

## 12E605 - POWER ELECTRONICS (ANSWER KEY)

## PART - A

## A1. UnControlled rectifier

A rectifier employing diodes is called an uncontrolled rectifier, because its average output voltage is a fixed dc voltage.

(2)

V<sub>avg</sub> is high compared to fully controlled Rectifier.

## Controlled rectifier

A rectifier employing Thyristor is called as controlled rectifier, because the average output voltage is a variable dc voltage.

(2)

V<sub>avg</sub> is less compared to uncontrolled Rectifier

## A2. Expression of average DC load voltage of 1φ fully controlled converter with R load.

$$V_{dc} \text{ (or) } V_o = \frac{V_m}{\pi} (1 + \cos \alpha) \quad (2)$$

## A3. Condition to make fully controlled rectifier to operate in inversion mode.

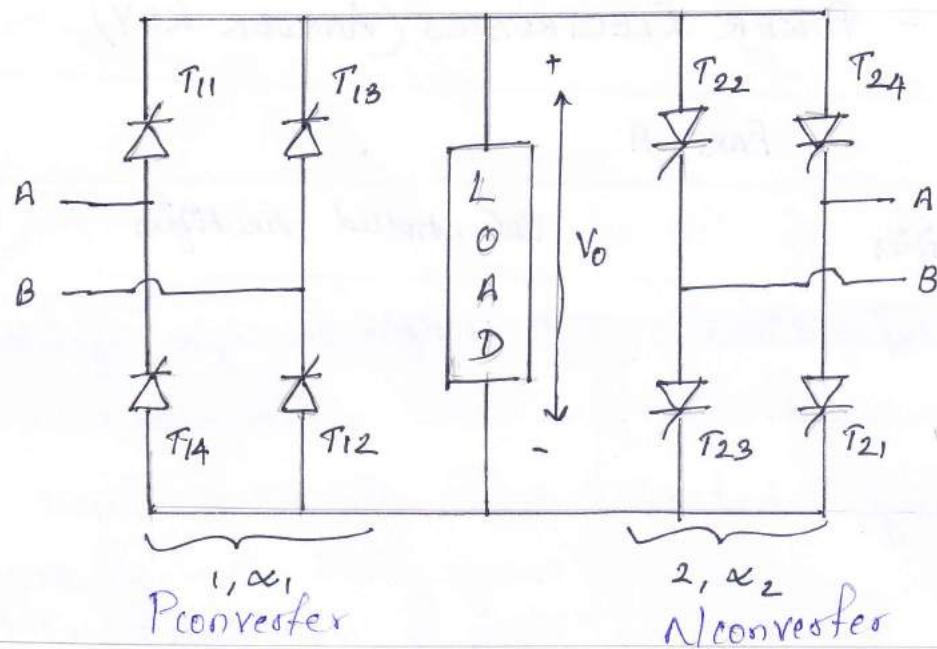
For  $\alpha > 90^\circ$ , the fully controlled rectifier operates in inversion mode and during the inversion operation the DC power at the output is converted to AC power at the source (or) i/p

## A4. Range of firing angle and extinction angle in fully controlled converter.

For fully controlled converter, the range of firing angle is  $(0^\circ - 180^\circ)$  & the range of extinction angle is  $(180^\circ \text{ to } 360^\circ)$ .

$\alpha \leq 180^\circ$  and  $\beta \geq 180^\circ$

A5. Circuit diagram of single phase dual converter.



A6. Constant frequency control and Variable frequency control.

In this control technique, the frequency of the chopper remains constant, but on period is changed to vary the output.

It should be noted that as 'ON' pulse width varies, 'OFF' pulse width also varies.

In this control technique, either on time ( $T_{on}$ ) or off time ( $T_{off}$ ) is kept constant. And frequency is varied to change the off voltage.

A7. Expression of average off voltage for step up and step down chopper.

$$\text{For step up chopper} \rightarrow V_{o(\text{av})(\text{on})} V_{dc} = \frac{V_s}{1-s}$$

$$\text{For stepdown chopper} \rightarrow V_{o(\text{av})(\text{on})} V_{dc} = s V_s$$

$$\delta \rightarrow \text{duty cycle} = \frac{T_{on}}{T} \quad V_s \rightarrow \text{ip source voltage}$$

### A8. Current limit control.

In this control technique, the output current is continuously sensed and is maintained within minimum & maximum limit.

If the output current falls below certain minimum level, then chopper turns-on. This increases the output as well as output current. When reaches maximum switch is turned-OFF.

A9.

Given :

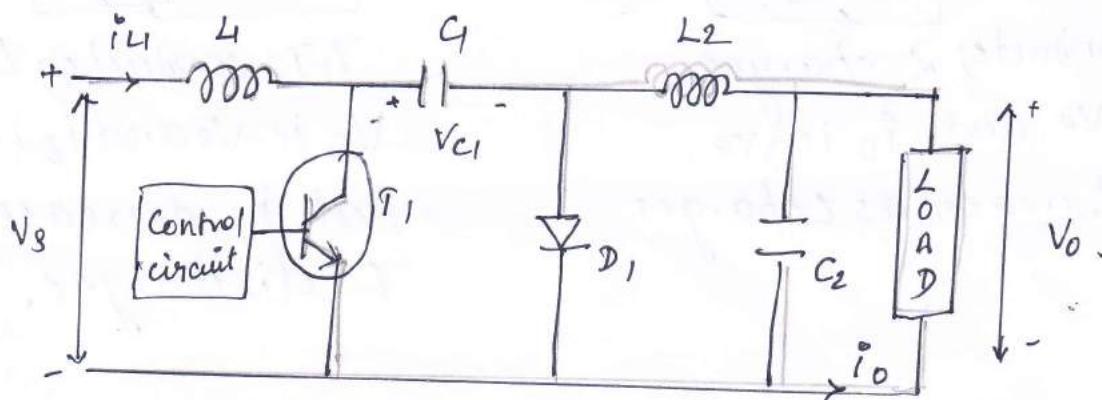
$$V_s = 80 \text{ V}, \delta (\text{duty cycle}) = 20\% = 0.2$$

To find : o/p voltage of boost converter  $V_o = \frac{V_s}{1 - \delta}$

$$= \frac{80}{1 - 0.2} = \frac{80}{0.8} = 100 \text{ V}$$

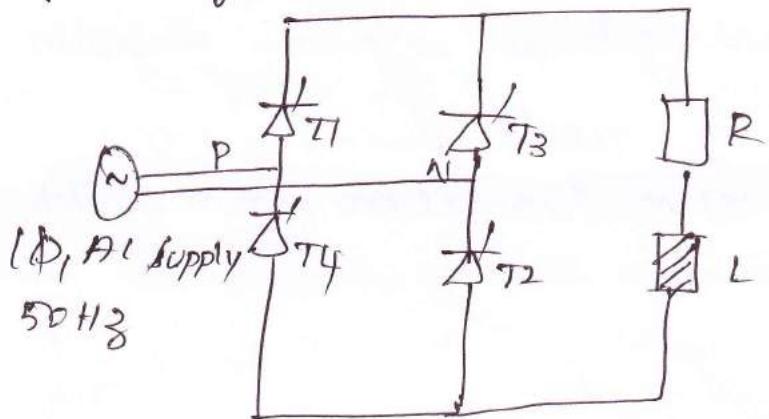
$$V_o = 100 \text{ V}$$

### A10. Circuit diagram of buck converter.



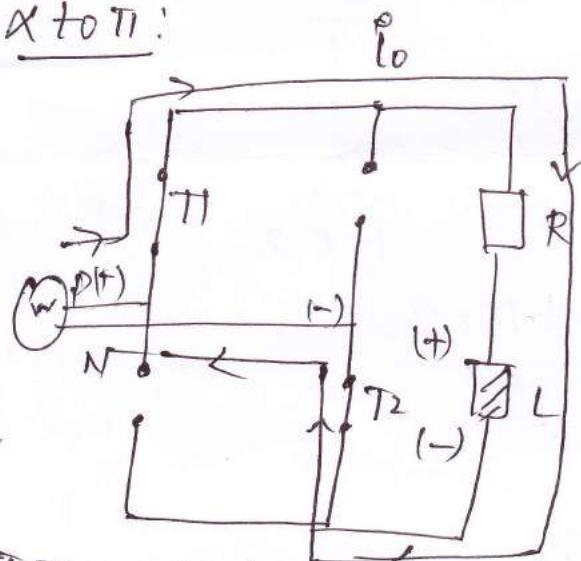
B1.a.i) Single phase fully controlled thyristor bridge rectifier with RL load:-

Circuit diagram: (1 mark)



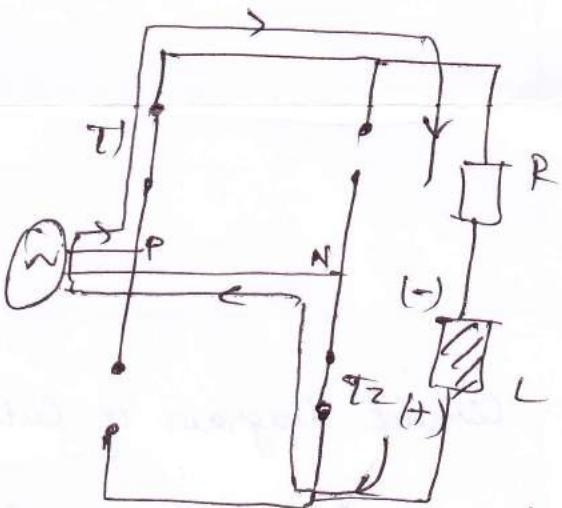
Operation: (2 mark).

$\alpha$  to  $\pi$ :

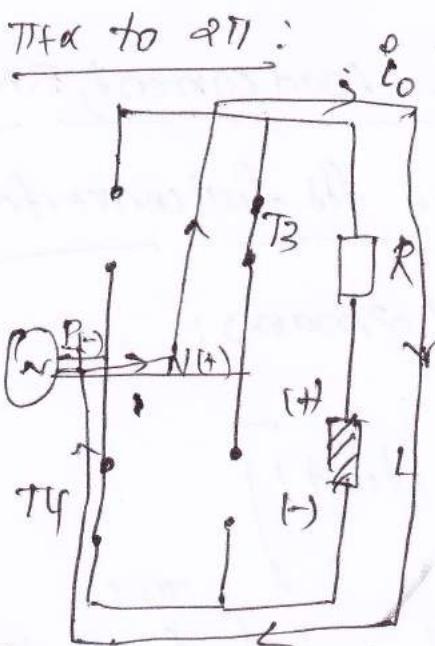


$T_1 T_2$  conducts;  $L$  charges.  
 $V_o$  is +ve and  $i_o$  is +ve.  
 $i_o$  is increased as  $L$  charges

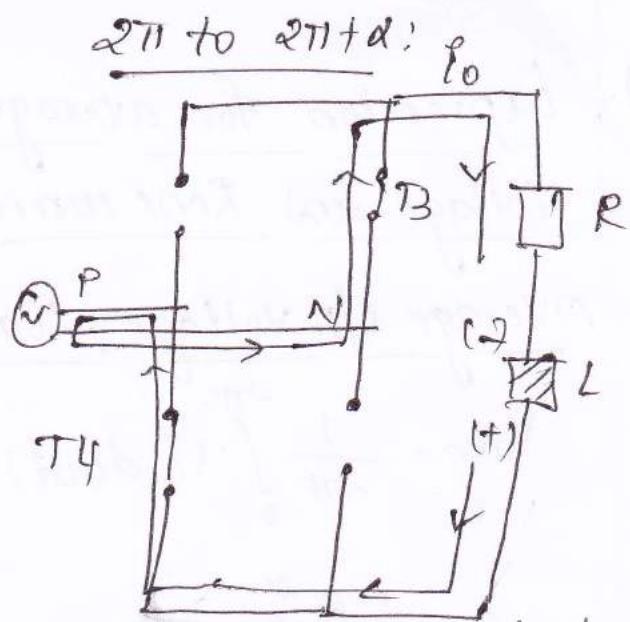
$\pi$  to  $\pi+\alpha$ :



$T_1 T_2$  conducts;  $L$  discharges  
 $V_o$  is -ve and  $i_o$  is +ve  
 But  $i_o$  decreases as  
 $L$  discharges.

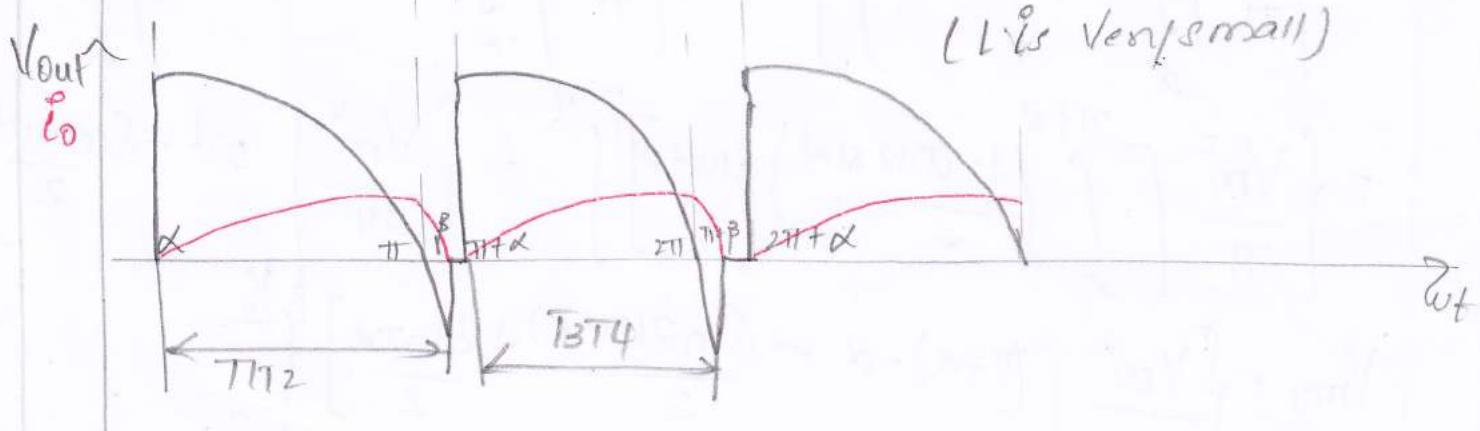
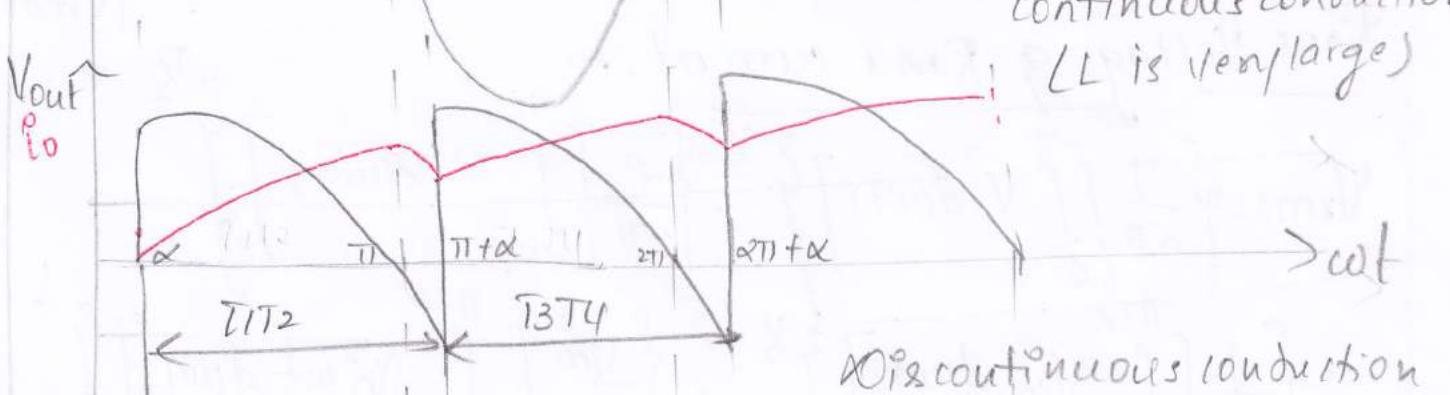
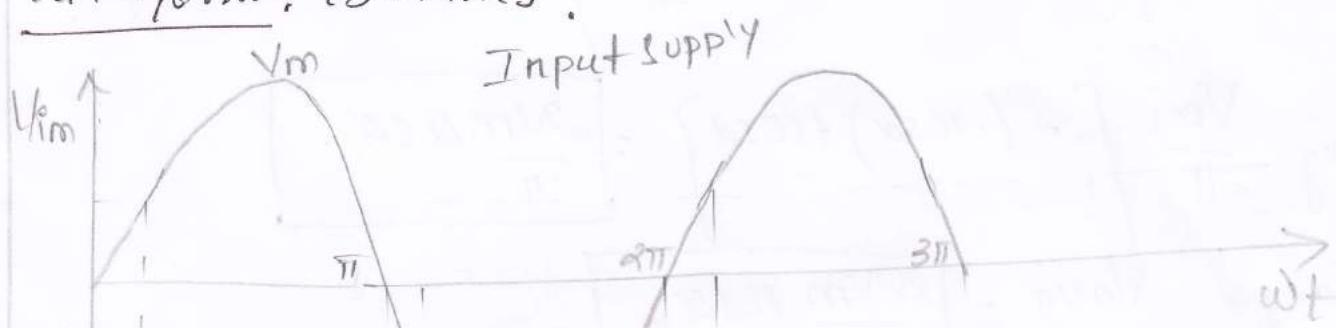


$T_3 T_4$  conducts,  $L$  charges  
 $V_o$  is +ve and  $i_o$  is +ve  
 $i_o$  increases as  $L$  charges



$T_3 T_4$  conducts,  $L$  discharges  
 $V_o$  is -ve and  $i_o$  is +ve  
 $i_o$  decreases as  $L$  discharges

### Waveforms! (3 marks)



ii)

Expression for average o/p voltage, load current, RMS voltage and RMS current for RL load fed full converter

Average O/P Voltage, Load current: (2marks)

$$\begin{aligned}
 V_{avg} &= \frac{1}{2\pi} \int_0^{2\pi} V \, d(\omega t) = \frac{1}{2\pi} \left[ \int_0^{\pi} V \, d(\omega t) \right] \\
 &= \frac{1}{\pi} \left[ \int_0^{\alpha} 0 \, d(\omega t) + \int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t) \, d(\omega t) \right] = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t) \, d(\omega t) \\
 &= \frac{V_m}{\pi} \left[ -\cos(\omega t) \right]_{\alpha}^{\pi+\alpha} = \frac{V_m}{\pi} \left[ -\cos(\pi+\alpha) + \cos(\alpha) \right].
 \end{aligned}$$

$$V_{avg} = \frac{V_m}{\pi} \left[ -(-\cos\alpha) + \cos\alpha \right] = \boxed{\frac{\alpha V_m \cos\alpha}{\pi}}$$

$$I_{avg} = \frac{V_{avg}}{R} = \boxed{\frac{\alpha V_m \cos\alpha}{\pi R}}$$

(3marks)

RMS Voltage & RMS current:

$$\begin{aligned}
 V_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V^2 \, d(\omega t)} = \sqrt{\frac{1}{2\pi} \left[ \int_0^{\pi} V^2 \, d(\omega t) \right]} \\
 &= \sqrt{\frac{1}{\pi} \left[ \int_{\alpha}^{\pi+\alpha} V_{in}^2 \, d(\omega t) \right]} = \sqrt{\frac{V_m^2}{\pi} \left[ \int_{\alpha}^{\pi+\alpha} \sin^2(\omega t) \, d(\omega t) \right]} \\
 &= \sqrt{\frac{V_m^2}{\pi} \left[ \int_{\alpha}^{\pi+\alpha} \left( \frac{1-\cos 2\omega t}{2} \right) \, d(\omega t) \right]} = \sqrt{\frac{V_m^2}{2\pi} \left[ \int_{\alpha}^{\pi+\alpha} \left( \omega t - \frac{\sin 2\omega t}{2} \right) \, d(\omega t) \right]} \\
 V_{rms} &= \sqrt{\frac{V_m^2}{2\pi} \left[ (\pi+\alpha) - \alpha - \frac{\sin 2(\pi+\alpha)}{2} + \frac{\sin 2\alpha}{2} \right]}
 \end{aligned}$$

$$= \sqrt{\frac{V_m^2}{2\pi}} \left[ \pi + \frac{\sin 2\alpha}{2} \right]^{\frac{1}{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2\pi}} \left[ \pi + \frac{\sin 2\alpha}{2} \right]^{\frac{1}{2}}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R\sqrt{2\pi}} \left( \pi + \frac{\sin 2\alpha}{2} \right)^{\frac{1}{2}}$$

### iii) Fully controlled Rectifier

i) All the devices or switches used are of controllable type

ii) Output voltage crosses zero and goes negative

iii) Regeneration is possible

(Ans)

### Half controlled Rectifiers

Half of the devices (or) switches are controllable and other are uncontrollable type.

ii) Output voltage is only positive

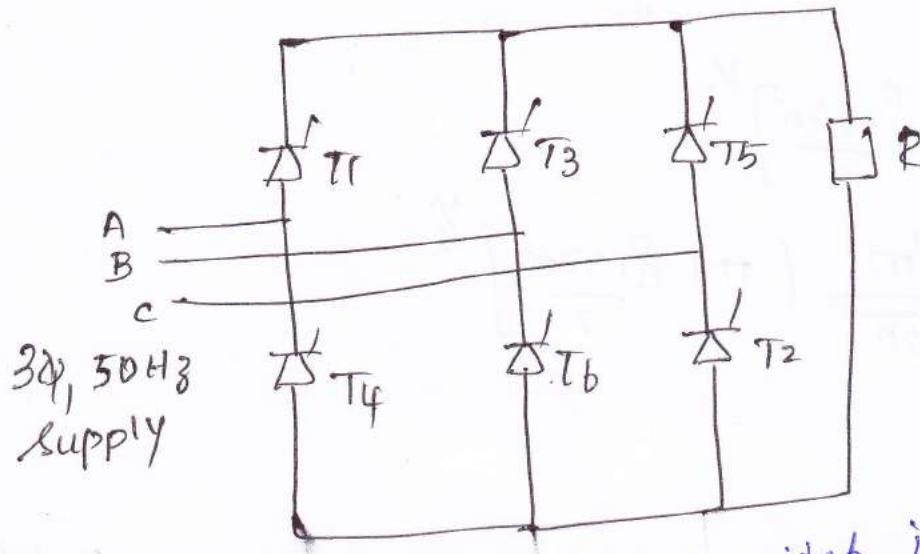
iii) Regeneration is not possible

### B1(b) Three-Phase fully controlled converter with R Load:

It has six switches and two devices

conduct simultaneously.

Circuit diagram: (2 marks)



commutating Voltages of each switch is given below.

$$\begin{array}{ll}
 T_1 \Rightarrow V_{DC} & T_4 \Rightarrow V_{CA} \\
 T_3 \Rightarrow V_{BA} & T_6 \Rightarrow V_{AB} \\
 T_5 \Rightarrow V_{CB} & T_2 \Rightarrow V_{BC}
 \end{array}$$

Switching table: (2 marks).

Degree of conduction interval	Devices in conduction	output Voltage	commuting device.
30°α to 90°α	- $T_6 T_1$	$V_{AB}$	- $T_1$
90°α to 150°α	- $T_1 T_2$	$V_{AC}$	- $T_2$
150°α to 210°α	- $T_2 T_3$	$V_{BC}$	- $T_3$
210°α to 270°α	- $T_3 T_4$	$V_{BA}$	- $T_4$
270°α to 330°α	- $T_4 T_5$	$V_{CA}$	- $T_5$
330°α to 30°α	- $T_5 T_6$	$V_{CB}$	- $T_6$

Where  $\alpha \Rightarrow$  firing angle

Equations of input supply: (3 mains)

$$V_{AN} = V_m \sin \omega t$$

$$V_{BN} = V_m \sin(\omega t - 120^\circ)$$

$$V_{CN} = V_m \sin(\omega t + 120^\circ)$$

$$V_{AB} = V_{AN} - V_{BN} = \sqrt{3} V_m \sin(\omega t + 30^\circ)$$

$$V_{AC} = V_{AN} - V_{CN} = \sqrt{3} V_m \sin(\omega t - 30^\circ)$$

$$V_{BC} = V_{BN} - V_{CN} = \sqrt{3} V_m \sin(\omega t - 90^\circ)$$

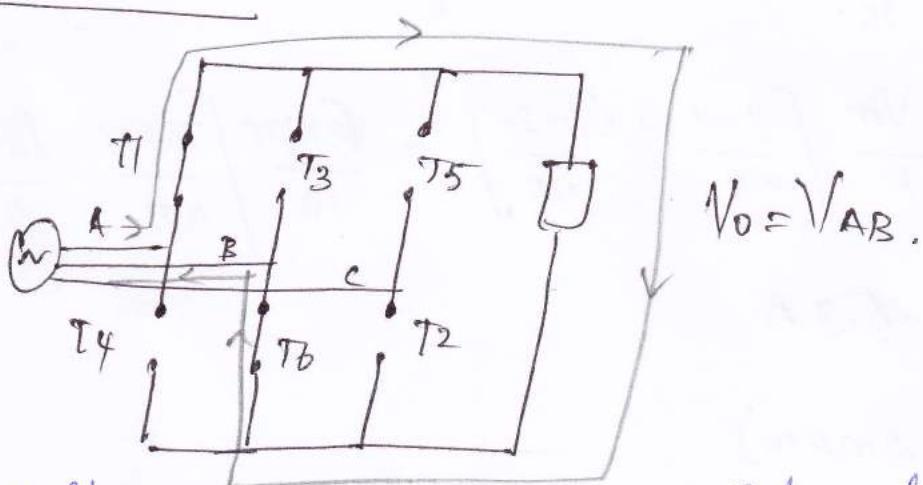
$$V_{BA} = V_{BN} - V_{AN} = \sqrt{3} V_m \sin(\omega t - 150^\circ)$$

$$V_{CA} = V_{CN} - V_{AN} = \sqrt{3} V_m \sin(\omega t + 150^\circ)$$

$$V_{CB} = V_{CN} - V_{BN} = \sqrt{3} V_m \sin(\omega t + 90^\circ)$$

Operation:-

$30^\circ\alpha$  to  $90^\circ\alpha$ :



Similarly for other conduction interval.

(ii) Given:

$$V_{in} = 120V \quad f = 60Hz \quad R = 10\Omega$$

$$V_{avg} = 25\% \text{ of } V_{avg(max)}$$

To find:

- (i) Delay angle ( $\alpha$ )
- (ii)  $R_{rms}$  and average output current

$$A6 \\ \alpha=0, V_{avg(max)} = \frac{2V_m}{\pi} \cos(0) = \frac{2V_m}{\pi}$$

$$= \frac{2 \times \sqrt{2} \times 120}{\pi} = \frac{2 \times 169.68}{\pi}$$

$$\boxed{V_{avg(max)} = 108 V}$$

$$V_{avg} = 0.25 \text{ of } V_{avg(max)} = 0.25 \times 108 = 27V$$

$$27 = \frac{120 \times \sqrt{2}}{\pi} (1 + \cos \alpha) \Rightarrow \boxed{\alpha = 120^\circ} \quad (2 \text{ mark})$$

$$I_{avg} = \frac{V_{avg}}{R} = \frac{27}{10} = 2.7A; \quad \boxed{I_{avg} = 2.7A} \quad (1 \text{ mark})$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R} \sqrt{\frac{\pi - \alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}^{\frac{1}{2}} = \frac{\sqrt{2} \times 120}{10} \sqrt{\frac{\pi - 120^\circ}{2\pi} + \frac{\sin 240^\circ}{4\pi}}$$

$$I_{rms} = \frac{53.9}{10} = 5.3A$$

$$\boxed{I_{rms} = 5.3A} \quad (2 \text{ mark})$$

(iii) Expression for Average load voltage, current and RMS voltage, current:

$$V_{avg} = \frac{1}{2\pi} \int_0^{2\pi} V d(\omega t) = \frac{1}{2\pi} \int_0^{\pi} V d(\omega t) = \frac{1}{\pi} \left[ \int_0^x 0 + \int_x^{\pi} V_m \sin \omega t d(\omega t) \right]$$

$$= \frac{1}{\pi} \left[ \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t) \right] = \frac{V_m}{\pi} \int_{\alpha}^{\pi} -\cos \omega t d(\omega t) = \frac{V_m}{\pi} \left[ \sin \omega t \right]_{\alpha}^{\pi} = \frac{V_m}{\pi} (\sin \pi - \sin \alpha)$$

$$\boxed{V_{avg} = \frac{V_m}{\pi} (1 + \cos \alpha)}$$

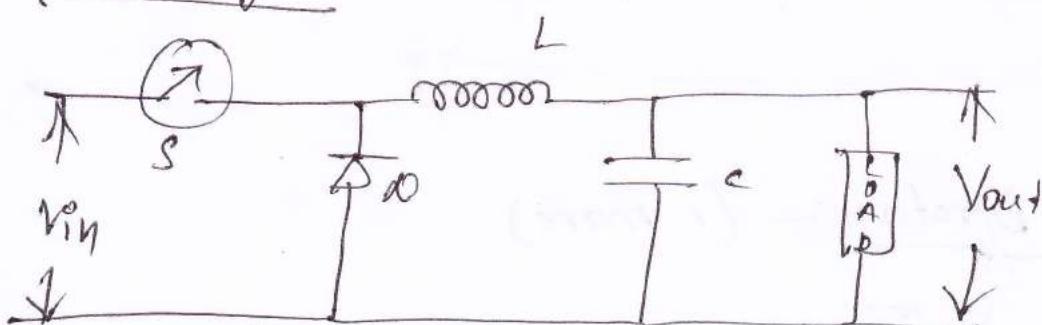
$$I_{avg} = \frac{V_{avg}}{R} = \frac{V_m}{\pi R} (1 + \cos \alpha), \quad (2 \text{ marks})$$

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{2\pi} \left[ \int_0^{2\pi} V^2 d\omega t \right]} = \sqrt{\frac{2}{\pi} \left[ \int_0^{\pi} V^2 d\omega t \right]} = \sqrt{\frac{1}{\pi} \left[ \int_{\alpha}^{\pi} V_{in}^2 d\omega t \right]} \\ &= \sqrt{\frac{V_m^2}{\pi} \left[ \int_{\alpha}^{\pi} \sin^2 \omega t d\omega t \right]} = \sqrt{\frac{V_m^2}{\pi} \left[ \int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} \right]} \\ &= \sqrt{\frac{V_m^2}{2\pi} \left[ \int_{\alpha}^{\pi} \omega t - \frac{\sin 2\omega t}{2} \right]} = \sqrt{\frac{V_m^2}{2\pi} \left[ \frac{\pi - \alpha}{2} - \frac{\sin 2\pi + \sin 2\alpha}{4} \right]} \\ V_{rms} &= V_m \sqrt{\frac{\pi - \alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}} \end{aligned}$$

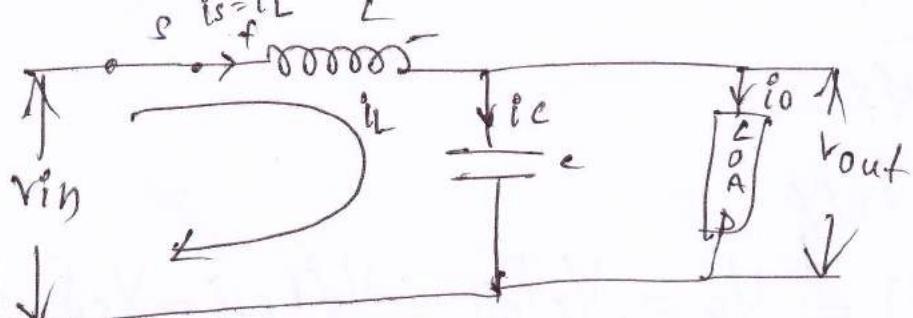
$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R} \sqrt{\frac{\pi - \alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}} \quad (2 \text{ marks})$$

B2.9.i) Buck converter:

Circuit diagram: (1 mark)

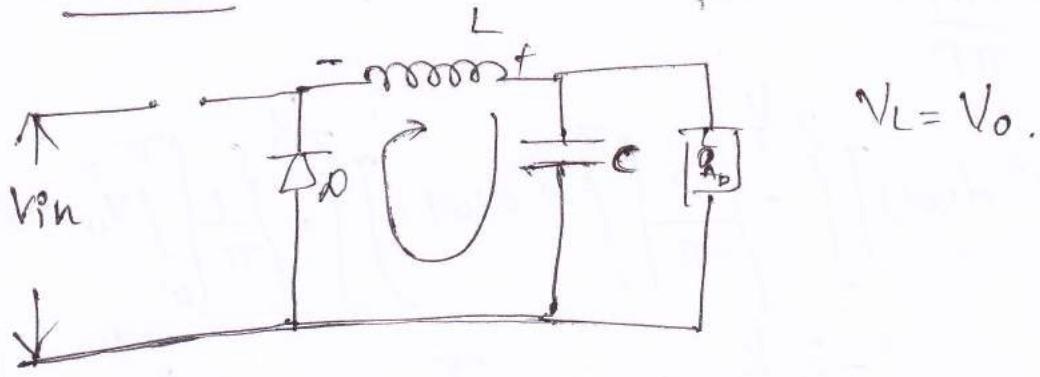


Model 1: Operation (2 marks)

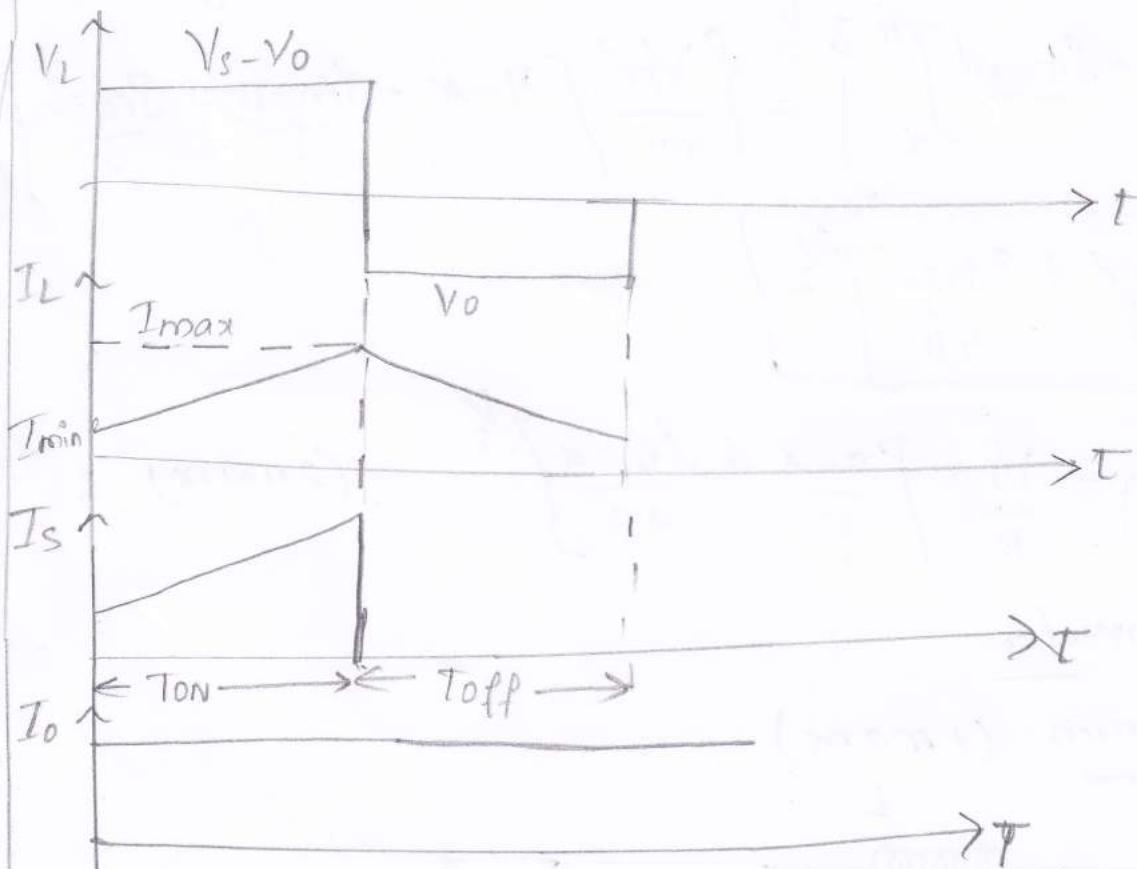


$$V_L = V_S - V_D$$

Mode II:-



waveform: (2 mark)



Output voltage equation: (1 mark)

$$(V_s - V_o) T_{on} = V_o T_{off}$$

$$V_s T_{on} - V_o T_{on} = V_o T_{off}$$

$$V_s T_{on} = V_o T_{on} + V_o T_{off}$$

$$V_o T_{on} = V_o (T_{on} + T_{off}) \Rightarrow V_o = \frac{V_s T_{on}}{T_{off} + T_{on}} = V_s \left( \frac{T_{on}}{T} \right) = V_s \delta$$

$\boxed{V_o = \delta V_s}$

ii) Output Voltage, Peak to Peak ripple current of Inductor  
and Peak to Peak ripple voltage of capacitor:-

$$V_L = L \frac{di}{dt}$$

Turn-on:

$$V_L = V_s - V_o \rightarrow (i)$$

$$V_L = L \frac{(i_2 - i_1)}{(t_2 - t_1)} = L \frac{(i_{2\max} - i_{1\min})}{T_{on}} \rightarrow (ii)$$

Equating (i) & (ii)

$$V_s - V_o = L \frac{\Delta I}{T_{on}} \Rightarrow \Delta I = \frac{(V_s - V_o) T_{on}}{L} \rightarrow \textcircled{A}$$

Turn-off: -

$$V_L = -V_o \rightarrow (iii)$$

$$V_L = L \frac{(i_1 - i_2)}{(t_1 - t_2)} = L \frac{(i_{1\min} - i_{2\max})}{T_{off}} \rightarrow (iv)$$

Equating (iii) & (iv)

$$-V_o = L \frac{-\Delta I}{T_{off}} \Rightarrow \Delta I = \frac{V_o T_{off}}{L} \rightarrow \textcircled{B}$$

Equating \textcircled{A} & \textcircled{B}.

$$\frac{(V_s - V_o) T_{on}}{L} = \frac{V_o T_{off}}{L}$$

$$V_s T_{on} - V_o T_{on} = V_o T_{off}$$

$$V_o T_{on} = V_o [T_{on} + T_{off}] = V_o T$$

$$V_o = \frac{V_s T_{on}}{T} = V_s \delta; \boxed{V_o = V_s \delta}$$

$$\Delta I = \frac{(V_g - V_o) T_{on}}{L} \rightarrow \textcircled{A}$$

$\therefore$  by  $T$

$$\Delta I = \frac{(V_s - \delta V_s) T_{on} f}{L_f} = \frac{V_s (1-\delta) \delta}{L_f}$$

$$\boxed{\Delta I = \frac{\delta V_s (1-\delta)}{L_f}}$$

$$V_c = \frac{\Delta I}{8Cf} = \frac{\delta V_s (1-\delta)}{8LCf^2}$$

iii) Given:

$$V_s = V_{in} = 10V; V_{avg} = 15V; I_{avg} = 0.4A; f = 20kHz$$

$$d = 100\%$$

To find:

(i) Duty cycle ratio, ii) Ripple current

Solution:

$$V_{avg} = \frac{V_s}{1-\delta}$$

$$1-\delta = \frac{V_s}{V_{avg}} \Rightarrow \delta = 1 - \frac{V_s}{V_{avg}} = 1 - \frac{10}{15} = 0.333$$

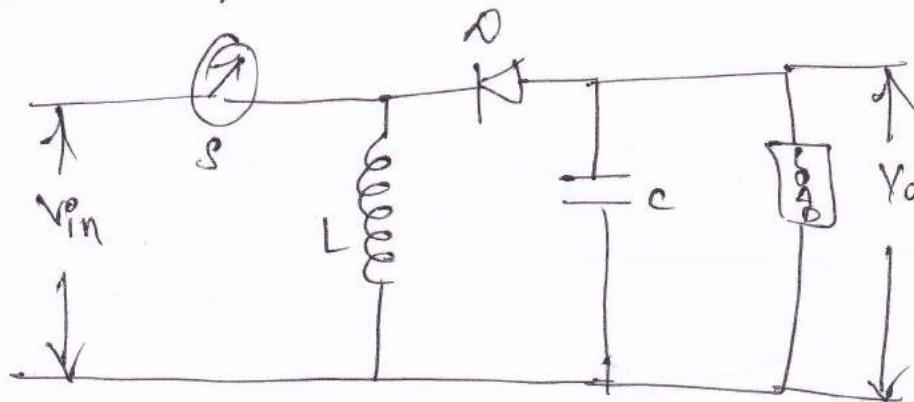
$$\boxed{\delta = 0.333} \quad (\text{2 marks})$$

$$\Delta I = \frac{V_s \delta}{L_f} = \frac{10 \times 0.333}{100 \times 20 \times 10^3} = 1.67 \mu A$$

(2 marks)

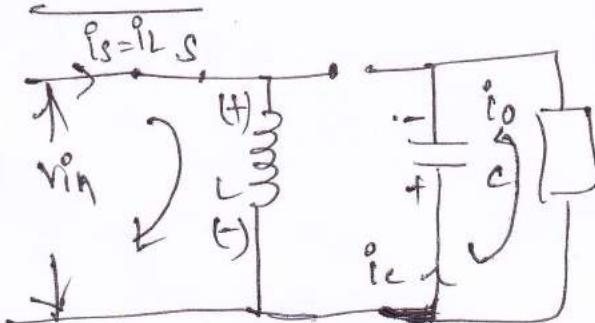
## b) Buck-Boost converter:

Circuit diagram:- (1mark)



Operation: (2 marks)

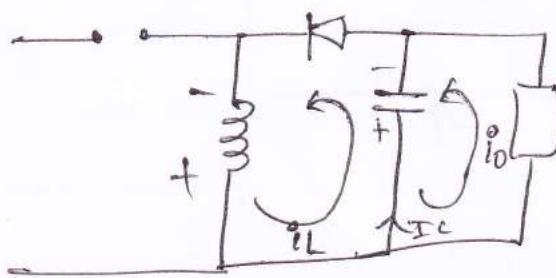
Mode I: S is on



S is on, L charges  $i_L$  increases

$$i_C = -i_a; V_L = V_s$$

Mode II: S is OFF



S is off, L discharges  $i_L$  decreases

$$i_C = i_L - i_o = i_a - i_o.$$

$$V_L = V_o$$

Output Voltage, Ripple current & Voltage:

$$V_L = \alpha \frac{di}{dt}$$

Turn-on:

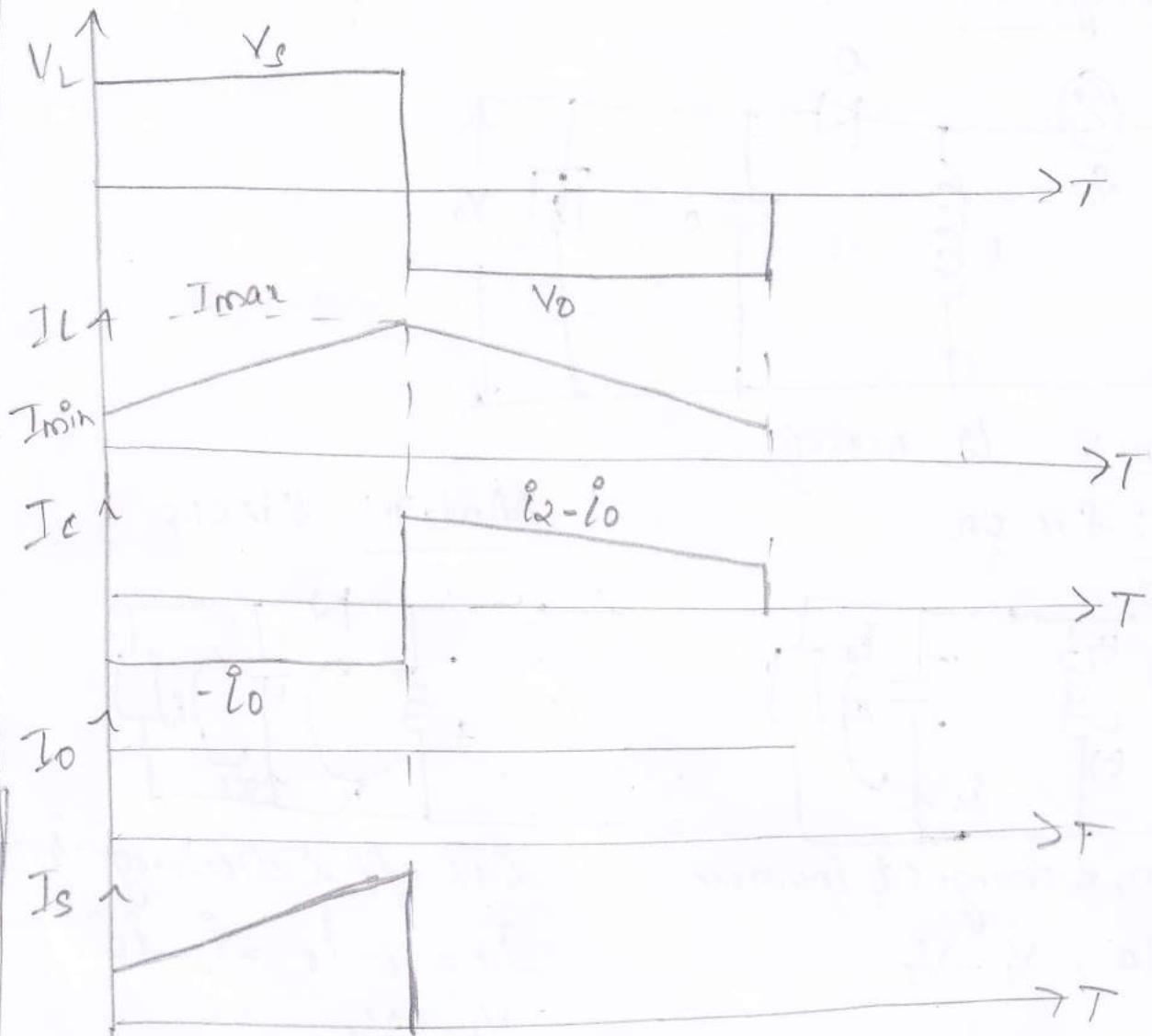
$$V_L = V_s - (i)$$

$$V_L = \alpha \frac{(i_2 - i_1)}{(t_2 - t_1)} = \alpha \frac{(I_{max} - I_{min})}{T_{on}} \rightarrow (ii)$$

Equating (i) & (ii)

$$V_s = \frac{\alpha (\Delta I)}{T_{on}} \Rightarrow \boxed{\Delta I = \frac{V_s T_{on}}{L}} \rightarrow \textcircled{A}$$

waveform! (3marks)



Turn-off:

$$V_L = V_0 \rightarrow (iii)$$

$$V_L = \alpha (i_2 - i_2) = \frac{L (I_{min} - I_{max})}{T_{off}} \rightarrow (iv)$$

Equating (iii) & (iv)

$$V_0 = \frac{L (\Delta I)}{T_{off}} \Rightarrow \boxed{\Delta I = -\frac{V_0 T_{off}}{L}} \rightarrow \textcircled{B}$$

Equating \textcircled{A} & \textcircled{B}

$$\frac{V_{off}}{K} = \frac{V_s T_{on}}{K}$$

$$V_0 = \frac{-V_s T_{on}}{\frac{T_{off}}{T_{on}}} = \frac{-V_s T_{on}}{T - T_{on}}$$

$$= \frac{-V_s}{\frac{T - T_{on}}{T_{on}}} = \frac{-V_s}{\frac{T}{T_{on}} - 1} = \frac{-V_s}{\frac{f}{f-1}} = \frac{-V_s}{\frac{1-f}{f}} = \frac{-V_s f}{1-f}$$

$$\boxed{V_0 = -\frac{\delta V_s}{1-f}} \Rightarrow \delta \neq$$

$$\Delta I = \frac{V_s T_{on}}{L} \rightarrow \textcircled{A}$$

$$\therefore \textcircled{A} \text{ by } L \quad \Delta I = \frac{V_s T_{on}}{L} = \frac{V_s \delta}{Lf}$$

$$V_c = \frac{1}{C} \int_0^{T_{on}} i_c dt + V_c(t=0)$$

$$V_c - V_c(t=0) = \frac{1}{C} \int_0^{T_{on}} i_c dt$$

$$\Delta V_c = \frac{I_c}{C} [t_{on}] = \frac{I_c}{C} \left[ \frac{L \Delta I}{V_s} \right] = \frac{I_c}{C} \left[ \frac{L}{V_s} \left[ \frac{V_s \delta}{Lf} \right] \right]$$

$$\Delta V_c = \frac{I_c f}{C f} = -\frac{I_0 \delta}{C f}$$

$$\boxed{\Delta V_c = -\frac{I_0 \delta}{C f}}$$

iii) Expression of output voltage, Ripple current & Voltage of boost converter.

$$V_L = L \frac{di}{dt}$$

Turn-on:

$$V_L = V_s \rightarrow (i)$$

$$V_L = L \frac{(i_2 - i_1)}{t_2 - t_1} = L \frac{(I_{max} - I_{min})}{T_{on}} \rightarrow (i_1)$$

$$\text{Equating (i) } \& \text{ (ii)} ; V_s = \frac{L \Delta I}{T_{on}} \Rightarrow \Delta I = \frac{V_s T_{on}}{L} \rightarrow (A)$$

Turn-off:

$$V_L = V_s - V_o \rightarrow (iii)$$

$$V_L = L \frac{(i_4 - i_2)}{t_4 - t_3} = L \frac{(I_{min} - I_{max})}{T_{off}} \rightarrow (iv)$$

$$\text{Equating (iii) } \& \text{ (iv)} ; V_s - V_o = \frac{-L \Delta I}{T_{off}} \Rightarrow \Delta I = \frac{-(V_s - V_o) T_{off}}{L} \rightarrow (B)$$

$$\text{Equating (A) } \& \text{ (B)} \quad \frac{V_s T_{on}}{L} = - \frac{(V_s - V_o) T_{off}}{L}$$

$$V_s T_{on} = -V_s T_{off} + V_o T_{off}$$

$$V_o = \frac{V_s T_{on}}{T_{off}} = \frac{V_s T_{on}}{T - T_{on}} = \frac{V_s}{\frac{T}{T_{on}} - 1} = \frac{V_s}{\frac{1}{f} - 1} = \frac{V_s}{1 - f}$$

$$V_o = \frac{V_s}{1 - f} = \frac{V_s}{1 - \delta} \Rightarrow V_o = \frac{V_s}{1 - \delta} \quad (\text{2 marks})$$

$$\Delta I = \frac{V_s T_{on}}{L} \div \text{by } T$$

$$\Delta I = \frac{V_s T_{on}/T}{L/T} = \frac{V_s \delta}{L f} \quad \boxed{\Delta I = \frac{V_s f}{L f}} \quad (\text{1 mark})$$

$$\Delta V_c = \frac{I_0}{C} \left[ \frac{L \Delta I}{V_s} \right]$$

$$= \frac{I_0}{C} \left( \frac{L}{V_s} \left[ \frac{V_{sf}}{L_f} \right] \right)$$

$$\boxed{\Delta V_c = \frac{I_0 f}{C f}}$$

(1 mark)

ii)  $V_{in} = 12V$   $L = 150H$   $C = 220F$   $I_{avg} = 1.25A$   $f = 0.25$   
 $f = 25kHz$

$$V_o = -\frac{V_s f}{1 - f} = -\frac{12 \times 0.25}{1 - 0.25} = -4V \quad (1 \text{ mark})$$

$$V_c = \frac{I_0 f}{f C} = \frac{1.25 \times 0.25}{25 \times 10^3 \times 220} = 5.68 \times 10^{-8} V$$

$$\boxed{V_c = 0.0568mV} \quad (2 \text{ marks})$$

$$\Delta I = \frac{V_s f}{f L} = \frac{12 \times 0.25}{25 \times 10^3 \times 150} = 8 \times 10^{-7} A$$

$$\boxed{\Delta I = 0.8 \mu A} \quad (2 \text{ mark})$$