times \vec{a_{R12}}}{4\pi R^2} = \frac{I\,dz \times \vec{a_z}}{4\pi(r^2+z^2)} \times \frac{r\vec{a_r} - z\vec{a_z}}{\left(\sqrt{r^2+z^2}\right)}$$

$$a_z \times a_r = a_\phi \qquad a_z \times a_z = 0$$

$$\boxed{\vec{dH} = \frac{I r\,dz}{4\pi(r^2+z^2)^{3/2}}\,a_\phi} \qquad (1)$$

Thus total Flux intensity $\vec{H}$ can be obtained by integrating. $\vec{dH}$ over the entire length of the conductor

$$\vec{H} = \int_{z=-\infty}^{\infty} \vec{dH} = \int_{z=-\infty}^{\infty} \frac{I r dz \, \vec{a_\phi}}{4\pi (r^2+z^2)^{3/2}}$$

By Subsitution method,

$$z = r \tan\theta \qquad dz = r \sec^2 d\theta$$

$$\therefore \vec{H} = \int_{\theta=-\pi/2}^{\pi/2} \frac{I r \, r \sec^2\theta \, d\theta \, \vec{a_\phi}}{4\pi (r^2 + r^2 \tan^2\theta)^{3/2}}$$

$$\boxed{\vec{H} = \frac{I}{2\pi r} a_\phi} \quad A/m \qquad (1)$$

$$B = \mu H$$

$$\boxed{\vec{B} = \frac{\mu I}{2\pi r} a_\phi} \quad wb/m^2 \qquad (1)$$

B2) a) iii)

Given:

$N = 500$ turns

$A = 3\,cm^2 = 3 \times 10^{-4}\,m^2$

$R = 10\,cm = 10 \times 10^{-2}\,m$

$\mu_r = 800$

To find:

Self Inductance

Solution:

$$L = \frac{\mu N^2 A}{2\pi R} \qquad (2)$$

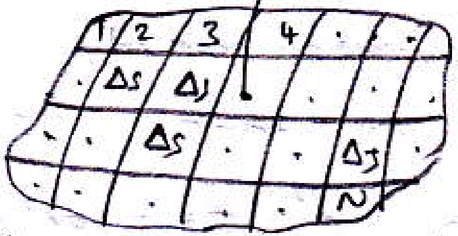
$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 800$$

$$L = \frac{4\pi \times 10^{-7} \times 800 \times (500)^2 \times 3 \times 10^{-4}}{2\pi \times 10 \times 10^{-2}} = 0.12\,H \quad (1)$$

$$(1) \quad \boxed{L = 120\,mH}$$

B2) b) i) Stokes Theorem:

The stokes theorem states that "the line integral of Vector $\vec{A}$ around a closed path L is equal to the integral of curl of $\vec{A}$ over the open surface S enclosed by the closed path L".



(1)

$$\oint \vec{H}\cdot\vec{dL} = \int_1 \vec{H}\cdot\vec{dL} + \int_2 \vec{H}\cdot\vec{dL} + \cdots + \int_N \vec{H}\cdot\vec{dL} \quad (1)$$

from curl definition,

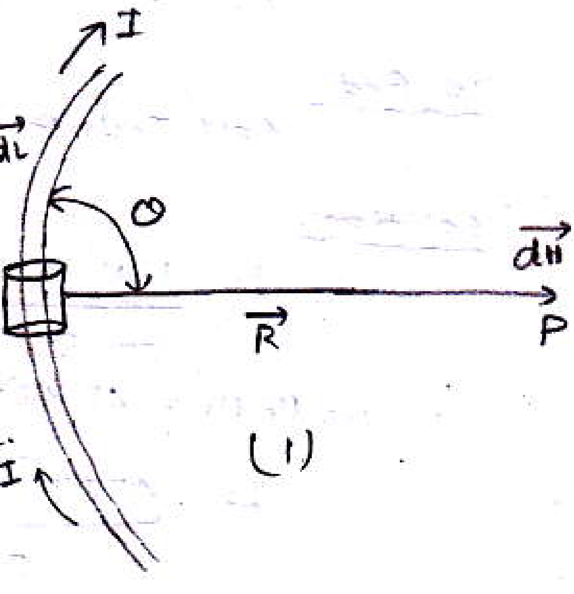
$$\lim_{\Delta s \to 0} \frac{\oint \vec{H}\cdot\vec{dL}}{\Delta S} = (\nabla \times \vec{H})_N \quad \text{—— } 1 \quad (1)$$

$$\oint \vec{H}\cdot\vec{dL} = \lim_{\Delta S \to 0} (\nabla\times\vec{H})_N \, \Delta S \quad \text{—— } 2 \quad (1)$$

$$\oint \vec{H}\cdot\vec{dL} = (\nabla\times\vec{H})\,\vec{a_N}\;\Delta S$$

$$\boxed{\oint H\cdot\vec{dL} = (\nabla\times\vec{H})\;\Delta S} \quad (2)$$

B2) b) ii) Biot - Savarts law:



Biot Savarts law states that,

The Magnetic field intensity $\vec{H}$ produced at a point P due to a differential current element IdL is,

1. proportional to the product of current I and differential length dL.

(1)

2. The sine of the angle b/w the element and the line joining point P to the element ⑦

3. And inversely proportional to the square of the distance R b/w point P and the element. (1)

Mathematically, the Biot - Savarts law can be stated as,

$$\vec{dH} \propto \frac{I\, dL \sin\theta}{R^2} \quad \text{--- (1)}$$

$$\vec{dH} = \frac{K I\, dL \sin\theta}{R^2} \quad \text{--- (2)} \quad K \rightarrow \text{constant of Proportionality}$$

$$K = \frac{1}{4\pi}$$

$$\vec{dH} = \frac{I\, dL \sin\theta}{4\pi R^2} \quad \text{--- (3)} \qquad (1)$$

from rule of cross product

$$\vec{dL} \times \vec{a_R} = dL\, |\vec{a_R}| \sin\theta = dL \sin\theta \quad \text{--- (4)} \quad (1)$$

using (4) in (3) we get

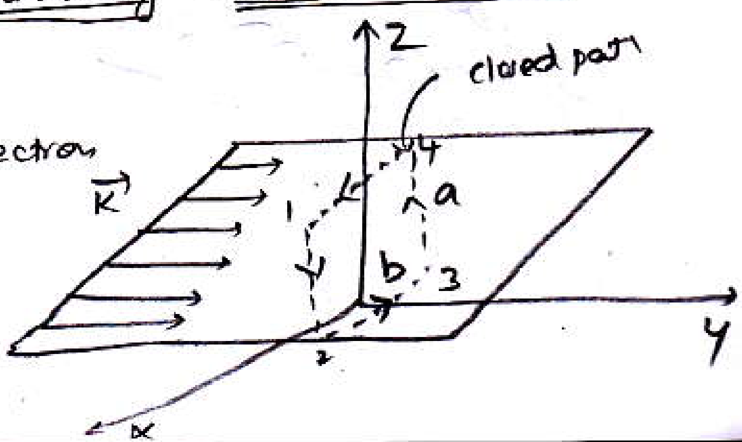
$$\vec{dH} = \frac{I\, \vec{dL} \times \vec{a_R}}{4\pi R^2} \quad A/m$$

$$\vec{a_R} = \frac{\vec{R}}{|\vec{R}|} = \frac{\vec{R}}{R}$$

$$\boxed{\vec{dH} = \frac{I\, \vec{dL} \times \vec{R}}{4\pi R^3}} \quad A/m \qquad (1)$$

The above eq. is the mathematical form of Biot Savarts law.

B2) b) iii) Magnetic field intensity - Infinite sheet:



Current Flowing in y direction

$K = K_y a_y$

Applying Amperes circuital law

$$\oint \vec{H}\cdot\vec{dL} = I_{enclosed}$$ (1)

$$\oint \vec{H}\cdot\vec{dL} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 (\vec{H}\cdot\vec{dL})$$

from closed path, 1-2 and 3-4 the integrals

$$\int_1^2 \vec{H}\cdot\vec{dL} + \int_3^4 \vec{H}\cdot\vec{dL} = 0$$

consider path 2-3,

$$\int_2^3 \vec{H}\cdot\vec{dL} = \int_2^3 (-H_x\,\vec{a_x})\cdot(dx\,\vec{a_x}) = b\,H_x$$ (1)

consider path 4-1

$$\int_4^1 \vec{H}\cdot\vec{dL} = \int_4^1 (H_x\,\vec{a_x})\cdot(dx\,\vec{a_x}) = b\,H_x$$ (1)

$$\oint H\cdot dL = b\,H_x + b\,H_2 = 2b\,H_x = I_{enclosed}$$

$$2bH_x = K_y\,b$$

$$\boxed{H_x = \frac{1}{2}K_y}$$ (1)

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