Two-Port Networks

CHAPTER 16

16.1 TWO-PORT NETWORK

Generally any network may be represented schematically by a rectangular box. A network may be used for representing either source or load, or for a variety of purposes. A pair of terminals at which a signal may enter or leave a network is called a port. A *port* is defined as any pair of terminals into which energy is supplied, or from which energy is withdrawn, or where the network variables may be measured. One such network having only one pair of terminals (1-1′) is shown in Fig. 16.1 (a).

A two-port network is simply a network inside a black box, and the network has only two pairs of accessible terminals; usually one pair represents the input and the other represents the output. Such a building block is very common in electronic systems, communication systems, transmission and distribution systems. Figure 16.1(b) shows a two-port network, or two terminal pair network, in which the four terminals have been paired into ports 1-1′ and 2-2′. The terminals 1-1′ together constitute a port. Similarly, the terminals 2-2′ constitute another port. Two ports containing no sources in their branches are called *passive ports*; among them are power transmission lines and transformers. Two ports containing sources in their branches are called *active ports*. A voltage and current assigned to each of the two ports. The voltage and current at the input terminals are V_1 and I_1; whereas V_2 and I_2 are specified at the output port. It is also assumed that the currents I_1 and I_2 are entering into the network at the upper terminals 1 and 2, respectively. The variables of the two-port network are V_1, V_2, and I_1, I_2. Two of these are dependent variables, the other two are independent variables. The number of possible combinations generated by the four variables, taken two at a time, is six. Thus, there are six possible sets of equations describing a two-port network.

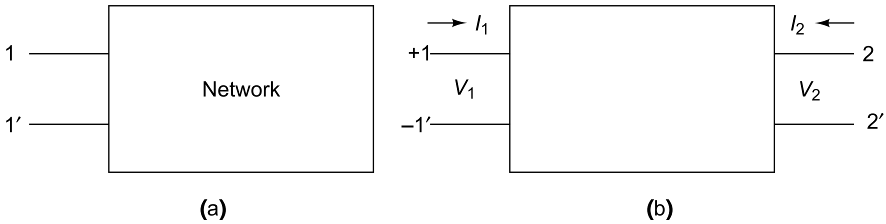

Fig. 16.1

16.2 OPEN CIRCUIT IMPEDANCE (Z) PARAMETERS

A general linear two-port network defined in Section 16.1 which does not contain any independent sources is shown in Fig. 16.2.

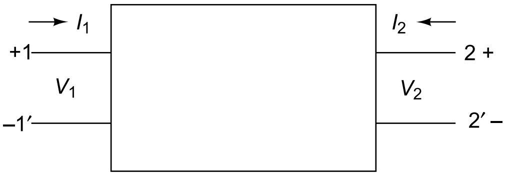

Fig. 16.2

The Z parameters of a two-port for the positive directions of voltages and currents may be defined by expressing the port voltages V_1 and V_2 in terms of the currents I_1 and I_2. Here V_1 and V_2 are dependent variables, and I_1, I_2 are independent variables. The voltage at port 1-1′ is the response produced by the two currents I_1 and I_2. Thus

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \tag{16.1}$$

Similarly, $$V_2 = Z_{21} I_1 + Z_{22} I_2 \tag{16.2}$$

Z_{11}, Z_{12}, Z_{21} and Z_{22} are the network functions, and are called impedance (Z) parameters, and are defined by Eqs 16.1 and 16.2. These parameters can be represented by matrices.

We may write the matrix equation $[V] = [Z][I]$

where V is the column matrix $= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Z is the square matrix $= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

and we may write $|I|$ in the column matrix $= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Thus, $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero. Suppose port 2-2′ is left open-circuited, then $I_2 = 0$.

Thus, $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

where Z_{11} is the driving-point impedance at port 1-1′ with port 2-2′ open circuited. It is called the open circuit input impedance.

Similarly, $Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$

where Z_{21} is the transfer impedance at port 1-1′ with port 2-2′ open circuited. It is also called the open circuit forward transfer impedance. Suppose port 1-1′ is left open circuited, then $I_1 = 0$.

Thus, $Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$

where Z_{12} is the transfer impedance at port 2-2′, with port 1-1′ open circuited. It is also called the open circuit reverse transfer impedance.

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

where Z_{22} is the open circuit driving point impedance at port 2-2′ with port 1-1′ open circuited. It is also called the open circuit output impedance. The equivalent circuit of the two-port networks governed by the Eqs 16.1 and 16.2, i.e. open circuit impedance parameters is shown in Fig. 16.3.

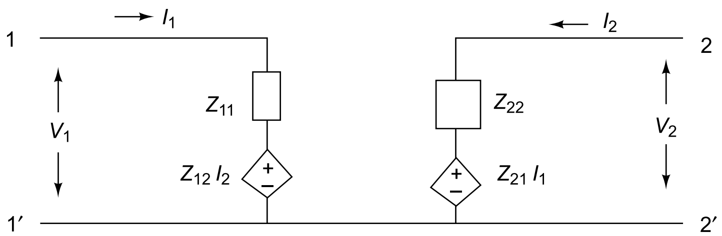

Fig. 16.3

If the network under study is reciprocal or bilateral, then in accordance with the reciprocity principle

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

or $$Z_{21} = Z_{12}$$

It is observed that all the parameters have the dimensions of impedance. Moreover, individual parameters are specified only when the current in one of the ports is zero. This corresponds to one of the ports being open circuited from which the Z parameters also derive the name *open circuit impedance parameters*.

Example 16.1 *Find the Z parameters for the circuit shown in Fig. 16.4.*

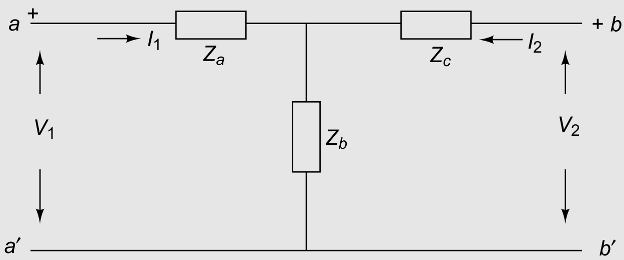

Fig. 16.4

Solution The circuit in the problem is a T network. From Eqs 16.1 and 16.2 we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

When port b-b' is open circuited, $Z_{11} = \dfrac{V_1}{I_1}$

where $V_1 = I_1(Z_a + Z_b)$

$$\therefore \quad Z_{11} = (Z_a + Z_b)$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

where $V_2 = I_1 Z_b$

$$\therefore \quad Z_{21} = Z_b$$

When port a-a' is open circuited, $I_1 = 0$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

where $V_2 = I_2(Z_b + Z_c)$

$$\therefore \quad Z_{22} = (Z_b + Z_c)$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

where $V_1 = I_2 Z_b$

$$\therefore \quad Z_{12} = Z_b$$

It can be observed that $Z_{12} = Z_{21}$, so the network is a bilateral network which satisfies the principle of reciprocity.

16.3 SHORT CIRCUIT ADMITTANCE (Y) PARAMETERS

A general two-port network which is considered in Section 16.2 is shown in Fig. 16.5.

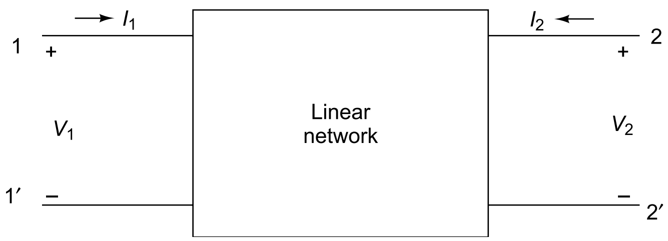

Fig. 16.5

The Y parameters of a two-port for the positive directions of voltages and currents may be defined by expressing the port currents I_1 and I_2 in terms of the voltages V_1 and V_2. Here I_1, I_2 are dependent variables and V_1 and V_2 are independent variables. I_1 may be considered to be the superposition of two components, one caused by V_1 and the other by V_2.

Thus,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \tag{16.3}$$

Similarly, $$I_2 = Y_{21} V_1 + Y_{22} V_2 \tag{16.4}$$

Y_{11}, Y_{12}, Y_{21} and Y_{22} are the network functions and are also called the admittance (Y) parameters. They are defined by Eqs 16.3 and 16.4. These parameters can be represented by matrices as follows

$$[I] = [Y]\,[V]$$

where $I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}; Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

and $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Thus, $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The individual Y parameters for a given network can be defined by setting each port voltage to zero. If we let V_2 be zero by short circuiting port 2-2′, then

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Y_{11} is the driving point admittance at port 1-1′, with port 2-2′ short circuited. It is also called the short circuit input admittance.

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Y_{21} is the transfer admittance at port 1-1′ with port 2-2′ short circuited. It is also called short circuited forward transfer admittance. If we let V_1 be zero by short circuiting port 1-1′, then

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

Y_{12} is the transfer admittance at port 2-2′ with port 1-1′ short circuited. It is also called the short circuit reverse transfer admittance.

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Y_{22} is the short circuit driving point admittance at port 2-2′ with port 1-1′ short circuited. It is also called the short circuit output admittance. The equivalent circuit of the network governed by Eqs 16.3 and 16.4 is shown in Fig . 16.6.

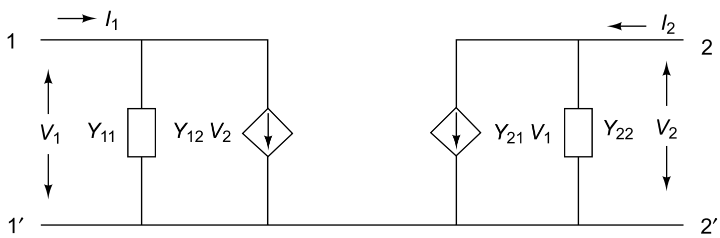

Fig. 16.6

If the network under study is reciprocal, or bilateral, then

$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

or $$Y_{12} = Y_{21}$$

It is observed that all the parameters have the dimensions of admittance which are obtained by short circuiting either the output or the input port from which the parameters also derive their name, i.e. the *short circuit admittance parameters*.

Example 16.2 *Find the Y parameters for the network shown in Fig. 16.7.*

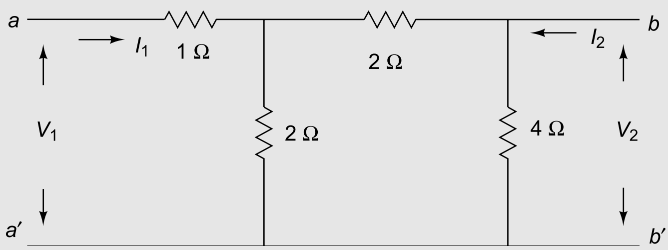

Fig. 16.7

Solution $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

When *b-b′* is short circuited, $V_2 = 0$ and the network looks as shown in Fig. 16.8 (a).

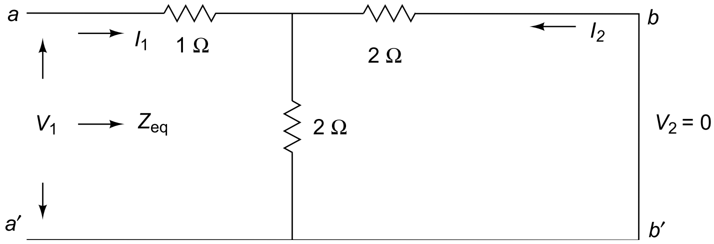

Fig. 16.8 (a)

$$V_1 = I_1 Z_{eq}$$

$$Z_{eq} = 2\ \Omega$$

$$\therefore \quad V_1 = I_1 2$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{2}\ ℧$$

$$Y_{21} = \left.\frac{I_2}{V_2}\right|_{V_2=0}$$

With port *b-b′* short circuited, $-I_2 = I_1 \times \frac{2}{4} = \frac{I_1}{2}$

$$\therefore \quad -I_2 = \frac{V_1}{4}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{4}\ ℧$$

Similarly, when port *a-a′* is short circuited, $V_1 = 0$ and the network looks as shown in Fig. 16.8 (b).

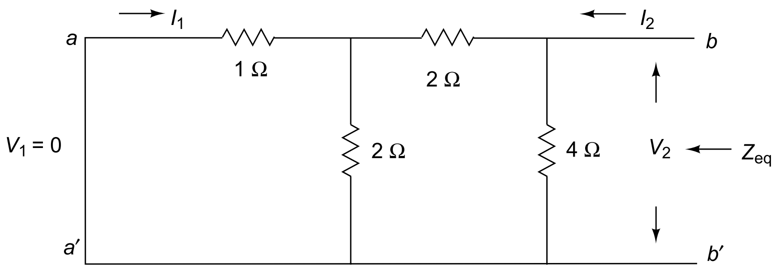

Fig. 16.8 (b)

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$V_2 = I_2\, Z_{eq}$$

where Z_{eq} is the equivalent impedance as viewed from b-b'.

$$Z_{eq} = \frac{8}{5}\ \Omega$$

$$V_2 = I_2 \times \frac{8}{5}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{5}{8}\ \mho$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

With a-a' short circuited, $-I_1 = \frac{2}{5} I_2$

Since $I_2 = \frac{5V_2}{8}$

$$-I_1 = \frac{2}{5} \times \frac{5}{8} V_2 = \frac{V_2}{4}$$

$$\therefore\quad Y_{12} = \frac{I_1}{V_2} = -\frac{1}{4}\ \mho$$

The describing equations in terms of the admittance parameters are

$$I_1 = 0.5\, V_1 - 0.25\, V_2$$

$$I_2 = -0.25\, V_1 + 0.625\, V_2$$

16.4 TRANSMISSION (ABCD) PARAMETERS

Transmission parameters, or *ABCD* parameters, are widely used in transmission line theory and cascade networks. In describing the transmission parameters, the input variables V_1 and I_1 at port 1-1′, usually called the *sending end*, are expressed in terms of the output variables V_2 and I_2 at port 2-2′, called the *receiving end*. The transmission parameters provide a direct relationship between input and output. Transmission parameters are also called general circuit parameters, or chain parameters. They are defined by

$$V_1 = AV_2 - BI_2 \tag{16.5}$$

$$I_1 = CV_2 - DI_2 \qquad (16.6)$$

The negative sign is used with I_2, and not for the parameter B and D. Both the port currents I_1 and $-I_2$ are directed to the right, i.e. with a negative sign in Eqs 16.5 and 16.6 the current at port 2-2′ which leaves the port is designated as positive. The parameters A, B, C and D are called the *transmission parameters*. In the matrix form, Eqs 16.5 and 16.6 are expressed as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called the *transmission matrix*.

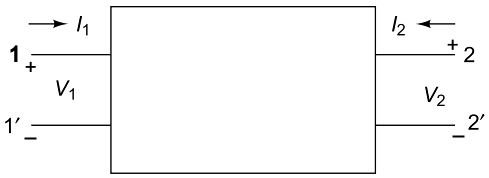

Fig. 16.9

For a given network, these parameters can be determined as follows. With port 2-2′ open, i.e. $I_2 = 0$; applying a voltage V_1 at the port 1-1′, using Eq. 16.5, we have

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \text{ and } C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = g_{21}\Big|_{I_2=0}$$

$1/A$ is called the open circuit voltage gain, a dimensionless parameter. And $\frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_{21}$, which is the open circuit transfer impedance. With port 2-2′ short circuited, i.e. with $V_2 = 0$, applying voltage V_1 at port 1-1′, from Eq. 16.6, we have

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \text{and} \quad -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$-\frac{1}{B} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = Y_{21}, \text{ which is the short circuit transfer admittance}$$

$$-\frac{1}{D} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \alpha_{21}\Big|_{V_2=0},$$ which is the short circuit current gain, a dimensionless parameter.

Cascade Connection

The main use of the transmission matrix is in dealing with a cascade connection of two-port networks as shown in Fig. 16.10.

Let us consider two two-port networks N_x and N_y connected in cascade with port voltages and currents as indicated in Fig. 16.10. The matrix representation of *ABCD* parameters for the network *X* is as under.

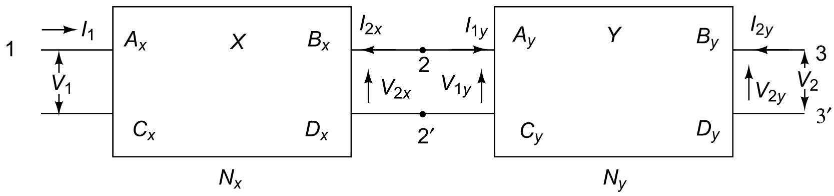

Fig. 16.10

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix}$$

And for the network Y, the matrix representation is

$$\begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

It can also be observed that at for 2-2′

$$V_{2x} = V_{1y} \text{ and } I_{2x} = -I_{1y}.$$

Combining the results, we have

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_1 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the transmission parameters matrix for the overall network.

Thus, the transmission matrix of a cascade of a two-port networks is the product of transmission matrices of the individual two-port networks. This property is used in the design of telephone systems, microwave networks, radars, etc.

Example 16.3 *Find the transmission or general circuit parameters for the circuit shown in Fig. 16.11.*

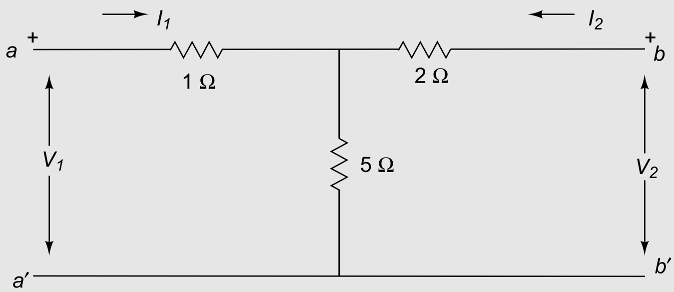

Fig. 16.11

Solution From Eqs 16.5 and 16.6 in Section 16.4, we have

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

When b-b' is open, $I_2 = 0$; $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$

where $V_1 = 6I_1$ and $V_2 = 5I_1$

$$\therefore \quad A = \frac{6}{5}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{5}\ \mho$$

When b-b' is short circuited; $V_2 = 0$ (See Fig. 16.12)

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}; \quad D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

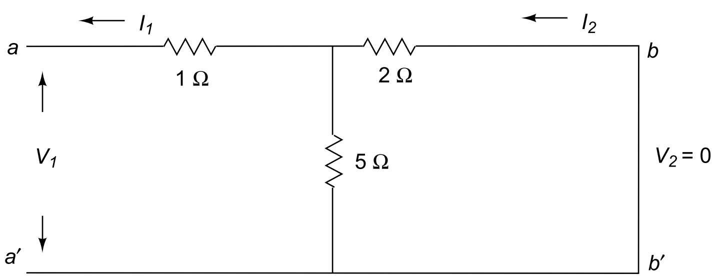

Fig. 16.12

In the circuit, $-I_2 = \dfrac{5}{17} V_1$

$$\therefore \quad B = \frac{17}{5}\ \Omega$$

Similarly, $I_1 = \dfrac{7}{17} V_1$ and $-I_2 = \dfrac{5}{17} V_1$

$$\therefore \quad D = \frac{7}{5}$$

16.5 INVERSE TRANSMISSION (A′ B′ C′ D′) PARAMETERS

In the preceding section, the input port voltage and current are expressed in terms of output port voltage and current to describe the transmission parameters. While defining the transmission parameters, it is customary to designate the input port as the sending end and output port as receiving end. The voltage and current at the receiving end can also be expressed in terms of the sending end voltage and current. If the voltage and current at port 2-2′ is expressed in terms of voltage and current at port 1-1′, we may write the following equations.

$$V_2 = A'V_1 - B'I_1 \tag{16.7}$$

$$I_2 = C'V_1 - D'I_1 \tag{16.8}$$

The coefficients A', B', C' and D' in the above equations are called inverse transmission parameters. Because of the similarities of Eqs 16.7 and 16.8 with Eqs 16.5 and 16.6 in Section 16.4, the A', B', C', D' parameters have properties similar to $ABCD$ parameters. Thus when port 1-1′ is open, $I_1 = 0$.

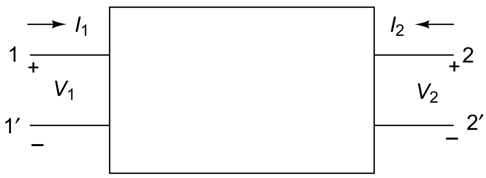

Fig. 16.13

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \quad ; \quad C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

If port 1-1′ is short circuited, $V_1 = 0$

$$B' = \left.\frac{-V_2}{I_1}\right|_{V_1=0} \quad ; \quad D = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$

16.6 HYBRID (h) PARAMETERS

Hybrid parameters, or h parameters find extensive use in transistor circuits. They are well suited to transistor circuits as these parameters can be most conveniently measured. The hybrid matrices describe a two-port, when the voltage of one port and the current of other port are taken as the independent variables. Consider the network in Fig. 16.14.

If the voltage at port 1-1′ and current at port 2-2′ are taken as dependent variables, we can express them in terms of I_1 and V_2.

$$V_1 = h_{11} I_1 + h_{12} V_2 \tag{16.9}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \tag{16.10}$$

The coefficients in the above equations are called hybrid parameters. In matrix notation

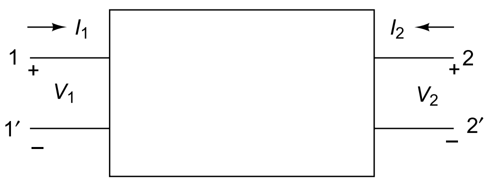

Fig. 16.14

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From Eqs 16.9 and 16.10, the individual h parameters may be defined by letting $I_1 = 0$ and $V_2 = 0$.

When $V_2 = 0$, the port 2-2′ is short circuited.

Then $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$ Short circuit input impedance $\left(\dfrac{1}{Y_{11}}\right)$

$h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ Short circuit forward current gain $\left(\dfrac{Y_{21}}{Y_{11}}\right)$

Similarly, by letting port 1-1′ open, $I_1 = 0$

$h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0}$ Open circuit reverse voltage gain $\left(\dfrac{Z_{12}}{Z_{22}}\right)$

$h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$ Open circuit output admittance $\left(\dfrac{1}{Z_{22}}\right)$

Since the *h* parameters represent dimensionally an impedance, an admittance, a voltage gain and a current gain, these are called hybrid parameters. An equivalent circuit of a two-port network in terms of hybrid parameters is shown in Fig. 16.15.

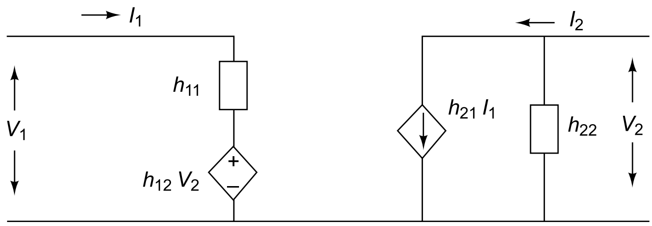

Fig. 16.15

Example 16.4 *Find the h parameters of the network shown in Fig. 16.16.*

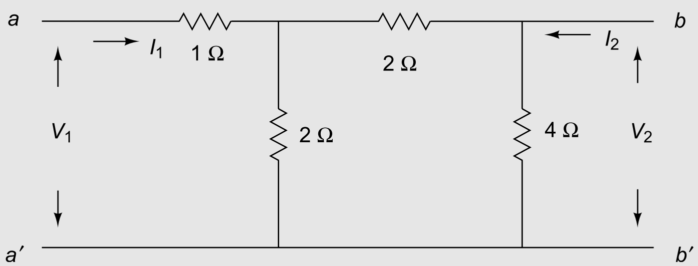

Fig. 16.16

Solution From Eqs 16.9 and 16.10, we have

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0};\quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0};\quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0};\quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

If port b-b' is short circuited, $V_2 = 0$. The circuit is shown in Fig. 16.17 (a).

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} ; \; V_1 = I_1 Z_{eq}$$

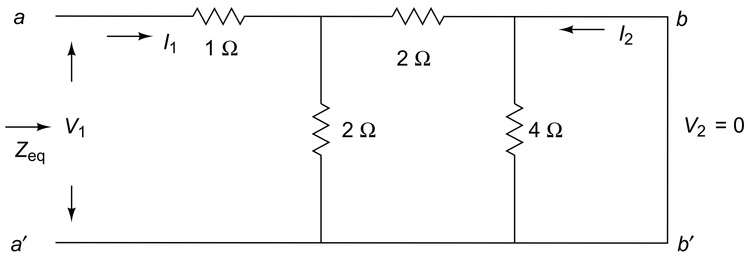

Fig. 16.17 (a)

Z_{eq} the equivalent impedance as viewed from the port a-a' is 2 Ω

$$\therefore \quad V_1 = I_1 \, 2\text{V}$$

$$h_{11} = \frac{V_1}{I_1} = 2\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad \text{when} \quad V_2 = 0; -I_2 = \frac{I_1}{2}$$

$$\therefore \quad h_{21} = -\frac{1}{2}$$

If port a-a' is let open, $I_1 = 0$. The circuit is shown in Fig. 16.17 (b). Then

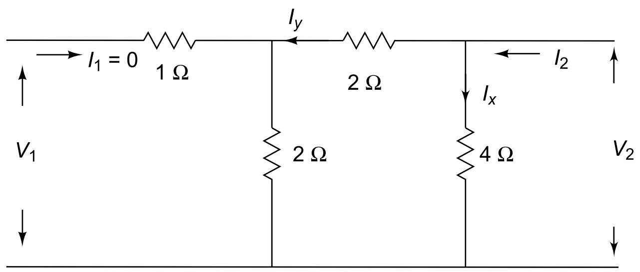

Fig. 16.17 (b)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$V_1 = I_y 2; \; I_y = \frac{I_2}{2}$$

$$V_2 = I_X 4; I_X = \frac{I_2}{2}$$

$$\therefore \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2}\ \mho$$

16.7 INVERSE HYBRID (g) PARAMETERS

Another set of hybrid matrix parameters can be defined in a similar way as was done in Section 16.6. This time the current at the input port I_1 and the voltage at the output port V_2 can be expressed in terms of I_2 and V_1. The equations are as follows.

$$I_1 = g_{11} V_1 + g_{12} I_2 \tag{16.11}$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \tag{16.12}$$

The coefficients in the above equations are called the inverse hybrid parameters. In matrix notation

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

It can be verified that $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$

The individual g parameters may be defined by letting $I_2 = 0$ and $V_1 = 0$ in Eqs 16.11 and 16.12.

Thus, when $I_2 = 0$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \text{Open circuit input admittance} \left(\frac{1}{Z_{11}}\right)$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \text{Open circuit voltage gain}$$

When $V_1 = 0$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \text{Short circuit reverse current gain}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \text{Short circuit output impedance} \left(\frac{1}{Y_{22}}\right)$$

16.8 INTER RELATIONSHIPS OF DIFFERENT PARAMETERS

Expression of Z Parameters in Terms of Y Parameters and Vice-versa

From Eqs 16.1, 16.2, 16.3 and 16.4, it is easy to derive the relation between the open circuit impedance parameters and the short circuit admittance parameters by means of two matrix equations of the respective parameters. By solving Eqs 16.1 and 16.2 for I_1 and I_2, we get

$$I_1 = \begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix} \Big/ \Delta_z; \text{ and } I_2 = \begin{vmatrix} Z_{11} & V_1 \\ V_{21} & V_2 \end{vmatrix} \Big/ \Delta_z$$

where Δ_z is the determinant of Z matrix

$$\Delta_z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$I_1 = \frac{Z_{22}}{\Delta_z} V_1 - \frac{Z_{12}}{\Delta_z} V_2 \qquad (16.13)$$

$$I_2 = \frac{-Z_{21}}{\Delta_z} V_1 + \frac{Z_{11}}{\Delta_z} V_2 \qquad (16.14)$$

Comparing Eqs 16.13 and 16.14 with Eqs 16.3 and 16.4 we have

$$Y_{11} = \frac{Z_{22}}{\Delta_z}; Y_{12} = \frac{-Z_{12}}{\Delta_z}$$

$$Y_{21} = \frac{Z_{21}}{\Delta_z}; Y_{22} = \frac{Z_{11}}{\Delta_z}$$

In a similar manner, the Z parameters may be expressed in terms of the admittance parameters by solving Eqs 16.3 and 16.4 for V_1 and V_2

$$V_1 = \begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix} \Big/ \Delta_y; \text{ and } V_2 = \begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix} \Big/ \Delta_y$$

where Δ_y is the determinant of the Y matrix

$$\Delta_y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}$$

$$V_1 = \frac{Y_{22}}{\Delta_y} I_1 - \frac{Y_{12}}{\Delta_y} I_2 \qquad (16.15)$$

$$V_2 = \frac{-Y_{21}}{\Delta_y} I_1 + \frac{Y_{11}}{\Delta_y} I_2 \qquad (16.16)$$

Comparing Eqs 16.15 and 16.16 with Eqs 16.1 and 16.2, we obtain

$$Z_{11} = \frac{Y_{22}}{\Delta_y}; Z_{12} = \frac{-Y_{12}}{\Delta_y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta_y}; Z_{22} = \frac{Y_{11}}{\Delta_y}$$

Example 16.5 *For a given, $Z_{11} = 3\ \Omega$; $Z_{12} = 1\ \Omega$; $Z_{21} = 2\ \Omega$ and $Z_{22} = 1\ \Omega$, find the admittance matrix, and the product of Δ_y and Δ_z.*

Solution The admittance matrix $= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{22}}{\Delta_z} & \dfrac{-Z_{12}}{\Delta_z} \\ \dfrac{-Z_{21}}{\Delta_z} & \dfrac{Z_{11}}{\Delta_z} \end{bmatrix}$

$$\text{given } Z = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\therefore \quad \Delta_z = 3 - 2 = 1$$

$$\therefore \quad \Delta_y = \begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix} = 1$$

$$(\Delta_y)\ (\Delta_z) = 1$$

General Circuit Parameters or ABCD Parameters in Terms of Z Parameters and Y Parameters

We know that

$$V_1 = AV_2 - BI_2;\ V_1 = Z_{11} I_1 + Z_{12} I_2;\ I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_1 = CV_2 - DI_2;\ V_2 = Z_{21} I_1 + Z_{22} I_2;\ I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0};\ C = \left.\frac{I_1}{V_2}\right|_{I_2=0};\ B = \left.\frac{-V_1}{I_2}\right|_{V_2=0};\ D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

Substituting the condition $I_2 = 0$ in Eqs 16.1 and 16.2 we get

$$\left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11}}{Z_{21}} = A$$

Substituting the condition $I_2 = 0$ in Eq. 16.4 we get,

$$\left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{-Y_{22}}{Y_{21}} = A$$

Substituting the condition $I_2 = 0$ in Eq. 16.2

we get $\left.\dfrac{I_1}{V_2}\right|_{I_2=0} = \dfrac{1}{Z_{21}} = C$

Substituting the condition $I_2 = 0$ in Eqs 16.3 and 16.4, and solving for V_2 gives $\dfrac{-I_1 Y_{21}}{\Delta_y}$

where Δy is the determinant of the admittance matrix

$$\left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{-\Delta_y}{Y_{21}} = C$$

Substituting the condition $V_2 = 0$ in Eq. 16.4, we get

$$\left.\frac{V_1}{I_2}\right|_{V_2=0} = -\frac{1}{Y_{21}} = B$$

Substituting the condition $V_2 = 0$ in Eqs 16.1 and 16.2 and solving for $I_2 = \frac{-V_1 Z_{21}}{\Delta_z}$

$$-\left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{\Delta_z}{Z_{21}} = B$$

where Δ_z is the determinant of the impedance matrix.
Substituting $V_2 = 0$ in Eq. 16.2

we get $$\left.\frac{-I_1}{I_2}\right|_{V_2=0} = \frac{Z_{22}}{Z_{21}} = D$$

Substituting $V_2 = 0$ in Eqs 16.3 and 16.4, we get

$$\left.\frac{-I_1}{I_2}\right|_{V_2=0} = \frac{-Y_{11}}{Y_{21}} = D$$

The determinant of the transmission matrix is given by

$$-AD + BC$$

Substituting the impedance parameters in A, B, C and D, we have

$$BC - AD = \frac{\Delta_z}{Z_{21}}\frac{1}{Z_{21}} - \frac{Z_{11}}{Z_{21}}\frac{Z_{22}}{Z_{21}}$$

$$= \frac{\Delta_z}{(Z_{21})^2} - \frac{Z_{11}Z_{22}}{(Z_{21})^2}$$

$$BC - AD = \frac{-Z_{12}}{Z_{21}}$$

For a bilateral network, $Z_{12} = Z_{21}$

$\therefore \quad BC - AD = -1$

or $\quad AD - BC = 1$

Therefore, in a two-port bilateral network, if three transmission parameters are known, the fourth may be found from equation $AD - BC = 1$.

In a similar manner the h parameters may be expressed in terms of the admittance parameters, impedance parameters or transmission parameters. Transformations of this nature are possible between any of the various parameters. Each parameter has its own utility. However, we often find that it is necessary to convert from one set of parameters to another. Transformations between different parameters, and the condition under which the two-port network is reciprocal are given in Table 16.1.

Table 16.1 Reciprocalty Condition for a Two Port Network

	Z	Y	$ABCD$	$A'B'C'D'$	h	g
Z	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta_y} & \frac{-Y_{12}}{\Delta_y} \\ \frac{-Y_{21}}{\Delta_y} & \frac{Y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D'}{C'} & \frac{1}{C'} \\ \frac{\Delta_{T'}}{C'} & \frac{A'}{C'} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{22}} & \frac{h_{22}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{matrix}$
Y	$\begin{matrix} \frac{Z_{22}}{\Delta_z} & \frac{-Z_{12}}{\Delta_z} \\ \frac{-Z_{21}}{\Delta_z} & \frac{-Z_{11}}{\Delta_z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{A'}{B'} & \frac{-1}{B'} \\ \frac{-\Delta_{T'}}{B'} & \frac{D'}{B'} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$
AB CD	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta_z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{D'}{\Delta_{T'}} & \frac{B'}{\Delta_{T'}} \\ \frac{C'}{\Delta_{T'}} & \frac{A'}{\Delta_{T'}} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{21}} & \frac{h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{matrix}$
$A'\ B'$ $C'\ D'$	$\begin{matrix} \frac{Z_{22}}{Z_{12}} & \frac{\Delta_z}{Z_{12}} \\ \frac{1}{Z_{12}} & \frac{Z_{11}}{Z_{12}} \end{matrix}$	$\begin{matrix} \frac{-Y_{11}}{Y_{12}} & \frac{-1}{Y_{12}} \\ \frac{-\Delta_Y}{Y_{12}} & \frac{-Y_{22}}{Y_{12}} \end{matrix}$	$\begin{matrix} \frac{D}{\Delta_T} & \frac{B}{\Delta_T} \\ \frac{C}{\Delta_T} & \frac{A}{\Delta_T} \end{matrix}$	$\begin{matrix} A' & B' \\ C' & D' \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta_h}{h_{12}} \end{matrix}$	$\begin{matrix} \frac{-\Delta_g}{g_{12}} & \frac{-g_{22}}{g_{12}} \\ \frac{-g_{11}}{g_{12}} & \frac{-1}{g_{12}} \end{matrix}$
h	$\begin{matrix} \frac{\Delta_z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta_Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} \frac{B'}{A'} & \frac{1}{A'} \\ \frac{\Delta_{T'}}{A'} & \frac{C'}{A'} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\Delta_g} & \frac{-g_{12}}{\Delta_g} \\ \frac{-g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{matrix}$
g	$\begin{matrix} \frac{1}{Z_{11}} & \frac{-Z_{12}}{Z_{11}} \\ \frac{Z_{21}}{Z_{11}} & \frac{\Delta_z}{Z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_Y}{Y_{22}} & \frac{Y_{12}}{Y_{22}} \\ \frac{-Y_{21}}{Y_{22}} & \frac{1}{Y_{22}} \end{matrix}$	$\begin{matrix} \frac{C}{A} & \frac{-\Delta_T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{C'}{D'} & \frac{-1}{D'} \\ \frac{\Delta_{T'}}{D'} & \frac{B'}{D'} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta_h} & \frac{-h_{12}}{\Delta_h} \\ \frac{-h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$
The two port is reciprocal If	$Z_{12} = Z_{21}$	$Y_{12} = Y_{21}$	The determinant of the transmission matrix = 1 ($\Delta_T = 1$)	The determinant of the inverse transmission matrix = 1	$h_{12} = -h_{21}$	$g_{12} = -g_{21}$

Example 16.6 *The impedance parameters of a two port network are $Z_{11} = 6\ \Omega$; $Z_{22} = 4\ \Omega$; $Z_{12} = Z_{21} = 3\ \Omega$. Compute the Y parameters and ABCD parameters and write the describing equations.*

Solution *ABCD* parameters are given by

$$A = \frac{Z_{11}}{Z_{21}} = \frac{6}{3} = 2;\ B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = 5\ \Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{3}\ \mho;\quad D = \frac{Z_{22}}{Z_{21}} = \frac{4}{3}$$

Y parameters are given by

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{4}{15}\ \mho;\quad Y_{12} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{-1}{5}\ \mho$$

$$Y_{21} = Y_{12} = \frac{-Z_{12}}{\Delta_Z} = \frac{-1}{5}\ \mho;\qquad Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{2}{5}\ \mho$$

The equations, using *Z* parameters are

$$V_1 = 6I_1 + 3I_2$$

$$V_2 = 3I_1 + 4I_2$$

Using *Y* parameters

$$I_1 = \frac{4}{15}V_1 - \frac{1}{5}V_2$$

$$I_2 = \frac{-1}{5}V_1 + \frac{2}{5}V_2$$

Using *ABCD* parameters

$$V_1 = 2V_2 - 5I_2$$

$$I_1 = \frac{1}{3}V_2 - \frac{4}{3}I_2$$

16.9 INTERCONNECTION OF TWO-PORT NETWORKS

Series Connection of a Two-port Network

It has already been shown in Section 16.4.1 that when two-port networks are connected in cascade, the parameters of the interconnected network can be conveniently expressed with the help of *ABCD* parameters. In a similar way, the *Z*-parameters can be used to describe the parameters of series connected two-port networks; and *Y* parameters can be used to describe parameters of parallel connected two-port networks. A series connection of two-port networks is shown in Fig. 16.18.

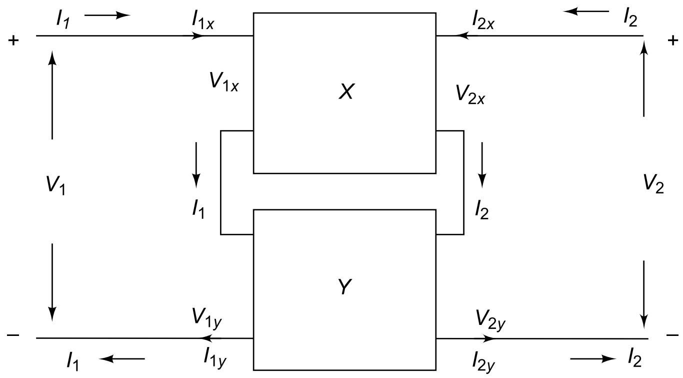

Fig. 16.18

Let us consider two, two-port networks, connected in series as shown. If each port has a common reference node for its input and output, and if these references are connected together then the equations of the networks X and Y in terms of Z parameters are

$$V_{1X} = Z_{11X} I_{1X} + Z_{12X} I_{2X}$$

$$V_{2X} = Z_{21X} I_{1X} + Z_{22X} I_{2X}$$

$$V_{1Y} = Z_{11Y} I_{1Y} + Z_{12Y} I_{2Y}$$

$$V_{2Y} = Z_{21Y} I_{1Y} + Z_{22Y} I_{2Y}$$

From the inter-connection of the networks, it is clear that

$$I_1 = I_{1X} = I_{1Y}; I_2 = I_{2X} = I_{2Y}$$

and $V_1 = V_{1X} + V_{1Y}; V_2 = V_{2X} + V_{2Y}$

$$\therefore \quad V_1 = Z_{11X} I_1 + Z_{12X} I_2 + Z_{11Y} I_1 + Z_{12Y} I_2$$

$$= (Z_{11X} + Z_{11Y})I_1 + (Z_{12X} + Z_{12Y}) I_2$$

$$V_2 = Z_{21X} I_1 + Z_{22X} I_2 + Z_{21Y} I_1 + Z_{22Y} I_2$$

$$= (Z_{21X} + X_{21Y})I_1 + (Z_{22X} + Z_{22Y})I_2$$

The describing equations for the series connected two-port network are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

where $Z_{11} = Z_{11X} + Z_{11Y}; Z_{12} = Z_{12X} + Z_{12Y}$

$$Z_{21} = Z_{21X} + Z_{21Y}; Z_{22} = Z_{22X} + Z_{22Y}$$

Thus, we see that each Z parameter of the series network is given as the sum of the corresponding parameters of the individual networks.

Parallel Connection of Two Two-port Networks

Let us consider two two-port networks connected in parallel as shown in Fig. 16.19. If each two-port has a reference node that is common to its input and output port, and if the two ports are connected so that they have a common reference node, then the equations of the networks *X* and *Y* in terms of *Y* parameters are given by

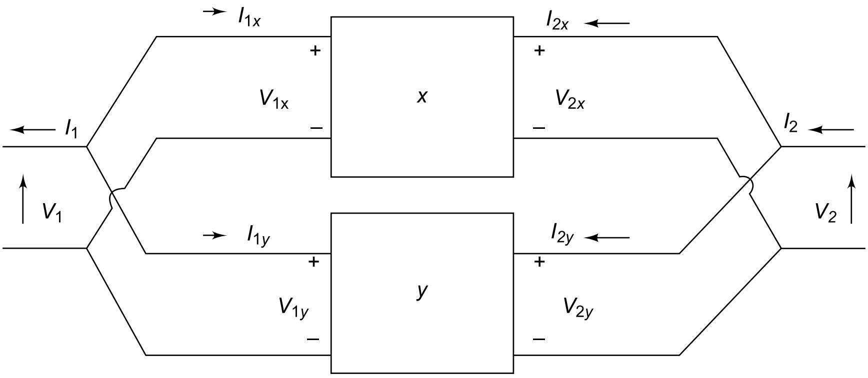

Fig. 16.19

$$I_{1X} = Y_{11X} V_{1X} + Y_{12X} V_{2X}$$

$$I_{2X} = Y_{21X} V_{1X} + Y_{22X} V_{2X}$$

$$I_{1Y} = Y_{11Y} V_{1Y} + Y_{12Y} V_{2Y}$$

$$I_{2Y} = Y_{21Y} V_{1Y} + Y_{22Y} V_{2Y}$$

From the interconnection of the networks, it is clear that

$$V_1 = V_{1X} = V_{1Y};\ V_2 = V_{2X} = V_{2Y}$$

and $$I_1 = I_{1X} + I_{1Y};\ I_2 = I_{2X} + I_{2Y}$$

$$\therefore\quad I_1 = Y_{11X} V_1 + Y_{12X} V_2 + Y_{11Y} V_1 + Y_{12Y} V_2$$

$$= (Y_{11X} + Y_{11Y}) V_1 + (Y_{12X} + Y_{12Y}) V_2$$

$$I_2 = Y_{21X} V_1 + Y_{22X} V_2 + Y_{21Y} V_1 + Y_{22Y} V_2$$

$$= (Y_{21X} + Y_{21Y}) V_1 + (Y_{22X} + Y_{22Y}) V_2$$

The describing equations for the parallel connected two-port networks are

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

where $$Y_{11} = Y_{11X} + Y_{11Y};\ Y_{12} = Y_{12X} + Y_{12Y}$$

$$Y_{21} = Y_{21X} + Y_{21Y};\ Y_{22} = Y_{22X} + Y_{22Y}$$

Thus we see that each Y parameter of the parallel network is given as the sum of the corresponding parameters of the individual networks.

Example 16.7 *Two networks shown in Figs 16.20 (a) and (b) are connected in series. Obtain the Z parameters of the combination. Also verify by direct calculation.*

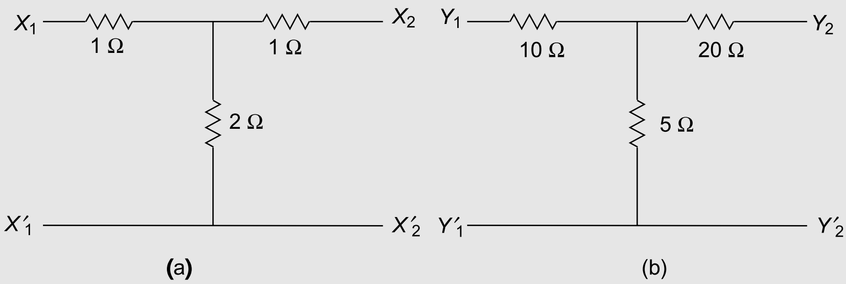

Fig. 16.20

Solution The Z parameters of the network in Fig. 16.20 (a) are

$$Z_{11X} = 3\ \Omega \quad Z_{12X} = Z_{21X} = 2\ \Omega \quad Z_{22X} = 3\ \Omega$$

The Z parameters of the network in Fig. 16.20 (b) are

$$Z_{11Y} = 15\ \Omega \quad Z_{21Y} = 5\ \Omega \quad Z_{22Y} = 25\ \Omega \quad Z_{12Y} = 5\ \Omega$$

The Z parameters of the combined network are

$$Z_{11} = Z_{11X} + Z_{11Y} = 18\ \Omega$$

$$Z_{12} = Z_{12X} + Z_{12Y} = 7\ \Omega$$

$$Z_{21} = Z_{21X} + Z_{21Y} = 7\ \Omega$$

$$Z_{22} = Z_{22X} + Z_{22Y} = 28\ \Omega$$

Check If the two networks are connected in series as shown in Fig. 16.20 (c), the Z parameters are

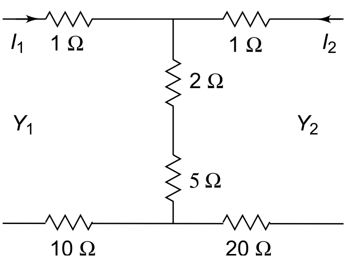

Fig. 16.20 (c)

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 18\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 7\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 28\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 7\ \Omega$$

Example 16.8 *Two identical sections of the network shown in Fig. 16.21 are connected in parallel. Obtain the Y parameters of the combination.*

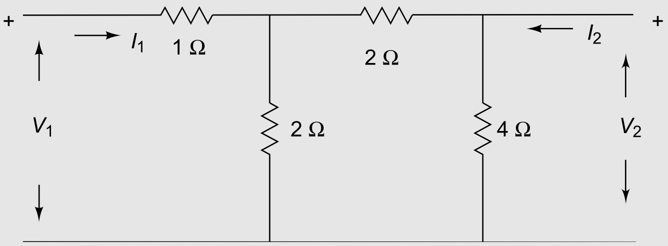

Fig. 16.21

Solution The Y parameters of the network in Fig. 16.21 are (See Ex. 16.2).

$$Y_{11} = \frac{1}{2}\,\mho \quad Y_{21} = \frac{-1}{4}\,\mho \quad Y_{22} = \frac{5}{8}\,\mho \quad Y_{12} = \frac{-1}{4}\,\mho$$

If two such networks are connected in parallel then the Y parameters of the combined network are

$$Y_{11} = \frac{1}{2} + \frac{1}{2} = 1\,\mho \quad Y_{21} = \frac{-1}{4} \times 2 = \frac{-1}{2}\,\mho$$

$$Y_{22} = \frac{5}{8} \times 2 = \frac{5}{4}\,\mho \quad Y_{12} = \frac{-1}{4} \times 2 = \frac{-1}{2}\,\mho$$

16.10 T AND Π REPRESENTATION

A two-port network with any number of elements may be converted into a two-port three-element network. Thus, a two-port network may be represented by an equivalent T-network, i.e. three impedances are connected together in the form of a T as shown in Fig. 16.22.

It is possible to express the elements of the T-network in terms of Z parameters, or $ABCD$ parameters as explained below.

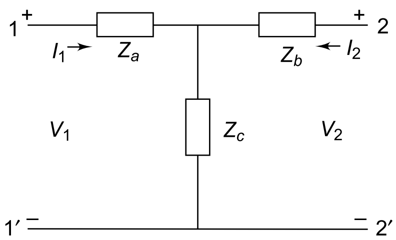

Fig. 16.22

Z parameters of the network

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_c$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_c$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_b + Z_c$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_c$$

From the above relations, it is clear that

$$Z_a = Z_{11} - Z_{21}$$

$$Z_b = Z_{22} - Z_{12}$$

$$Z_c = Z_{12} = Z_{21}$$

ABCD parameters of the network

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_a + Z_c}{Z_c}$$

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$$

When 2-2′ is short circuited

$$-I_2 = \frac{V_1 Z_c}{Z_b Z_c + Z_a (Z_b + Z_c)}$$

$$B = (Z_a + Z_b) + \frac{Z_a Z_b}{Z_c}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_c}$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

When 2-2′ is short circuited

$$-I_2 = I_1 \frac{Z_c}{Z_b + Z_c}$$

$$D = \frac{Z_b + Z_c}{Z_c}$$

From the above relations we can obtain

$$Z_a = \frac{A-1}{C}; \quad Z_b = \frac{D-1}{C}; \quad Z_c = \frac{1}{C}$$

Example 16.9 *The Z parameters of a two-port network are $Z_{11} = 10\ \Omega$; $Z_{22} = 15\ \Omega$; $Z_{12} = Z_{21} = 5\ \Omega$. Find the equivalent T network and ABCD parameters.*

Solution The equivalent *T* network is shown in Fig. 16.23,

where $Z_a = Z_{11} - Z_{21} = 5\ \Omega$

$Z_b = Z_{22} - Z_{12} = 10\ \Omega$

and $Z_c = 5\ \Omega$

The *ABCD* parameters of the network are

$$A = \frac{Z_a}{Z_c} + 1 = 2; \quad B = (Z_a + Z_b) + \frac{Z_a Z_b}{Z_c} = 25\,\Omega$$

$$C = \frac{1}{Z_c} = 0.2\,℧ \quad D = 1 + \frac{Z_b}{Z_c} = 3$$

In a similar way, a two-port network may be represented by an equivalent π-network, i.e. three impedances or admittances are connected together in the form of π as shown in Fig. 16.24.

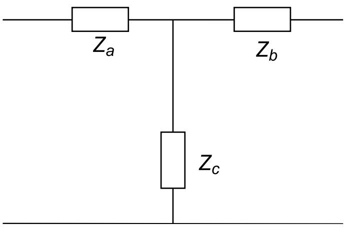

Fig. 16.23

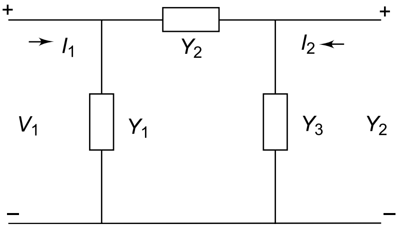

Fig. 16.24

It is possible to express the elements of the π-network in terms of *Y* parameters or *ABCD* parameters as explained below.

Y parameters of the network

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_1 + Y_2$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_3 + Y_2$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_2$$

From the above relations, it is clear that

$$Y_1 = Y_{11} + Y_{21}$$

$$Y_2 = -Y_{12}$$

$$Y_3 = Y_{22} + Y_{21}$$

Writing *ABCD* parameters in terms of *Y* parameters yields the following results.

$$A = \frac{-Y_{22}}{Y_{21}} = \frac{Y_3 + Y_2}{Y_2}$$

$$B = \frac{-1}{Y_{21}} = \frac{1}{Y_2}$$

$$C = \frac{-\Delta y}{Y_{21}} = Y_1 + Y_3 + \frac{Y_1 Y_3}{Y_2}$$

$$D = \frac{-Y_{11}}{Y_{21}} = \frac{Y_1 + Y_2}{Y_2}$$

From the above results, we can obtain

$$Y_1 = \frac{D-1}{B}$$

$$Y_2 = \frac{1}{B}$$

$$Y_3 = \frac{A-1}{B}$$

Example 16.10 *The port currents of a two-port network are given by*

$$I_1 = 2.5V_1 - V_2$$

$$I_2 = -V_1 + 5V_2$$

Find the equivalent π-network.

Solution Let us first find the Y parameters of the network

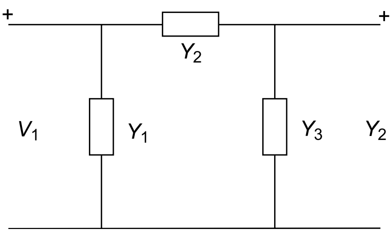

Fig. 16.25

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 2.5\,\mho; \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -1\,\mho$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -1\,\mho; \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 5\,\mho$$

The equivalent π-network is shown in Fig. 16.25.

where $Y_1 = Y_{11} + Y_{21} = 1.5\,\mho$;

$Y_2 = -Y_{12} = -1\mho$

and $Y_3 = Y_{22} + Y_{12} = 4\,\mho$

16.11 TERMINATED TWO-PORT NETWORK

Driving Point Impedance at the Input Port of a Load Terminated Network

Figure 16.26 shows a two-port network connected to an ideal generator at the input port and to a load impedance at the output port. The input impedance of this network can be expressed in terms of parameters of the two port network.

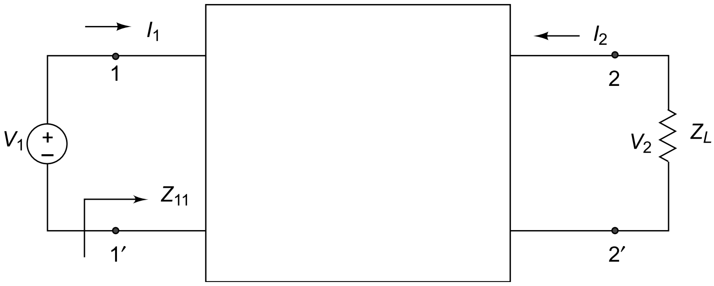

Fig. 16.26

In Terms of Z Parameters

The load at the output port 2-2′ impose the following constraint on the port voltage and current,

i.e., $V_2 = -Z_L I_2$

Recalling Eqs 16.1 and 16.2, we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Substituting the value of V_2 in Eq. 16.2, we have

$$-Z_L I_2 = Z_{21} I_1 + Z_{22} I_2$$

from which $I_2 = \dfrac{-I_1 Z_{21}}{Z_L + Z_{22}}$

Substituting the value of I_2 in Eq. 16.1 gives

$$V_1 = Z_{11} I_1 - \frac{Z_{12} Z_{21} I_1}{Z_L + Z_{22}}$$

$$V_1 = I_1 \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}} \right)$$

Hence the driving point impedance at 1-1′ is

$$\frac{V_1}{I_1} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}$$

If the output port is open, i.e. $Z_L \to \infty$, the input impedance is given by $V_1/I_1 = Z_{11}$
If the output port is short circuited, i.e. $Z_L \to 0$,
The short circuit driving point impedance is given by

$$\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} = \frac{1}{Y_{11}}$$

In Terms of Y Parameters

If a load admittance Y_L is connected across the output port. The constraint imposed on the output port voltage and current is

$$-I_2 = V_2 Y_L, \text{ where } Y_L = \frac{1}{Z_L}$$

Recalling Eqs 16.3 and 16.4 we have

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Substituting the value of I_2 in Eq. 16.4, we have

$$-V_2 Y_L = Y_{21} V_1 + Y_{22} V_2$$

$$V_2 = -\left(\frac{Y_{21}}{Y_L + Y_{22}}\right) V_1$$

Substituting V_2 value in Eq. 16.3, we have

$$I_1 = Y_{11} V_1 - \frac{Y_{12} Y_{21} V_1}{Y_L + Y_{22}}$$

From which $\quad \dfrac{I_1}{V_1} = Y_{11} - \dfrac{Y_{12} Y_{21}}{Y_L + Y_{22}}$

Hence the driving point impedance is given by

$$\frac{V_1}{I_1} = \frac{Y_{22} + Y_L}{Y_{11}(Y_1 + Y_{22}) - Y_{12} Y_{21}}$$

If the output port is open, i.e., $Y_L \to 0$

$$\frac{V_1}{I_1} = \frac{Y_{22}}{\Delta_y} = Z_{11}$$

If the output port is short circuited, i.e. $Y_L \to \infty$

Then $\quad Y_{in} = Y_{11}$

In a similar way, the input impedance of the load terminated two port network may be expressed in terms of other parameters by simple mathematical manipulations. The results are given in Table 16.2.

Driving Point Impedance at the Output Port with Source Impedance at the Input Port

Let us consider a two-port network connected to a generator at input port with a source impedance Z_s as shown in Fig. 16.27. The output impedance, or the driving point impedance, at the output port can be evaluated in terms of the parameters of two-port network.

In terms of Z parameters

If I_1 is the current due to V_s at port 1-1′

From Eqs 16.1 and 16.2, we have

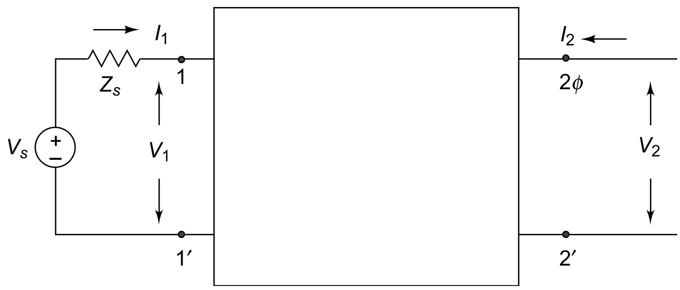

Fig. 16.27

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$V_1 = V_s - I_1Z_s$$

$$= Z_{11}\, I_1 + Z_{12}I_2 - (I_1)\,(Z_s + Z_{11}) = Z_{12}I_2 - V_s$$

$$-I_1 = \frac{Z_{12}I_2 - V_s}{Z_s + Z_{11}}$$

Substituting I_1 in Eq. 16.2, we get

$$V_2 = -Z_{21}\frac{(Z_{12}I_2 - V_s)}{Z_s + Z_{11}} + Z_{22}I_2$$

With no source voltage at port 1-1′, i.e. if the source V_s is short circuited

$$V_2 = \frac{-Z_{21}Z_{12}}{Z_s + Z_{11}}I_2 + Z_{22}I_2$$

Hence the driving point impedance at port 2-2′ $= \dfrac{V_2}{I_2}$

$$\frac{V_2}{I_2} = \frac{Z_{22}Z_s + Z_{22}Z_{11} - Z_{21}Z_{12}}{Z_s + Z_{11}} \quad \text{or} \quad \frac{\Delta_z + Z_{22}Z_s}{Z_s + Z_{11}}$$

If the input port is open, i.e. $Z_s \to \infty$

Then $$\frac{V_2}{I_2} = \left[\frac{\dfrac{\Delta_z}{Z_s} + Z_{22}}{1 + \dfrac{Z_{11}}{Z_s}}\right]_{Z_s=\infty} = Z_{22}$$

If the source impedance is zero with a short circuited input port, the driving point impedance at output port is given by

$$\frac{V_2}{I_2} = \frac{\Delta_z}{Z_{11}} = \frac{1}{Y_{22}}$$

In terms of Y parameters

Let us consider a two-port network connected to a current source at input port with a source admittance Y_s as shown in Fig. 16.28.

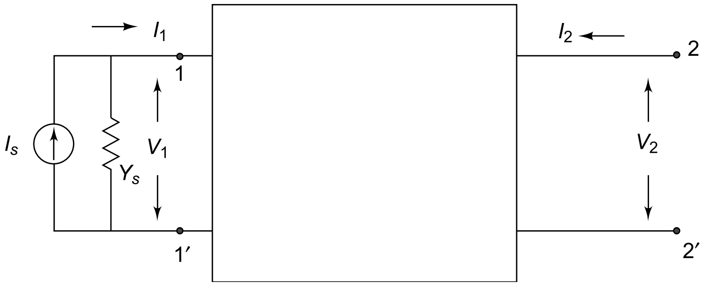

Fig. 16.28

At port 1-1′ $\quad I_1 = I_s - V_1 Y_s$

Recalling Eqs 16.3 and 16.4, we have

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Substituting I_1 in Eq. 16.3, we get

$$I_s - V_1 Y_s = Y_{11} V_1 + Y_{12} V_2$$

$$-V_1(Y_s + Y_{11}) = Y_{12} V_2 - I_s$$

$$-V_1 = \frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}}$$

Substituting V_1 in Eq. 16.4, we get

$$I_2 = -Y_{21}\left(\frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}}\right) + Y_{22}V_2$$

With no source current at 1-1′, i.e. if the current source is open circuited

$$I_2 = \frac{-Y_{21}Y_{12}V_2}{Y_s + Y_{11}} + Y_{22}V_2$$

Hence the driving point admittance at the output port is given by

$$\frac{I_2}{V_2} = \frac{Y_{22}Y_s + Y_{22}Y_{11} - Y_{21}Y_{12}}{Y_s + Y_{11}} \quad \text{or} \quad \frac{\Delta_y + Y_{22}Y_s}{Y_s + Y_{11}}$$

If the source admittance is zero, with an open circuited input port, the driving point admittance at the output port is given by

$$\frac{I_2}{V_2} = \frac{\Delta_y}{Y_{11}} = \frac{1}{Z_{22}} = Y_{22}$$

In a similar way, the output impedance may be expressed in terms of the other two port parameters by simple mathematical manipulations. The results are given in Table 16.2.

Table 16.2 Output Impedance

			In terms of			
	Z parameters	Y parameters	*ABCD*	*A′B′C′D′*	*h* parameter	*g* parameter
Driving point impedance at input port, or input impedance $\left(\frac{V_1}{I_1}\right)$	$\frac{\Delta_z + Z_{11}Z_L}{Z_{22}+Z_L}$	$\frac{Y_{22}+Y_L}{\Delta_y + Y_{11}Y_L}$	$\frac{AZ_L + B}{CZ_L + D}$	$\frac{B' - D'Z_L}{C'Z_L - A'}$	$\frac{\Delta_h Z_L + h_{11}}{1 + h_{22}Z_L}$	$\frac{1 + g_{22}Y_L}{\Delta_{gYL} + g_{11}}$
Driving point impedance at output port, or output impedance $\left(\frac{V_2}{I_2}\right)$	$\frac{\Delta_z + Z_{22}Z_s}{Z_1 + Z_{11}}$	$\frac{Y_{11}+Y_s}{\Delta_y + Y_s Y_{22}}$	$\frac{DZ_s + B}{CZ_s + A}$	$\frac{A'Z_s + B'}{C'Z_s + D'}$	$\frac{h_{11}+Z_s}{\Delta_h + h_{22}Z_s}$	$\frac{g_{22}+\Delta_s}{1+g_{11}Z_s}$

Note The above relations are obtained, when $V_s = 0$ and $I_s = 0$ at the input port.

Example 16.11 *Calculate the input impedance of the network shown in Fig. 16.29.*

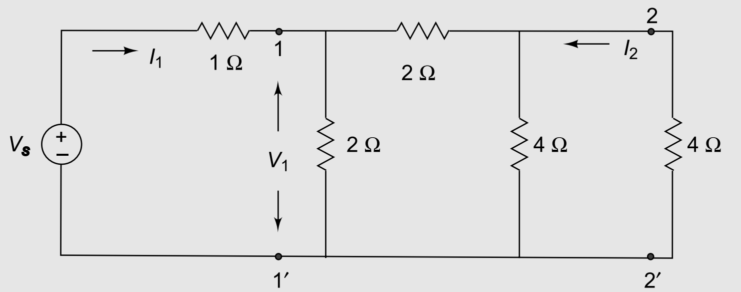

Fig. 16.29

Solution Let us calculate the input impedance in terms of Z parameters. The Z parameters of the given network (see Solved Problem 16.1) are $Z_{11} = 2.5\ \Omega$; $Z_{21} = 1\ \Omega$; $Z_{22} = 2\ \Omega$; $Z_{12} = 1\ \Omega$

From Section 16.11.1 we have the relation

$$\frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_L + Z_{22}}$$

where Z_L is the load impedance $= 2\ \Omega$

$$\frac{V_1}{I_1} = 2.5 - \frac{1}{2+2} = 2.25\ \Omega$$

The source resistance is 1 Ω

$$\therefore \quad Z_{in} = 1 + 2.25 = 3.25\ \Omega$$

Example 16.12 *Calculate the output impedance of the network shown in Fig. 16.30 with a source admittance of 1 ℧ at the input port.*

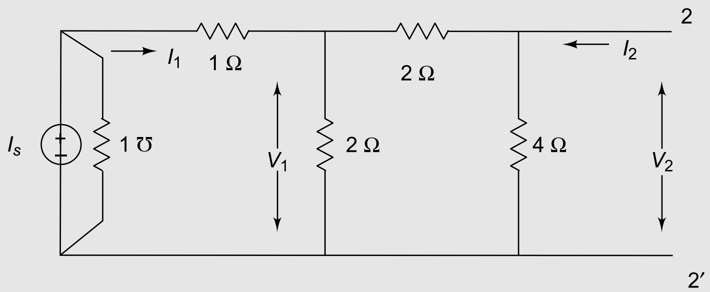

Fig. 16.30

Solution Let us calculate the output impedance in terms of Y parameters. The Y parameters of the given network (see Ex. 16.2) are

$$Y_{11} = \frac{1}{2}\ ℧;\ Y_{22} = \frac{5}{8}\ ℧;\ Y_{21} = Y_{12} = \frac{-1}{4}\ ℧$$

From Section 16.11.2, we have the relation

$$\frac{I_2}{V_2} = \frac{Y_{22}Y_s + Y_{22}Y_{11} - Y_{21}Y_{12}}{Y_s + Y_{11}}$$

where Y_s is the source admittance = 1 mho

$$Y_{22} = \frac{I_2}{V_2} = \frac{\frac{5}{8}\times 1 + \frac{5}{8}\times\frac{1}{2} - \frac{1}{16}}{1+\frac{1}{2}} = \frac{7}{12}\ ℧$$

$$\text{or} \quad Z_{22} = \frac{12}{7}\ ℧$$

16.12 LATTICE NETWORKS

One of the common four-terminal two-port network is the lattice, or bridge network shown in Fig. 16.31 (a). Lattice networks are used in filter sections and are also used as attenuaters. Lattice structures are sometimes used in preference to ladder structures in some special applications. Z_a and Z_d are called series arms, Z_b and Z_c are called the diagonal arms. It can be observed that, if Z_d is zero, the lattice structure becomes a π-section. The lattice network is redrawn as a bridge network as shown in Fig. 16.31 (b).

Z Parameters

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

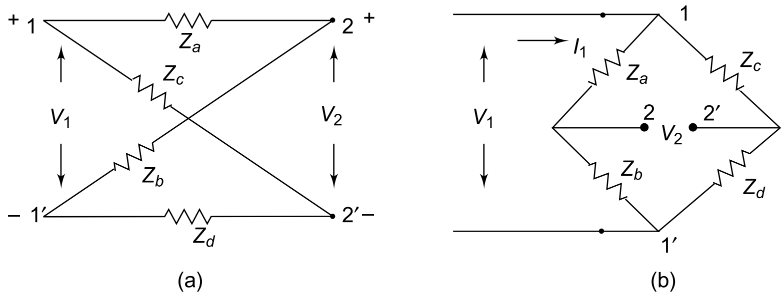

Fig. 16.31

When $\quad I_2 = 0;\ V_1 = I_1 \dfrac{(Z_a + Z_b)(Z_d + Z_c)}{Z_a + Z_b + Z_c + Z_d}$ (16.17)

$$\therefore \quad Z_{11} = \frac{(Z_a + Z_b)(Z_d + Z_c)}{Z_a + Z_b + Z_c + Z_d}$$

If the network is symmetric, then $Z_a = Z_d$ and $Z_b = Z_c$

$$\therefore \quad Z_{11} = \frac{Z_a + Z_b}{2}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

When $I_2 = 0$, V_2 is the voltage across $2-2'$

$$V_2 = V_1\left[\frac{Z_b}{Z_a + Z_b} - \frac{Z_d}{Z_c + Z_d}\right]$$

Substituting the value of V_1 from Eq. 16.17, we have

$$V_2 = \left[\frac{I_1(Z_a + Z_b)(Z_d + Z_c)}{Z_a + Z_b + Z_c + Z_d}\right]\left[\frac{Z_b(Z_c + Z_d) - Z_d(Z_a + Z_b)}{(Z_a + Z_b)(Z_c + Z_d)}\right]$$

$$\frac{V_2}{I_1} = \frac{Z_b(Z_c + Z_d) - Z_d(Z_a + Z_b)}{Z_a + Z_b + Z_c + Z_d} = \frac{Z_b Z_c - Z_a Z_d}{Z_a + Z_b + Z_c + Z_d}$$

$$\therefore \quad Z_{21} = \frac{Z_b Z_c - Z_a Z_d}{Z_a + Z_b + Z_c + Z_d}$$

If the network is symmetric, $Z_a = Z_d$, $Z_b = Z_c$

$$Z_{21} = \frac{Z_b - Z_a}{2}$$

When the input port is open, $I_1 = 0$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

The network can be redrawn as shown in Fig. 16.31 (c).

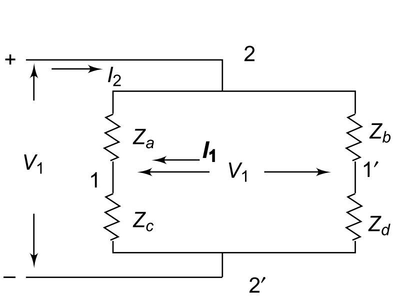

Fig. 16.31 (c)

$$V_1 = V_2\left[\frac{Z_c}{Z_a+Z_c} - \frac{Z_d}{Z_b+Z_d}\right] \tag{16.18}$$

$$V_2 = I_2\left[\frac{(Z_a+Z_c)(Z_d+Z_b)}{Z_a+Z_b+Z_c+Z_d}\right] \tag{16.19}$$

Substituting the value of V_2 in Eq. 16.18, we get

$$V_1 = I_2\left[\frac{Z_c(Z_b+Z_d) - Z_d(Z_a+Z_c)}{Z_a+Z_b+Z_c+Z_d}\right]$$

$$\frac{V_1}{I_2} = \frac{Z_cZ_b - Z_aZ_d}{Z_a+Z_b+Z_c+Z_d}$$

If the network is symmetric, $Z_a = Z_d$; $Z_b = Z_c$

$$\frac{V_1}{I_2} = \frac{Z_b^2 - Z_a^2}{2(Z_a+Z_b)}$$

$$\therefore \qquad Z_{12} = \frac{Z_b - Z_a}{2}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_2=0}$$

From Eq. 16.19, we have

$$\frac{V_2}{I_2} = \frac{(Z_a+Z_c)-(Z_d+Z_b)}{Z_a+Z_b+Z_c+Z_d}$$

If the network is symmetric,

$$Z_a = Z_d;\ Z_b = Z_c$$

$$Z_{22} = \frac{Z_a+Z_b}{2} = Z_{11}$$

From the above equations, $Z_{11} = Z_{22} = \dfrac{Z_a+Z_b}{2}$

and $\quad Z_{12} = Z_{21} = \dfrac{Z_b - Z_a}{2}$

$$\therefore \qquad Z_b = Z_{11} + Z_{12}$$

$$Z_a = Z_{11} - Z_{12}.$$

Example 16.13 *Obtain the lattice equivalent of a symmetrical T network shown in Fig. 16.32.*

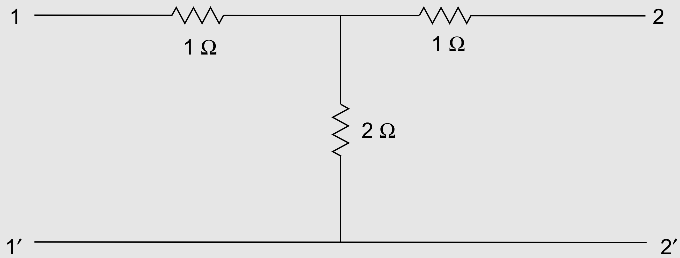

Fig. 16.32

Solution A two-port network can be realised as a symmetric lattice if it is reciprocal and symmetric. The Z parameters of the network are (see Ex. 16.1).

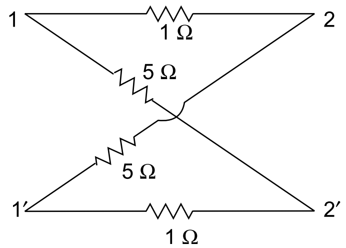

Fig. 16.33

$Z_{11} = 3\ \Omega$; $Z_{12} = Z_{21} = 2\ \Omega$; $Z_{22} = 3\ \Omega$.

Since $Z_{11} = Z_{22}$; $Z_{12} = Z_{21}$, the given network is symmetrical and reciprocal

∴ The parameters of the lattice network are

$$Z_a = Z_{11} - Z_{12} = 1\ \Omega$$

$$Z_b = Z_{11} + Z_{12} = 5\ \Omega$$

The lattice network is shown in Fig. 16.33.

Example 16.14 *Obtain the lattice equivalent of a symmetric π-network shown in Fig. 16.34.*

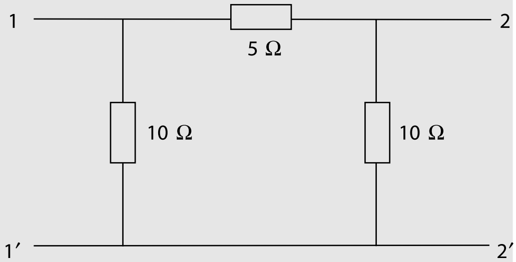

Fig. 16.34

Solution The Z parameters of the given network are

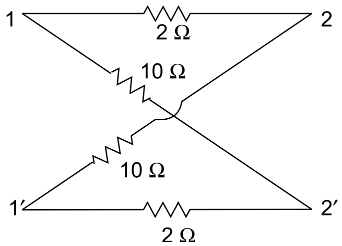

Fig. 16.35

$$Z_{11} = 6\ \Omega = Z_{22};\ Z_{12} = Z_{21} = 4\ \Omega$$

Hence the parameters of the lattice network are

$$Z_a = Z_{11} - Z_{12} = 2\ \Omega$$

$$Z_b = Z_{11} + Z_{12} = 10\ \Omega$$

The lattice network is shown in Fig.16.35

16.13 IMAGE PARAMETERS

The image impedance Z_{I1} and Z_{I2} of a two-port network shown in Fig. 16.36 are two values of impedance such that, if port 1-1′ of the network is terminated in Z_{I1}, the input impedance of port 2-2′ is Z_{I2}; and if port 2-2′ is terminated in Z_{I2}, the input impedance at port 1-1′ is Z_{I1}.

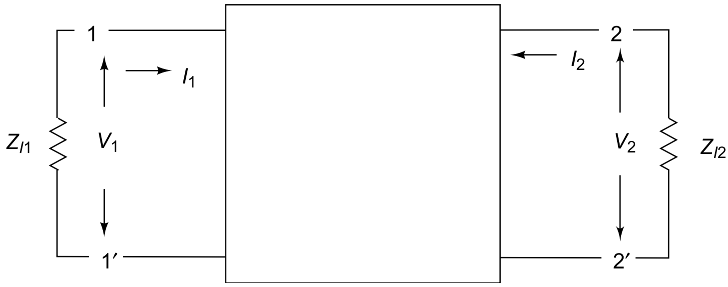

Fig. 16.36

Then, Z_{I1} and Z_{I2} are called image impedances of the two port network shown in Fig. 16.36. These parameters can be obtained in terms of two-port parameters. Recalling Eqs 16.5 and 16.6 in Section 16.4, we have

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

If the network is terminated in Z_{I2} at 2-2′ as shown in Fig. 16.37.

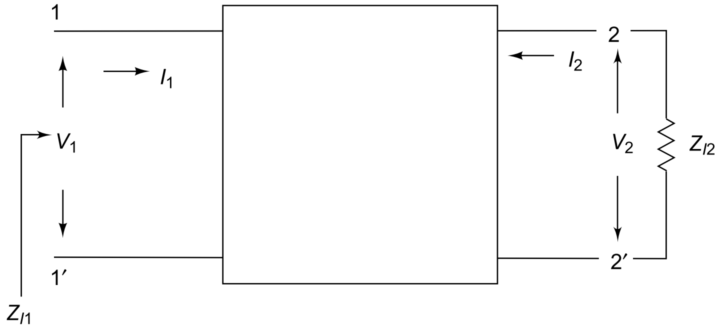

Fig. 16.37

$$V_2 = -I_2 Z_{I2}$$

$$\frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = Z_{I1}$$

$$Z_{I1} = \frac{-AI_2 Z_{I2} - BI_2}{-CI_2 Z_{I2} - DI_2}$$

$$Z_{I1} = \frac{-AZ_{I2} - B}{-CZ_{I2} - D}$$

or $$Z_{I1} = \frac{AZ_{I2} + B}{CZ_{I2} + D}$$

Similarly, if the network is terminated in Z_{I1} at port 1-1′ as shown in Fig. 16.38, then

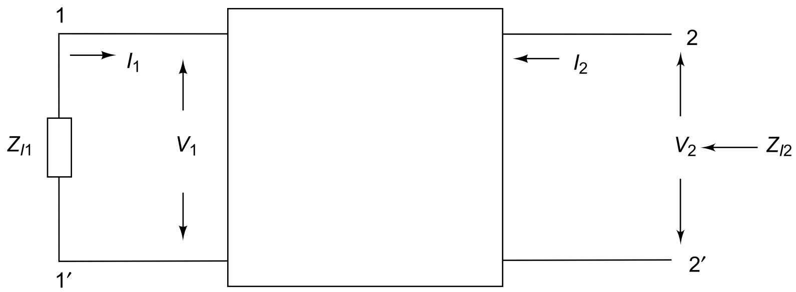

Fig. 16.38

$$V_1 = -I_1 Z_{I1}$$

$$\frac{V_2}{I_2} = Z_{I2}$$

$$\therefore \quad -Z_{I1} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$-Z_{I1} = \frac{AI_2 Z_{I2} - BI_2}{CI_2 Z_{I2} - DI_2}$$

$$-Z_{I1} = \frac{AZ_{I2} - B}{CZ_{I2} - D}$$

From which $$Z_{I2} = \frac{DZ_{I1} + B}{CZ_{I1} + A}$$

Substituting the value of Z_{I1} in the above equation

$$Z_{I2}\left[C\frac{(-AZ_{I2} + B)}{(CZ_{I2} - D)} + A\right] = D\left[\frac{-AZ_{I2} + B}{CZ_{I2} - D}\right] + B$$

From which $$Z_{I2} = \sqrt{\frac{BD}{AC}}$$

Similarly, we can find $Z_{I1} = \sqrt{\dfrac{AB}{CD}}$

If the network is symmetrical, then $A = D$

$$\therefore \quad Z_{I1} = Z_{I2} = \sqrt{\frac{B}{C}}$$

If the network is symmetrical, the image impedances Z_{I1} and Z_{I2} are equal to each other; the image impedance is then called the *characteristic* impedance, or the *iterative* impedance, i.e. if a symmetrical network is terminated in Z_L, its input impedance will also be Z_L, or its impedance transformation ratio is unity. Since a reciprocal symmetric network can be described by two independent parameters, the image parameters Z_{I1} and Z_{I2} are sufficient to characterise reciprocal symmetric networks. Z_{I1} and Z_{I2} the two image parameters do not completely define a network. A third parameter called *image transfer constant* ϕ is also used to describe reciprocal networks. This parameter may be obtained from the voltage and current ratios.

If the image impedance Z_{I2} is connected across port 2-2′, then

$$V_1 = AV_2 - BI_2 \tag{16.20}$$

$$V_2 = -I_2 Z_{I2} \tag{16.21}$$

$$\therefore \quad V_1 = \left[A + \frac{B}{Z_{I2}}\right] V_2 \tag{16.22}$$

$$I_1 = CV_2 - DI_2 \tag{16.23}$$

$$I_1 = -[CZ_{I2} + D]I_2 \tag{16.24}$$

From Eq. 16.22

$$\frac{V_1}{V_2} = \left[A + \frac{B}{Z_{I2}}\right] = A + B\sqrt{\frac{AC}{BD}}$$

$$\frac{V_1}{V_2} = A + \sqrt{\frac{ABCD}{D}} \tag{16.25}$$

From Eq. 16.24

$$\frac{-I_1}{I_2} = [CZ_{I2} + D] = D + C\sqrt{\frac{BD}{AC}}$$

$$\frac{-I_1}{I_2} = D + \sqrt{\frac{ABCD}{A}} \tag{16.26}$$

Multiplying Eqs 16.25 and 16.26 we have

$$\frac{-V_1}{V_2} \times \frac{I_1}{I_2} = \left(\frac{AD + \sqrt{ABCD}}{D}\right)\left(\frac{AD + \sqrt{ABCD}}{A}\right)$$

$$\frac{-V_1}{V_2} \times \frac{I_1}{I_2} = \left(\sqrt{AD} + \sqrt{BC}\right)^2$$

or $\quad \sqrt{AD} + \sqrt{BC} = \sqrt{\dfrac{-V_1}{V_2} \times \dfrac{I_1}{I_2}}$

$$\sqrt{AD} + \sqrt{AD - 1} = \sqrt{\frac{-V_1}{V_2} \times \frac{I_1}{I_2}} \quad (\because AD - BC = 1)$$

Let $\cos h\phi = \sqrt{AD}$; $\sin h\phi = \sqrt{AD-1}$

$$\tan h\phi = \frac{\sqrt{AD-1}}{\sqrt{AD}} = \sqrt{\frac{BC}{AD}}$$

$$\therefore \quad \phi = \tan h^{-1}\sqrt{\frac{BC}{AD}}$$

Also $e^{\phi} = \cos h\phi + \sin h\phi = \sqrt{-\frac{V_1 I_1}{V_2 I_2}}$

$$\phi = \log_e \sqrt{\left(-\frac{V_1 I_1}{V_2 I_2}\right)} = \frac{1}{2}\log_e\left(\frac{V_1}{V_2}\frac{I_1}{I_2}\right)$$

Since $V_1 = Z_{I1} I_1$; $V_2 = -I_2 Z_{I2}$

$$\phi = \frac{1}{2}\log_e\left[\frac{Z_{I1}}{Z_{I2}}\right] + \log\left[\frac{I_1}{I_2}\right]$$

For symmetrical reciprocal networks, $Z_{I1} = Z_{I2}$

$$\phi = \log_e\left[\frac{I_1}{I_2}\right] = \gamma$$

where γ is called the *propagation constant*.

Example 16.15 *Determine the image parameters of the T network shown in Fig.16.39.*

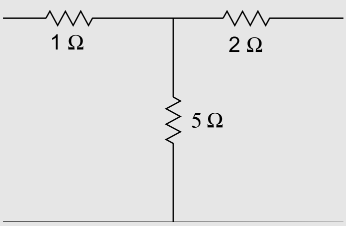

Fig. 16.39

Solution The *ABCD* parameters of the network are

$$A = \frac{6}{5};\ B = \frac{17}{5};\ C = \frac{1}{5};\ D = \frac{7}{5} \quad \text{(See Ex. 16.3)}$$

Since the network is not symmetrical, ϕ, Z_{I1} and Z_{I2} are to be evaluated to describe the network.

$$Z_{I1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{\frac{6}{5}\times\frac{17}{5}}{\frac{1}{5}\times\frac{7}{5}}} = 3.817\ \Omega$$

$$Z_{I2} = \sqrt{\frac{BC}{AC}} = \sqrt{\frac{\frac{17}{5} \times \frac{7}{5}}{\frac{6}{5} \times \frac{1}{5}}} = 4.453\,\Omega$$

$$\phi = \tan h^{-1}\sqrt{\frac{BC}{AD}} = \tan h^{-1}\sqrt{\frac{17}{42}}$$

or $\quad \phi = \ln\left[\sqrt{AD} + \sqrt{AD - 1}\right]$

$$\phi = 0.75$$

Additional Solved Problems

Problem 16.1 *Find the Z parameters for the circuit shown in Fig. 16.40.*

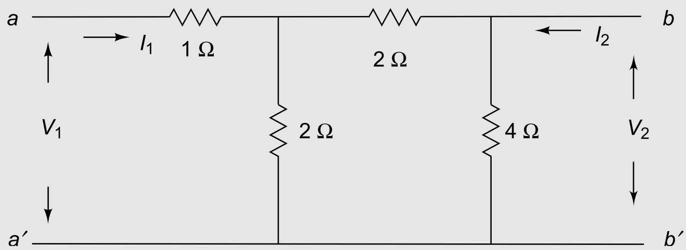

Fig. 16.40

Solution $\quad Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

When $I_2 = 0$; V_1 can be expressed in terms of I_1 and the equivalent impedance of the circuit looking from the terminal *a-a′* as shown in Fig. 16.41 (a).

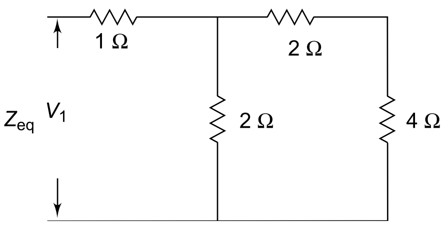

Fig. 16.41 (a)

$$Z_{eq} = 1 + \frac{6 \times 2}{6 + 2} = 2.5\ \Omega$$

$$V_1 = I_1 Z_{eq} = I_1\, 2.5$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 2.5\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

V_2 is the voltage across the 4 Ω impedance as shown in Fig. 16.41 (b).

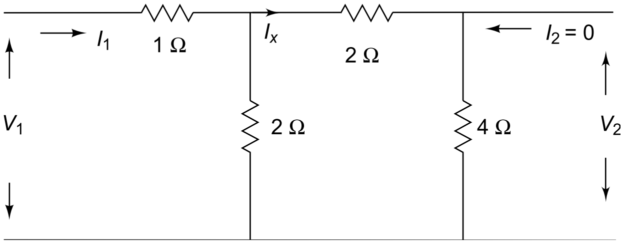

Fig. 16.41 (b)

Let the current in the 4 Ω impedance be I_x

$$I_x = I_1 \times \frac{2}{8} = \frac{I_1}{4}$$

$$V_2 = I_x 4 = \frac{I_1}{4} \times 4 = I_1$$

$$Z_{21} = \left.\frac{V_2}{I_2}\right|_{I_2=0} = 1\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

When port *a-a′* is open circuited the voltage at port *b-b′* can be expressed in terms of I_2, and the equivalent impedance of the circuit viewed from *b-b′* as shown in Fig. 16.41 (c).

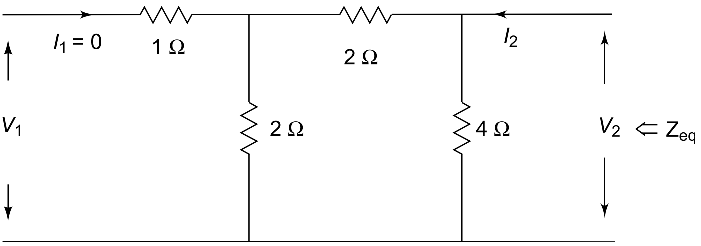

Fig. 16.41 (c)

$$V_2 = I_2 \times 2$$

$$\therefore \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 2\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

V_1 is the voltage across the 2 Ω (parallel) impedance, let the current in the 2 Ω (parallel) impedance is I_Y as shown in Fig. 16.41 (d).

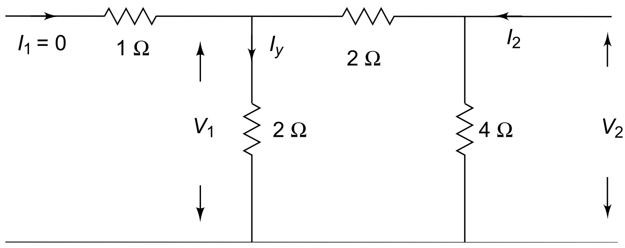

Fig. 16.41 (d)

$$I_Y = \frac{I_2}{2}$$

$$V_1 = 2I_Y$$

$$V_1 = 2\frac{I_2}{2}$$

$$\therefore \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 1\,\Omega$$

Here $Z_{12} = Z_{21}$, which indicates the bilateral property of the network. The describing equations for this two-port network in terms of impedance parameters are

$$V_1 = 2.5I_1 + I_2$$

$$V_2 = I_1 + 2I_2$$

Problem 16.2 *Find the short circuit admittance parameters for the circuit shown in Fig. 16.42.*

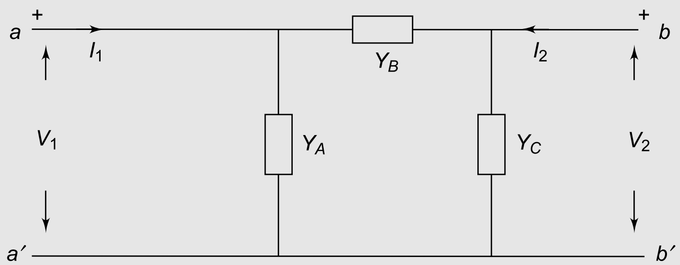

Fig. 16.42

Solution The elements in the branches of the given two-port network are admittances. The admittance parameters can be determined by short circuiting the two-ports.

When port *b*-*b*′ is short circuited, $V_2 = 0$. This circuit is shown in Fig. 16.43 (a).

$$V_1 = I_1 Z_{eq}$$

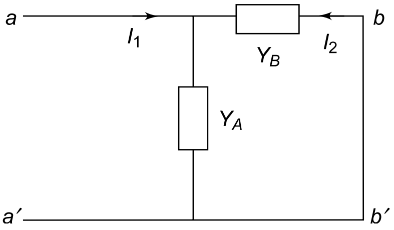

Fig. 16.43 (a)

where Z_{eq} is the equivalent impedance as viewed from *a*-*a*′.

$$Z_{eq} = \frac{1}{Y_{eq}}$$

$$Y_{eq} = Y_A + Y_B$$

$$V_1 = \frac{I_1}{Y_A + Y_B}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = (Y_A + Y_B)$$

With port *b*-*b*′ short circuited, the nodal equation at node 1 gives

$$-I_2 = V_1 Y_B$$

$$\therefore \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_B$$

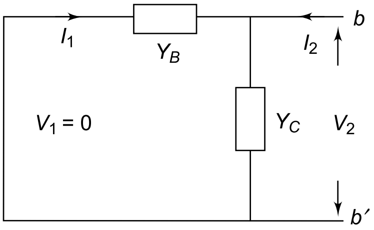

Fig. 16.43 (b)

when port *a*-*a*′ is short circuited; $V_1 = 0$ this circuit is shown in Fig. 16.43 (b).

$$V_2 = I_2 Z_{eq}$$

where Z_{eq} is the equivalent impedance as viewed from *b*-*b*′

$$Z_{eq} = \frac{1}{Y_{eq}}$$

$$Y_{eq} = Y_b + Y_c$$

$$\therefore \quad V_2 = \frac{I_2}{Y_B + Y_C}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = (Y_B + Y_C)$$

With port *a*-*a*′ short circuited, the nodal equation at node 2 gives

$$-I_1 = V_2 Y_B$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_B$$

The describing equations in terms of the admittance parameters are

$$I_1 = (Y_A + Y_B)V_1 - Y_B V_2$$

$$I_2 = -Y_B V_1 + (Y_C + Y_B)V_2$$

Problem 16.3 *Find the Z parameters of the RC ladder network shown in Fig. 16.44.*

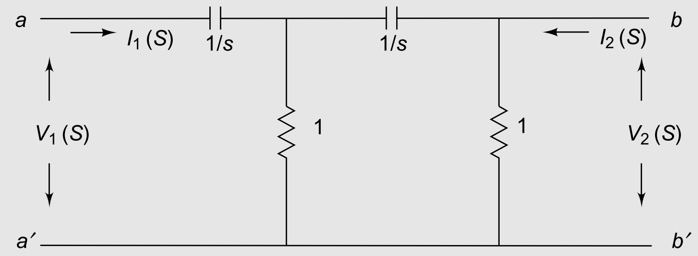

Fig. 16.44

Solution With port *b-b*′ open circuited and assuming mesh currents with $V_1(S)$ as the voltage at *a-a*′, the corresponding network is shown in Fig. 16.45 (a).

The KVL equations are as follows

$$V_2(S) = I_3(S) \tag{16.27}$$

$$I_3(S)\times\left(2+\frac{1}{S}\right)=I_1(S) \tag{16.28}$$

$$\left(1+\frac{1}{S}\right)I_1(S)-I_3(S)=V_1(S) \tag{16.29}$$

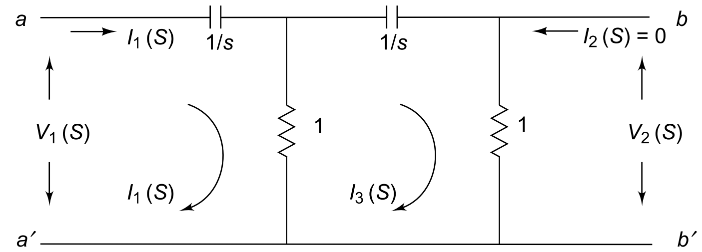

Fig. 16.45 (a)

From Eq. 16.28, $I_3(S)=I_1(S)\left(\dfrac{S}{1+2S}\right)$

From Eq. 16.29 $\left(\dfrac{S+1}{S}\right)I_1(S)-I_1(S)\dfrac{S}{1+2S}=V_1(S)$

$$I_1(S)\left(\frac{1+S}{S}-\frac{S}{1+2S}\right)=V_1(S)$$

$$I_1(S)\left(\frac{S^2+3S+1}{S(1+2S)}\right)=V_1(S)$$

$$Z_{11} = \left.\frac{V_1(S)}{I_1(S)}\right|_{I_2=0} = \frac{(S^2+3S+1)}{S(1+2S)}$$

Also $\quad V_2(S) = I_3(S) = I_1(S)\dfrac{S}{1+2S}$

$$Z_{21} = \left.\frac{V_2(S)}{I_1(S)}\right|_{I_2=0} = \frac{S}{1+2S}$$

With port *a*-*a*′ open circuited and assuming mesh currents with $V_2(S)$ as the voltage as *b*-*b*′, the corresponding network is shown in Fig. 16.45 (b).

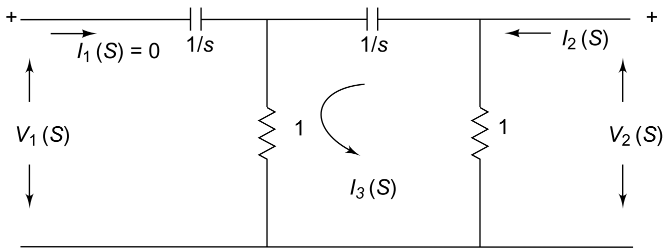

Fig. 16.45 (b)

The KVL equations are as follows

$$V_1(S) = I_3(S) \tag{16.30}$$

$$\left(2+\frac{1}{S}\right)I_3(S) = I_2(S) \tag{16.31}$$

$$V_2(S) = I_2(S) - I_3(S) \tag{16.32}$$

From Eq. 16.31 $\quad I_3(S) = I_2(S)\left(\dfrac{S}{2S+1}\right)$

From Eq. 16.32 $\quad V_2(S) = I_2(S) - I_2(S)\left(\dfrac{S}{2S+1}\right)$

$$V_2(S) = I_2(S)\left(1-\frac{S}{2S+1}\right)$$

$$Z_{22} = \left.\frac{V_2(S)}{I_2(S)}\right|_{I_1(S)=0} = \frac{S+1}{2S+1}$$

Also $\quad V_1(S) = I_3(S) = I_2(S)\left(\dfrac{S}{2S+1}\right)$

$$Z_{12} = \left.\frac{V_1(S)}{I_2(S)}\right|_{I_1(S)=0} = \left(\frac{S}{2S+1}\right)$$

The describing equations are

$$V_1(S) = \left[\frac{S^2+3S+1}{3(2S+1)}\right] I_1 + \left[\frac{S}{2S+1}\right] I_2$$

$$V_2(S) = \left[\frac{S}{2S+1}\right] I_1 + \left[\frac{S+1}{2S+1}\right] I_2$$

Problem 16.4 *Find the transmission parameters for the circuit shown in Fig. 16.46.*

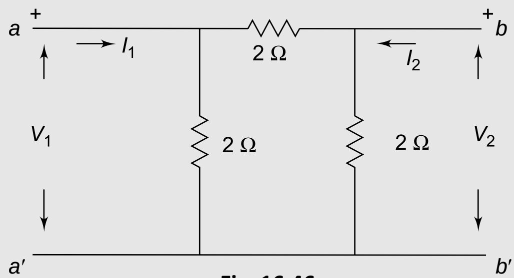

Fig. 16.46

Solution Recalling Eqs 16.5 and 16.6, we have

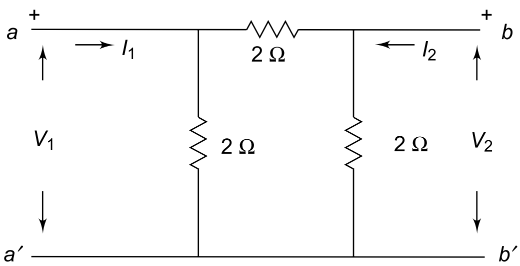

Fig. 16.47 (a)

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

When port b-b' is short circuited with V_1 across a-a', $V_2 = 0$ $B = \dfrac{-V_1}{I_2}$ and the circuit is as shown in Fig. 16.47 (a)

$$-I_2 = \frac{V_1}{2} I_1 = V_1$$

$\therefore$ $B = 2\ \Omega$

$$D = \frac{-I_1}{I_2} = 2$$

When port b-b' is open with V_1 across a-a', $I_2 = 0$

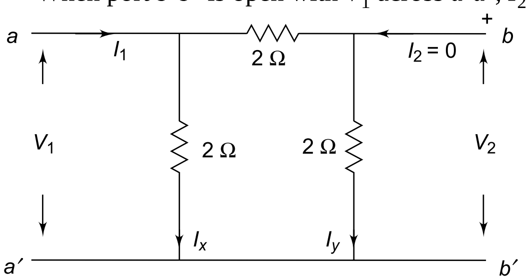

Fig. 16.47 (b)

$A = V_1/V_2$ and the circuit is as shown in Fig. 16.47 (b), where V_1 is the voltage across the 2 Ω resistor across port a-a' and V_2 is the voltage across the 2 Ω resistor across port b-b' when $I_2 = 0$.

From Fig. 16.47 (b), $I_Y = \dfrac{V_1}{4}$

$$V_2 = 2 \times I_Y = \frac{V_1}{2}$$

$$A = 2$$

From Fig. 16.47 (b) $I_x = \frac{V_1}{2}$

$$C = \frac{I_1}{V_2}$$

where $I_1 = \frac{3V_1}{4}$

Therefore $C = \frac{3}{2}\,\mho$

Problem 16.5 *Find h parameters for the network in Fig. 16.48.*

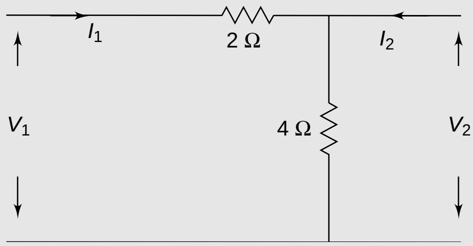

Fig. 16.48

Solution When $V_2 = 0$ the network is as shown in Fig. 16.49.

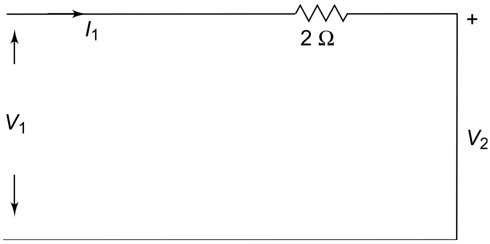

Fig. 16.49

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 2\,\Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} ; I_2 = -I_1$$

$$\therefore \quad h_{21} = -1$$

When $I_1 = 0; h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} ; h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

$$V_1 = I_2\,4,\; V_2 = I_2\,4$$

$$\therefore \quad h_{12} = 1\; h_{22} = \frac{1}{4}\,\mho$$

Problem 16.6 *For the hybrid equivalent circuit shown in Fig. 16.50, (a) determine the current gain, and (b) determine the voltage gain.*

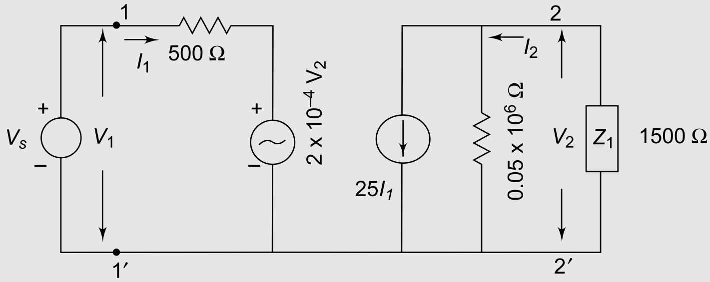

Fig. 16.50

Solution From port 2-2′ we can find

$$I_2 = \frac{(25I_1)(0.05\times10^6)}{(1500+0.05\times10^6)}$$

(a) current gain $\dfrac{I_2}{I_1} = \dfrac{1.25\times10^6}{0.0515\times10^6} = 24.3$

(b) applying KVL at port 1-1′

$$V_1 = 500\,I_1 + 2\times10^{-4}\,V_2$$

$$I_1 = \frac{V_1 - 2\times10^{-4}V_2}{500} \tag{16.33}$$

Applying KCL at port 2-2′

$$I_2 = 25I_1 + \frac{V_2}{0.05}\times10^{-6}$$

also $I_2 = \dfrac{-V_2}{1500}$

$$\frac{-V_2}{1500} = 25I_1 + \frac{V_2}{0.05}\times10^{-6}$$

Substituting the value of I_1 from Eq. 16.33, in the above equation, we get

$$\frac{-V_2}{1500} = 25\left(\frac{V_1 - 2\times10^{-4}V_2}{500}\right) + \frac{V_2}{0.05}\times10^{-6}$$

$$-6.6\times10^{-4}\,V_2 = 0.05V_1 - 0.1\times10^{-4}\,V_2 + 0.2\times10^{-4}\,V_2$$

$$\therefore \quad \frac{V_2}{V_1} = -73.89$$

The negative sign indicates that there is a 180° phase shift between input and output voltage.

Problem 16.7 *The hybrid parameters of a two-port network shown in Fig. 16.51 are h_{11} = 1 K; h_{12} = 0.003; h_{21} = 100; h_{22} = 50 μ℧. Find V_2 and Z parameters of the network.*

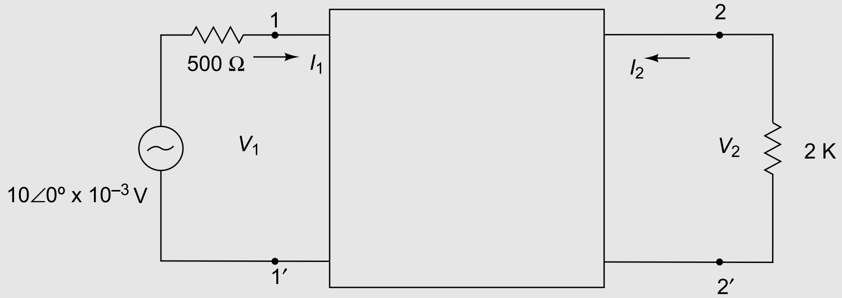

Fig. 16.51

Solution

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad (16.34)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad (16.35)$$

At port 2-2′ $\quad V_2 = -I_2\, 2000$

Substituting in Eq. 16.35, we have

$$I_2 = h_{21}I_1 - h_{22}I_2\, 2000$$

$$I_2\,(1 + h_{22}\, 2000) = h_{21}\, I_1$$

$$I_2(1 + 50 \times 10^{-6} \times 2000) = 100\, I_1$$

$$I_2 = \frac{100 I_1}{1.1}$$

Substituting the value of V_2 in Eq. 16.34, we have

$$V_1 = h_{11}\, I_1 - h_{12}\, I_2\, 2000$$

Also at port 1-1′, $\quad V_1 = V_S - I_1\, 500$

$$\therefore \quad V_s - I_1 500 = h_{11}I_1 - h_{12}\frac{100 I_1}{1.1} \times 2000$$

$$\left(10 \times 10^{-3}\right) - 500 I_1 = 1000 I_1 - 0.003 \times \frac{100}{1.1} I_1 \times 2000$$

$$954.54 I_1 = 10 \times 10^{-3}$$

$$I_1 = 10.05 \times 10^{-6}\ \text{A}$$

$$V_1 = V_S - I_1 \times 500$$

$$= 10 \times 10^{-3} - 10.5 \times 10^{-6} \times 500 = 4.75 \times 10^{-3}\ \text{V}$$

$$V_2 = \frac{V_1 - h_{11} I_1}{h_{12}}$$

$$V_2 = \frac{4.75\times10^{-3} - 1000\times10.5\times10^{-6}}{0.003} = -1.916 \text{ V}$$

(b) Z parameters of the network can be found from Table 16.1.

$$Z_{11} = \frac{\Delta_h}{h_{22}} = \frac{h_{11}h_{22} - h_{21}h_{12}}{h_{22}} = \frac{1\times10^3\times50\times10^{-6} - 100\times0.003}{50\times10^{-6}}$$

$$= -5000\ \Omega$$

$$Z_{12} = \frac{h_{12}}{h_{22}} = \frac{0.003}{50\times10^{-6}} = 60\ \Omega$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} = \frac{-100}{50\times10^{-6}} = -2\times10^6\ \Omega$$

$$Z_{22} = \frac{1}{h_{22}} = 20\times10^3\ \Omega$$

Problem 16.8 *The Z parameters of a two-port network shown in Fig. 16.52 are $Z_{11} = Z_{22} = 10\ \Omega$; $Z_{21} = Z_{12} = 4\ \Omega$. If the source voltage is 20 V, determine I_1, V_2, I_2 and input impedance.*

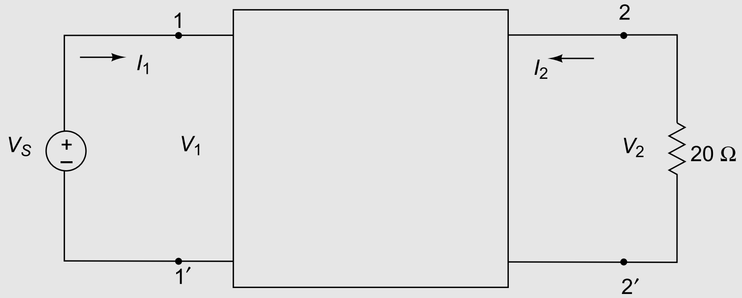

Fig. 16.52

Solution Given $V_1 = V_S = 20$ V

From Section 16.11.1, $V_1 = I_1\left(Z_{11} - \dfrac{Z_{12}Z_{21}}{Z_L + Z_{22}}\right)$

where $Z_L = 20\ \Omega$

$$\therefore \quad 20 = I_1\left(10 - \frac{4\times4}{20+10}\right)$$

$$I_1 = 2.11 \text{ A}$$

$$I_2 = -I_1\frac{Z_{21}}{Z_L + Z_{22}} = -2.11\times\frac{4}{20+10} = -0.281 \text{A}$$

At port 2-2′

$$V_2 = -I_2 \times 20 = 0.281 \times 20 = 5.626 \text{ V}$$

$$\text{Input impedance} = \frac{V_1}{I_1} = \frac{20}{2.11} = 9.478\ \Omega$$

Problem 16.9 *The Y parameters of the two-port network shown in Fig. 16.53 are $Y_{11} = Y_{22} = 6$ ℧ ; $Y_{12} = Y_{21} = 4$ ℧*

(a) determine the driving point admittance at port 2-2′ if the source voltage is 100 V and has an impedance of 1 ohm.

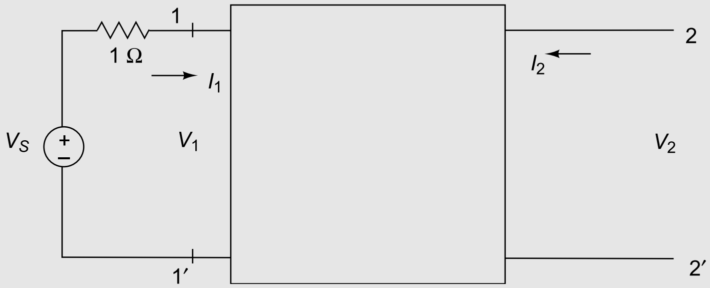

Fig. 16.53

Solution From Section 16.11.2,

$$\frac{I_2}{V_2} = \frac{Y_{22}Y_s + Y_{22}Y_{11} - Y_{21}Y_{12}}{Y_s + Y_{11}}$$

where Y_S is the source admittance = 1 ℧

$$\therefore \quad \text{The driving point admittance} = \frac{6\times1+6\times6-4\times4}{1+6} = 3.714\ ℧$$

$$\text{Or the driving point impedance at port 2-2}' = \frac{1}{3.714}\ \Omega$$

Problem 16.10 *Obtain the Z parameters for the two-port unsymmetrical lattice network shown in Fig. 16.54.*

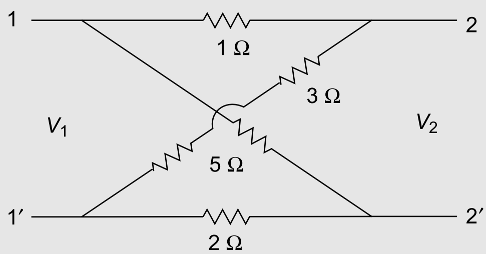

Fig. 16.54

Solution From Section 16.12, we have

$$Z_{11} = \frac{(Z_a + Z_b)(Z_d + Z_c)}{Z_a + Z_b + Z_c + Z_d} = \frac{(1+3)(2+5)}{1+3+5+2} = 2.545\ \Omega$$

$$Z_{21} = \frac{Z_b Z_c - Z_a Z_d}{Z_a + Z_b + Z_c + Z_d} = \frac{3\times 5 - 1\times 2}{11} = 1.181\,\Omega$$

$$Z_{21} = Z_{12}$$

$$Z_{22} = \frac{(Z_a + Z_c)(Z_d + Z_b)}{Z_a + Z_b + Z_c + Z_d} = \frac{(1+5)(2+3)}{11} = 2.727\,\Omega$$

Problem 16.11 *For the ladder two-port network shown in Fig. 16.55, find the open circuit driving point impedance at port 1-2.*

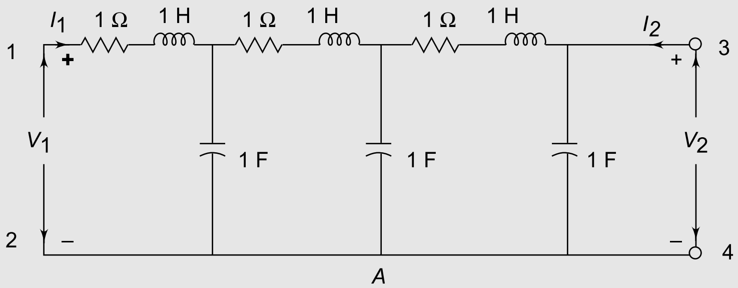

Fig. 16.55

Solution The Laplace transform of the given network is shown in Fig. 16.56.

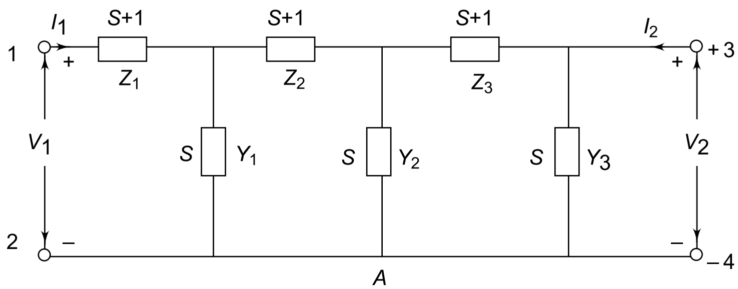

Fig. 16.56

Then the open circuit driving point impedance at port 1-2 is given by

$$Z_{11} = (s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s + \cfrac{1}{(s+1) + \cfrac{1}{s}}}}}$$

$$= \frac{s^6 + 3s^5 + 8s^4 + 11s^3 + 11s^2 + 6s + 1}{s^5 + 2s^4 + 5s^3 + 4s^2 + 3s}$$

Problem 16.12 *For the bridged T network shown in Fig. 16.57, find the driving point admittance y_{11} and transfer admittance y_{21} with a 2 Ω load resistor connected across port 2.*

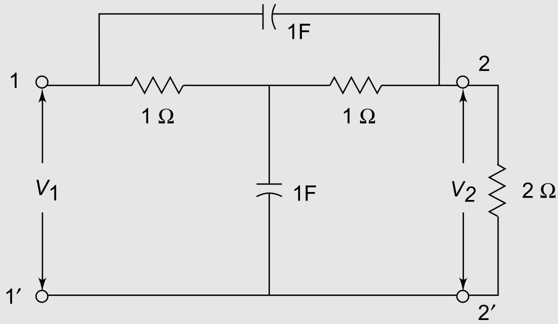

Fig. 16.57

Solution The corresponding Laplace transform network is shown in Fig. 16.58.

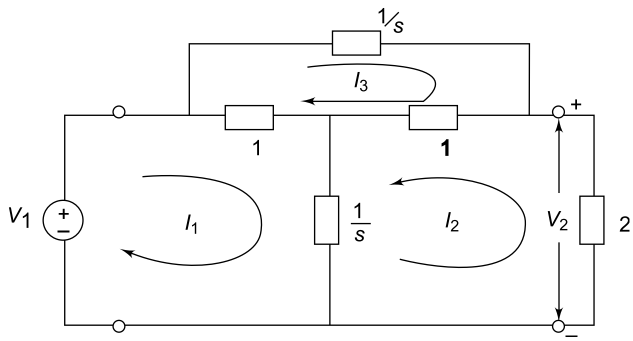

Fig. 16.58

The loop equations are

$$I_1\left(1+\frac{1}{s}\right)+I_2\left(\frac{1}{s}\right)-I_3=V_1$$

$$I_1\left(\frac{1}{s}\right)+I_2\left(1+\frac{1}{s}\right)+I_3=0$$

$$I_1(-1)+I_2+I_3\left(2+\frac{1}{s}\right)=0$$

Therefore,

$$\Delta=\begin{vmatrix}\left(1+\frac{1}{s}\right) & \frac{1}{s} & -1\\ \frac{1}{s} & 1+\frac{1}{s} & 1\\ -1 & 1 & 2+\frac{1}{s}\end{vmatrix}=\frac{s+2}{s^2}$$

Similarly, $$\Delta_{11} = \begin{vmatrix} \left(1+\frac{1}{s}\right) & \frac{1}{s} \\ 1 & \left(2+\frac{1}{s}\right) \end{vmatrix} = \frac{s^2+3s+1}{s^2}$$

and $$\Delta_{12} = \begin{vmatrix} \frac{1}{s} & +1 \\ +1 & \left(2+\frac{1}{s}\right) \end{vmatrix} = \frac{s^2+2s+1}{s^2}$$

Hence, $$Y_{11} = \frac{\Delta_{11}}{\Delta} = \frac{s^2+3s+1}{s+2}$$

and $$Y_{21} = \frac{\Delta_{12}}{\Delta} = \frac{-(s^2+2s+1)}{s+2}$$

Problem 16.13 *For the two port network shown in Fig. 16.59, determine the h-parameters. Using these parameters calculate the output (Port 2) voltage, V_2, when the output port is terminated in a 3 Ω resistance and a 1V(dc) is applied at the input port (V_1 = 1 V).*

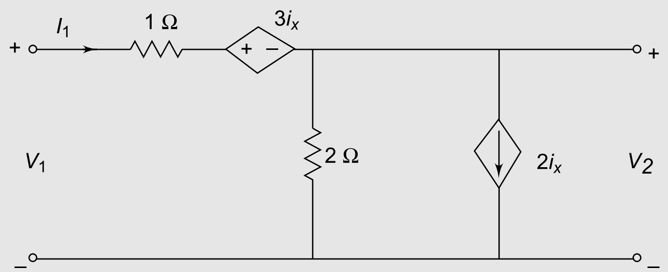

Fig. 16.59

Solution The *h* parameters are defined as

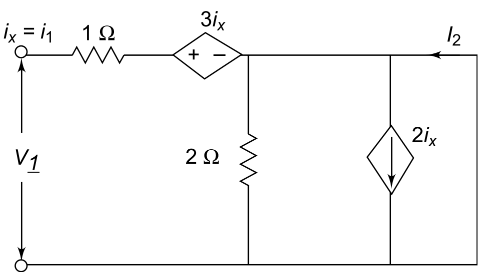

Fig. 16.60 (a)

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

For $V_2 = 0$, the circuit is redrawn as shown in Fig. 16.60 (a)

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{i_1 \times 1 + 3i_1}{i_1} = 4$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{i_2}{i_1} = \frac{2i_1 - i_1}{i_1} = 1$$

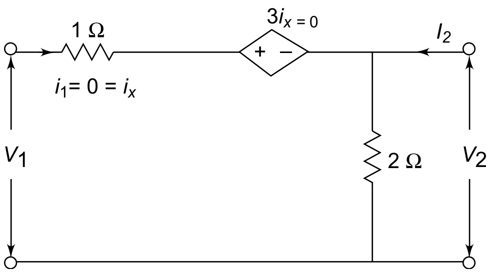

Fig. 16.60 (b)

For $I_1 = 0$, the circuit is redrawn as shown in Fig. 16.60 (b).

$$h_{12} = \frac{V_1}{V_2} = 1; \; h_{22} = \frac{I_2}{V_2} = \frac{1}{2} = 0.5$$

$$\text{Hence,} \quad h = \begin{bmatrix} 4 & 1 \\ 1 & 0.5 \end{bmatrix}$$

$$V_1 = 1\text{ V}$$

$$V_1 = 4I_1 + V_2$$

$$I_2 = I_1 + 0.5\,V_2$$

Eliminating I_1 from the above equations and putting

$$V_1 = 1 \text{ and } I_2 = \frac{-V_2}{3} \text{ we get, } V_2 = \frac{-3}{7}\text{V}$$

Problem 16.14 *Find the current transfer ratio* $\frac{I_2}{I_1}$ *for the network shown in Fig. 16.61.*

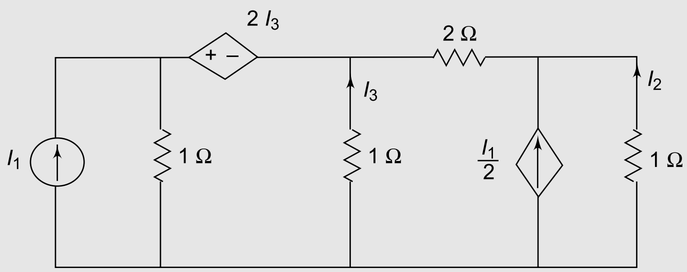

Fig. 16.61

Solution By transforming the current source into voltage source, the given circuit can be redrawn as shown in Fig. 16.62.

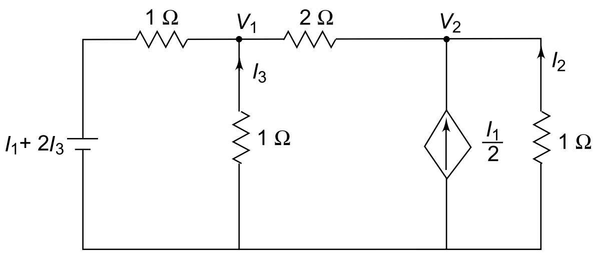

Fig. 16.62

Applying Kirchhoff's nodal analysis

$$\frac{V_1 - (I_1 + 2I_3)}{1} + \frac{V_1}{1} + \frac{V_1 - V_2}{2} = 0$$

$$\text{and} \quad \frac{V_2 - V_1}{2} - \frac{I_1}{2} - I_2 = 0$$

Putting $V_1 = -I_3$ and $V_2 = -I_2$

The above equations become

$$-I_3 - I_1 - 2I_3 - I_3 + \frac{I_2 - I_3}{2} = 0$$

$$\text{and} \quad \frac{I_2 - I_3}{2} - \frac{I_1}{2} - I_2 = 0$$

$$\text{or} \quad I_1 \; 0.5I_2 - 4.5\, I_3 = 0$$

$$\text{and} \quad -0.5\, I_1 - 1.5I_2 + 0.5I_3 = 0$$

By eliminating I_3, we get

$$\frac{I_2}{I_1} = \frac{-5.5}{13} = -0.42$$

Problem 16.15 *Obtain Y-parameters of the Two-port network shown in Fig. 16.63.*

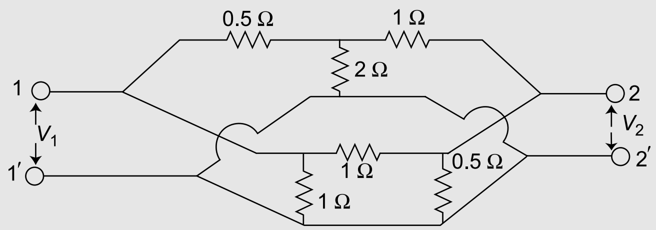

Fig. 16.63

Solution The Above network is the parallel connection of Two-port networks. Y-parameters of such networks can be found by finding individual Y-parameters of respective networks.

T and π networks of the above figure are shown separately.

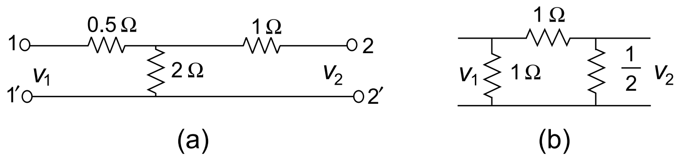

Fig. 16.64

Y-parameters of T- networks are given by

$$Y_{11} = \frac{6}{7};\; Y_{22} = \frac{5}{7};\; Y_{21} = Y_{12} = \frac{-4}{7}$$

The Y-parameters of the π network are given by

$$Y_{11} = 2;\ Y_{12} = 1;\ Y_{22} = 3;\ Y_{21} = -1$$

Y-parameters of the combination are given by

$$Y_{11} = \frac{6}{7} + 2 = \frac{20}{7},\ Y_{22} = \frac{5}{7} + 3 = \frac{26}{7}$$

$$Y_{12} = Y_{21} = \frac{-4}{7} - 1 = \frac{-5}{7}$$

Problem 16.16 *Find the transmission parameters for the network shown in Fig. 16.65.*

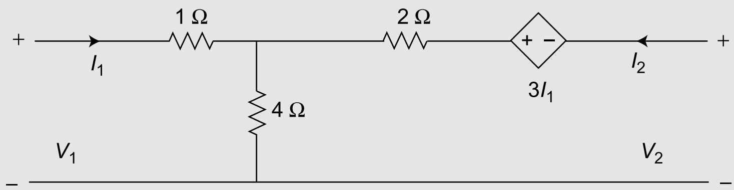

Fig. 16.65

Solution Equations of transmission parameters are

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0};\ -B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0};\ -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

When $I_2 = 0;\ V_1 = I_1 + 4I_1 = 5I_1$

and $-V_2 - 3I_1 + 4I_1 = 0$

$$V_2 = I_1 \Rightarrow \frac{I_1}{I_2} = 1$$

$$\therefore\ V_1 = 5V_2 \Rightarrow \frac{V_1}{V_2} = 5$$

When $V_2 = 0$; The network is shown in Fig. 16.66.

Loop equations $V_1 = 5I_1 + 4I_2$

$$-3I_1 + 2I_2 + 4I_2 + 4I_1 = 0$$

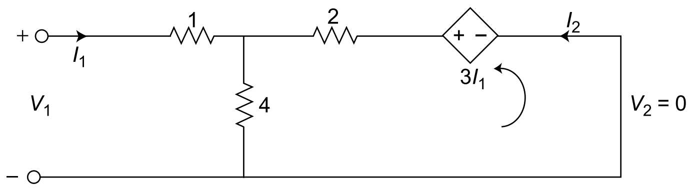

Fig. 16.66

From which $I_1 = -6I_2 \Rightarrow \frac{I_1}{I_2} = -6$

$$\therefore \quad V_1 = -30I_2 + 4I_2$$
$$= -26I_2$$

$$\frac{V_1}{I_2} = -26$$

$$A = \frac{V_1}{V_2} = 5; \; B = \frac{-V_1}{I_2} = 26$$

$$C = \frac{I_1}{V_2} = 1; \; D = \frac{-I_1}{I_2} = 6$$

Problem 16.17 *Determine Z and Y parameters for the circuit shown in Fig. 16.67.*

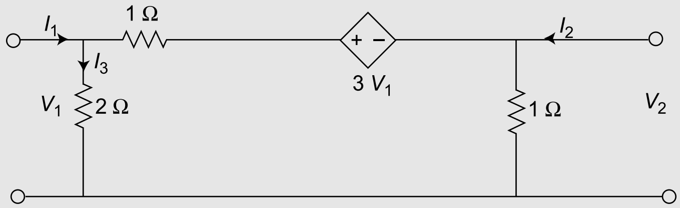

Fig. 16.67

Solution Z-Parameters

Let I_3 be the current in 2 Ω resistor

$$V_1 = 2I_3 \Rightarrow I_3 = V_1/2$$

Applying KVL to outer loop

$$2I_3 - (I_1 - I_3) - 3V_1 - (I_1 + I_2 - I_3) = 0$$

$$3V_1 = -2I_1 - I_2 + 4I_3$$

$$V_2 = I_1 - I_3 + I_2$$

$$V_2 = I_1 + I_2 - \frac{V_1}{2}$$

$$3V_1 = -2I_1 - I_2 + 2V_1$$
$$V_1 = -2I_1 - I_2$$
$$V_2 = I_1 + I_2 - \frac{1}{2}(-2I_1 - I_2)$$
$$V_2 = 2I_1 + \frac{3}{2}I_2$$
$$\therefore \quad Z_{11} = -2;\ Z_{12} = -1;\ Z_{21} = 2;\ Z_{22} = \frac{3}{2}$$

Y-Parameters

From the above equations

$$V_2 + V_1 = I_1 + I_2 - \frac{V_1}{2} - 2I_1 - I_2$$
$$I_1 = \frac{-3}{2}V_1 - V_2$$

Multiply equations V_2 with 2 and add equation V_1

$$2V_2 + V_1 = 2I_1 + 2I_2 - V_1$$
$$2V_2 + 2V_1 = 2I_1 + 2I_2$$
$$\text{Also } V_1 = -2I_1 - I_2$$
$$2V_2 + 3V_1 = I_2$$
$$\therefore \quad Y_{11} = \frac{-3}{2}\ \mho$$
$$Y_{12} = -1\ \mho$$
$$Y_{21} = 3\ \mho$$
$$Y_{22} = 2\ \mho$$

Problem 16.18 *Find Z_{21} and Z_{22} for the network shown in Fig. 16.68.*

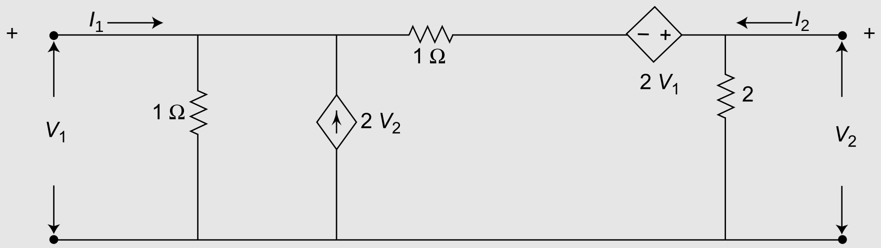

Fig. 16.68

Solution Transforming the dependent current source in to voltage source, the network is shown as

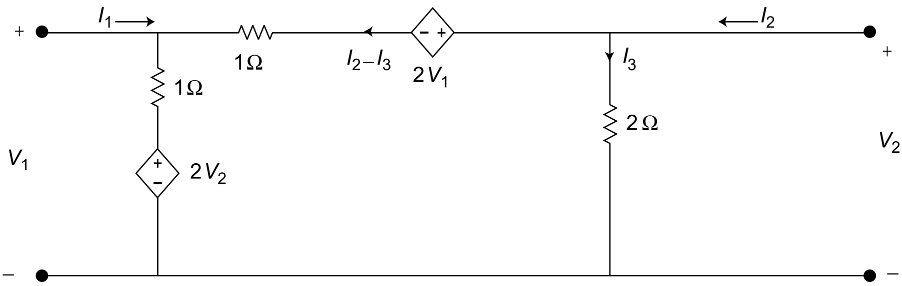

Fig. 16.69

Let I_3 be the current through 2 Ω. KVL to the outer loop.

$$-V_2 + 2V_1 + I_2 - I_3 + V_1 = 0$$

$$-V_2 + 3V_1 + I_2 - I_3 = 0$$

Also $\quad -V_1 + (I_1 + I_2 - I_3) + 2V_2 = 0$

$$V_1 = I_1 + I_2 - I_3 + 2V_2$$

From which

$$-7V_2 - 3I_1 - 2I_2 + 2I_3 = 0$$

where $\quad I_3 = \dfrac{V_2}{2}$

$$\therefore \quad V_2 = \frac{-I_1}{2} - \frac{I_2}{3}$$

Hence $\quad Z_{21} = \dfrac{-1}{2}; \; Z_{22} = \dfrac{-1}{3}$

Problem 16.19 *Obtain the transmission parameters for the following T-network and verify the reciprocity theorem.*

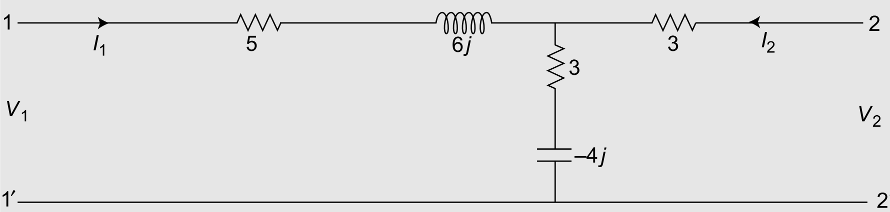

Fig. 16.70

Solution

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

when $I_2 = 0$

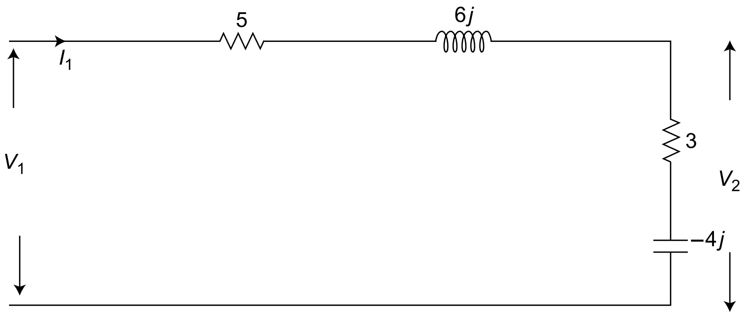

Fig. 16.71

$$V_1 = I_1\,(8 + 2j)$$

$$V_2 = I_1\,(3-4j)$$

$$A = \frac{V_1}{V_2} = \frac{I_1(8+2j)}{I_1(3-4j)} = \frac{8+2j}{3-4j} = 0.64 + 1.52j$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{I_1(3-4j)} = \frac{1}{3-4j} = 0.12 + 0.16j\ \mho$$

When $V_2 = 0$

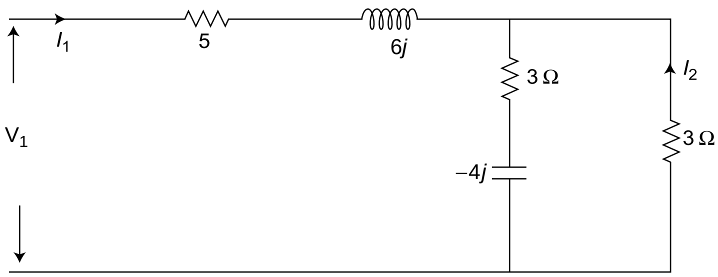

Fig. 16.72

$$B = \frac{-V_1}{I_2}$$

$$-I_2 = \frac{I_1(3-4j)}{6-4j}$$

$$-I_2 = I_1\,(0.65 - 0.23j)$$

$$V_1 = I_1\{(5 + 6j) + [(3 - 4j)\|3]\}$$

$$= I_1[6.96 + 5.3j]$$

$$B = \frac{-V_1}{I_2} = \frac{I_1(6.96+5.3j)}{I_1(0.65-0.23j)}$$

$$\therefore \quad B = 6.95 + 10.61j$$

$$D = -I_1 / I_2 = \frac{I_1}{I_1(0.65 - 0.23j)} = 1.367 + 0.48j$$

Reciprocity condition satisfy when $AD - BC = 1$.

$$\left[(0.64 + 1.52j)\ (1.367 + 0.48j)\right] - \left[(6.95 + 10.61j)(0.12 + 0.16j)\right]$$

$$= 1.00 - 1.6 \times 10^{-4} = 1.008\underline{|-0.009} \simeq 1$$ AQ1

Hence the condition of reciprocity is verified.

PSpice Problems

Problem 16.1 *Using PSpice, find Z-parameters for the circuit in Fig. 16.73.*

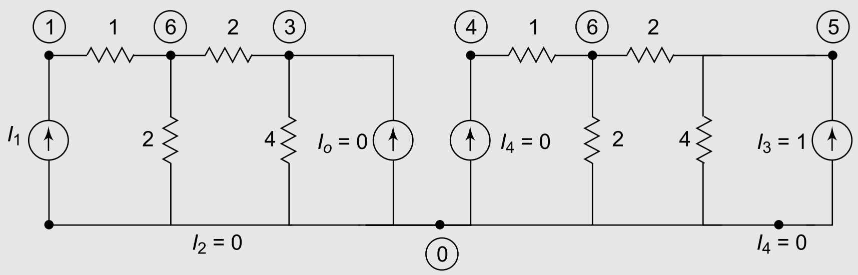

Fig. 16.73

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

```
* DETERMINATION OF Z PARAMETERS
    .SUBCKT AMP 1 3
    R1      1       6       1
    R2      6       3       2
    R3      0       6       2
    R4      0       3       4
    .ENDS
    I1      0       1       1
    I2      0       3       0
    I3      0       5       1
    I4      0       4       0
    X1      1       3 AMP
    X2      4       5 AMP
    .OP
    .END
    **** SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C
    ******************************************************************
```

NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE
(1) 2.5000 (3) 1.0000 (4) 1.0000 (5) 2.0000
(X1.6) 1.5000 (X2.6) 1.0000

Result

Sub circuit 1 is with $I_2 = 0$ ($I_2 = 0$)
⇒ output port is open circuited.

$$Z_{11} = \frac{V_1}{I_1} = 2.5\,\Omega$$

$$Z_{21} = \frac{V_3}{I_1} = 1\,\Omega$$

Sub circuit 2 is with $I_4 = 0$
⇒ input port open circuited.

$$Z_{12} = \frac{V_4}{I_3} = \frac{V_4}{1} = 1\,\Omega$$

$$Z_{22} = \frac{V_5}{I_3} = 2\,\Omega$$

Problem 16.2 *Using PSpice, find the transmission parameters for the circuit shown in Fig. 16.74.*

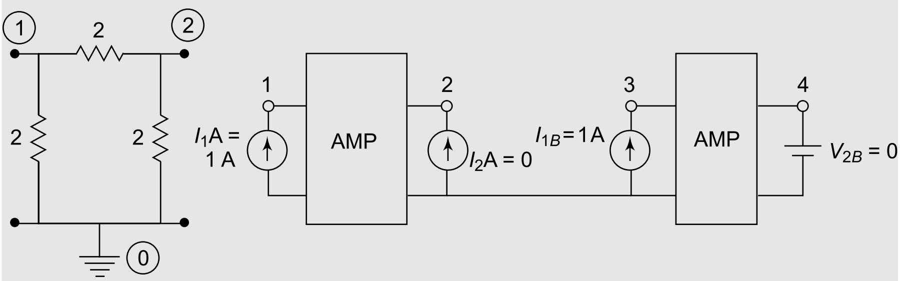

Fig. 16.74

$$V_1 = AV_2 - BI_2$$
$$V_1 = CV_2 - DI_2$$

```
* TRANSMISSION PARAMETERS
  .SUBCKT AMP 1 2
  R1    1   2   2
  R2    1   0   2
  R3    2   0   2
  .ENDS
  I1A   0   1   1
  I2A   0   2   0
  I1B   0   3   1
  V2B   4   0   0
```

```
X1      1       2 AMP
X2      3       4 AMP
.OP
.END
**** SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C
***********************************************************
NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE
(1)  1.3333    (2)  .6667    (3)  1.0000    (4)  0.0000
VOLTAGE SOURCE CURRENTS
NAME            CURRENT
V2B             5.000E – 01
```

Result

Sub circuit 1 is with output node open circuit.

$$A = \frac{V_1}{V_2} = \frac{1.333}{0.667} = 2$$

$$C = \frac{I_1}{V_2} = \frac{1}{0.667} = 1.5\,\mho$$

Sub circuit 2 is with output node short circuit.

$$B = \frac{-V_3}{I_2} = \frac{-1}{0.5} = 2\ \Omega$$

$$D = \frac{-I_1}{I_2} = \frac{-1}{0.5} = 2$$

Practice Problems

16.1 Find the Z parameters of the network shown in Fig. 16.75.

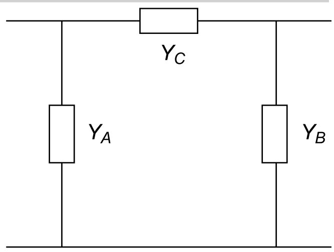

Fig. 16.75

16.2 Find the transmission parameters for the *R–C* network shown in Fig. 16.76.

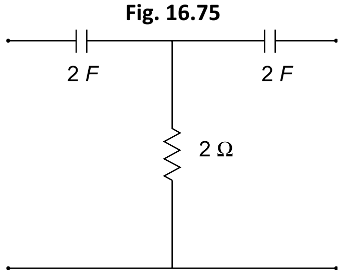

Fig. 16.76

16.3 Find the inverse transmission parameters for the network in Fig. 16.77.

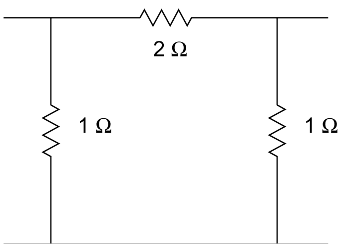

Fig. 16.77

16.4 Calculate the overall transmission parameters for the cascaded network shown in Fig. 16.78.

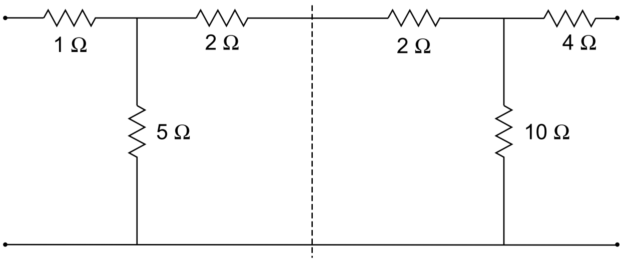

Fig. 16.78

16.5 For the two-port network shown in Fig. 16.79, find the *h* parameters and the inverse *h* parameters.

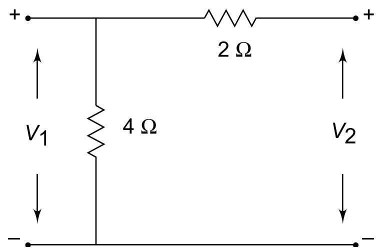

Fig. 16.79

16.6 Determine the impedance parameters for the *T* network shown in Fig. 16.80 and draw the *Z* parameter equivalent circuit.

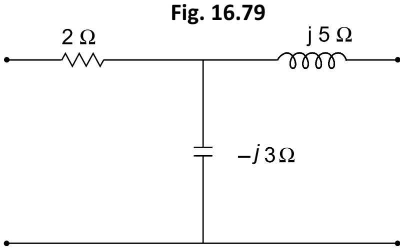

Fig. 16.80

16.7 Determine the admittance parameters for the π-network shown in Fig. 16.81 and draw the *Y* parameter equivalent circuit.

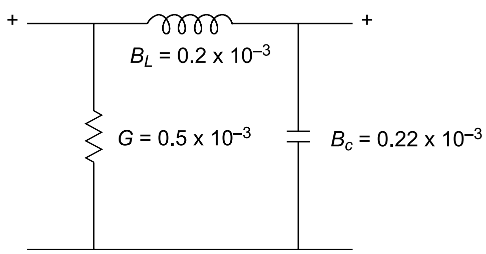

Fig. 16.81

16.8 Determine the impedance parameters and the transmission parameters for the network in Fig. 16.82.

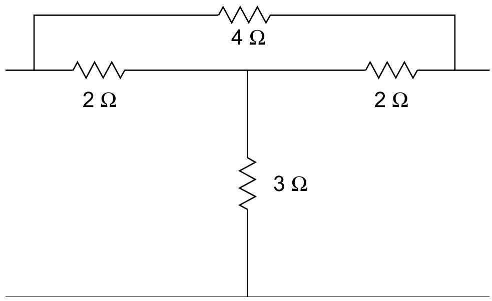

Fig. 16.82

16.9 For the hybrid equivalent circuit shown in Fig. 16.83, determine (a) the input impedance (b) the output impedance.

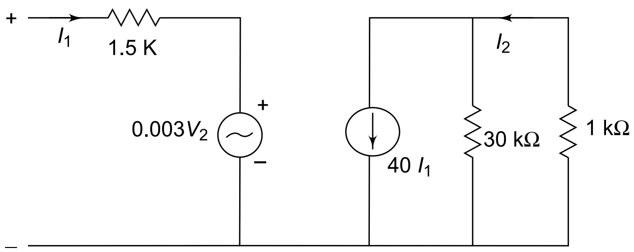

Fig. 16.83

16.10 Determine the input and output impedances for the Z parameter equivalent circuit shown in Fig. 16.84.

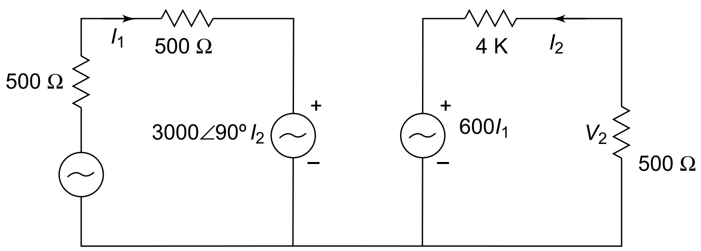

Fig. 16.84

16.11 The hybrid parameters of a two-port network shown in Fig. 16.85 are $h_{11} = 1.5$ K; $h_{12} = 2 \times 10^{-3}$; $h_{21} = 250$; $h_{22} = 150 \times 10^{-6}$ ℧ (a) Find V_2 (b). Draw the Z parameter equivalent circuit.

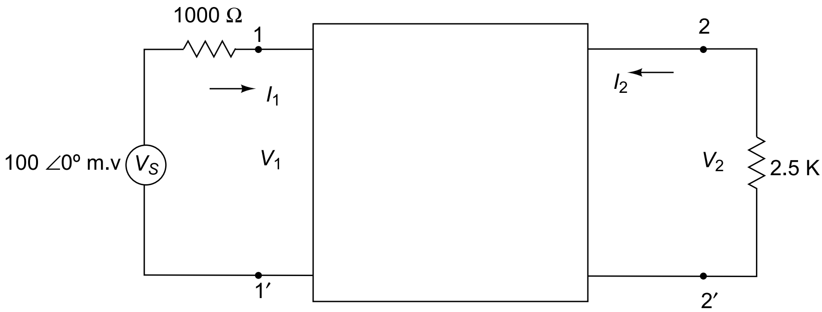

Fig. 16.85

16.12 The Z parameters of a two-port network shown in Fig. 16.86 are $Z_{11} = 5\ \Omega$; $Z_{12} = 4\ \Omega$; $Z_{22} = 10\ \Omega$; $Z_{21} = 5\ \Omega$. If the source voltage is 25 V, determine I_1, V_2 I_2, and the driving point impedance at the input port.

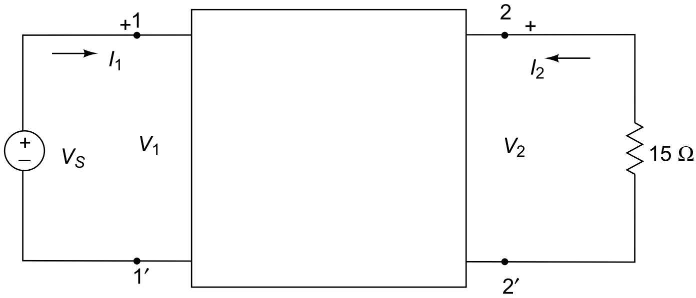

Fig. 16.86

16.13 Obtain the image parameters of the symmetric lattice network given in Fig. 16.87.

16.14 Determine the Z parameters and image parameters of a symmetric lattice network whose series arm impedance is 10 Ω and diagonal arm impedance is 20 Ω.

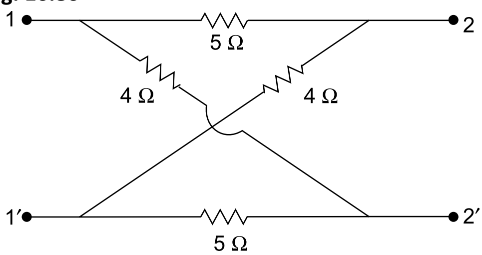

Fig. 16.87

16.15 For the network shown in Fig. 16.88, determine all four open circuit impedance parameters.

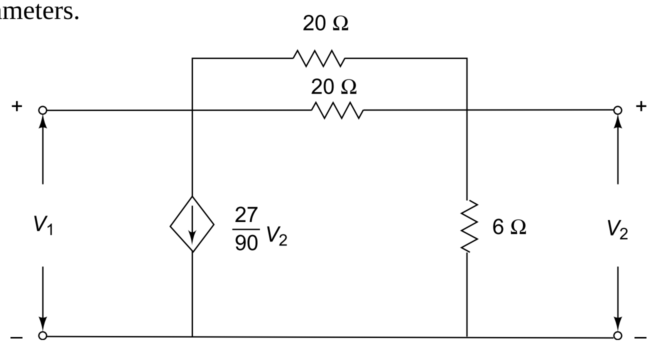

Fig. 16.88

16.16 For the network shown in Fig. 16.89, determine y_{12} and y_{21}.

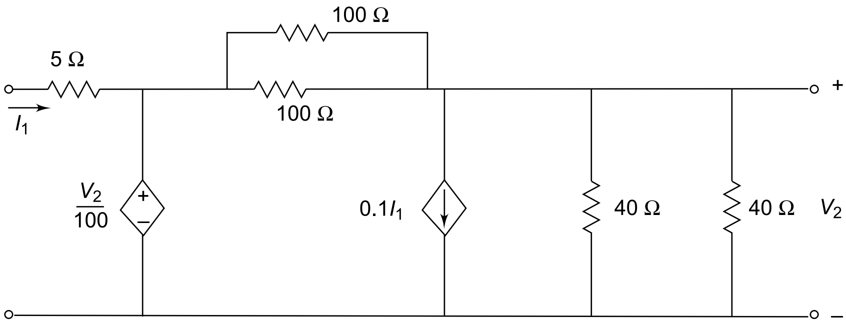

Fig. 16.89

16.17 For the network shown in Fig. 16.90, determine h parameters at $\omega = 10^8$ rad/sec.

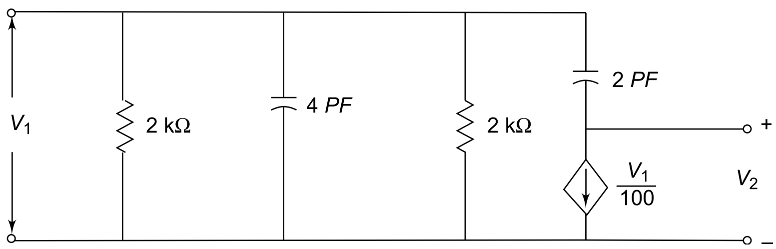

Fig. 16.90

16.18 For the network shown in Fig. 16.91, determine y parameters.

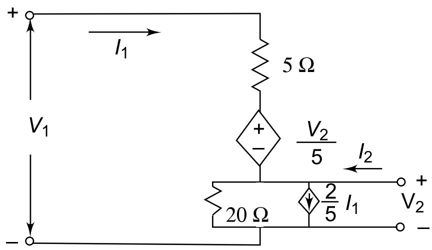

Fig. 16.91

16.19 Using PSpice, obtain γ parameters of the two-port network shown in Fig. 16.92.

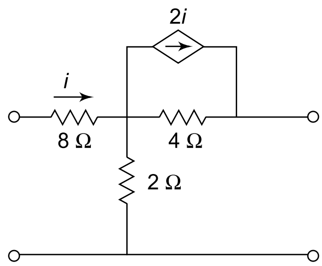

Fig. 16.92

16.20 Using PSpice, find transmission parameters of the network shown in Fig. 16.93.

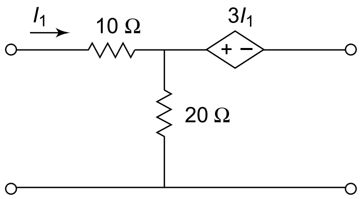

Fig. 16.93

16.21 Using PSpice, find hybrid parameters of the network shown in Fig. 16.94.

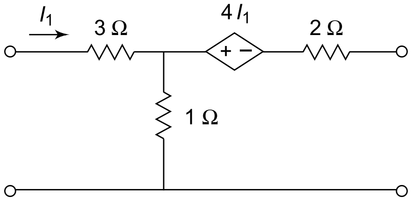

Fig. 16.94

16.22 Find *Y* parameters of the network shown in Fig. 16.95.

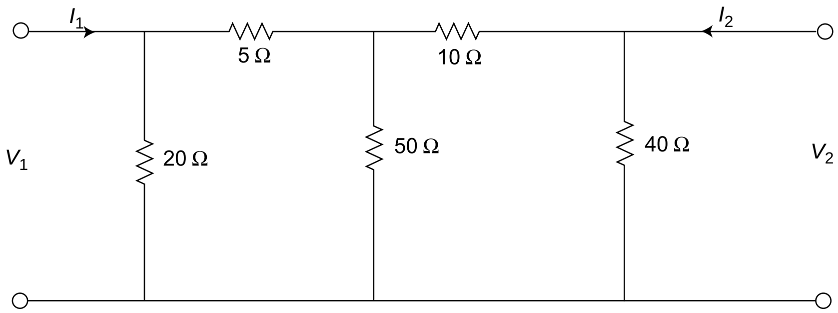

Fig. 16.95

16.23 Find *Z* and *Y* parameters of the given π - network.

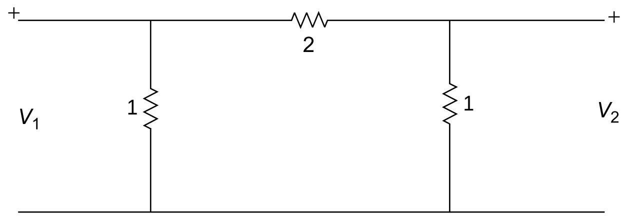

Fig. 16.96

16.24 Obtain a π- equivalent circuit for the following Fig. 16.97.

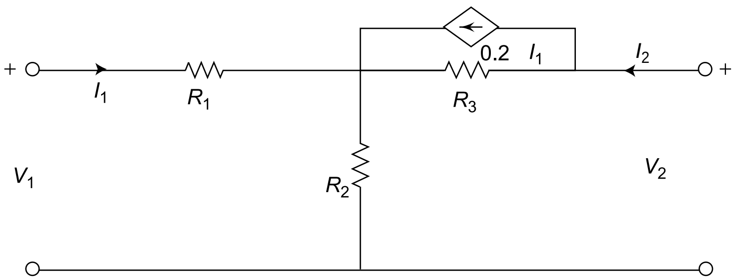

Fig. 16.97

16.25 Find the Y parameters of the Two-port network shown in Fig. 16.98.

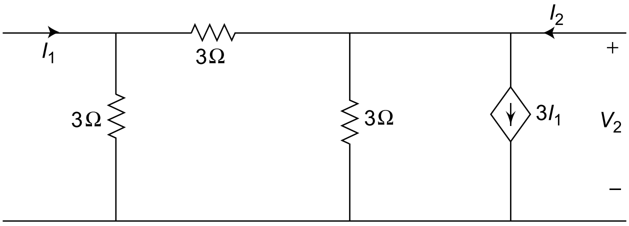

Fig. 16.98

Answers to Practice Problems

16.1 $Z_{11} = \dfrac{Y_B + Y_C}{\Delta Y}; Z_{12} = Z_{21} = \dfrac{Y_C}{\Delta Y}; Z_{22} = \dfrac{Y_A + Y_C}{\Delta Y}$

$\Delta_Y = Y_A Y_B + Y_B Y_C + Y_C Y_A$

16.3 $A' = 3; B' = 2; C' = 4; D' = 3$

16.5 $h_{11} = \dfrac{4}{3}; h_{21} = \dfrac{-2}{3}; h_{22} = \dfrac{1}{6}; h_{12} = \dfrac{2}{3}$

$g_{11} = \dfrac{1}{4}; g_{12} = -1; g_{21} = 1; g_{22} = 2$

16.7 $Y_{11} = (0.5 - j0.2)10^{-3};$

$Y_{12} = Y_{21} = (j0.2 \times 10^{-3})$

$Y_{22} = j(0.02 \times 10^{-3})$

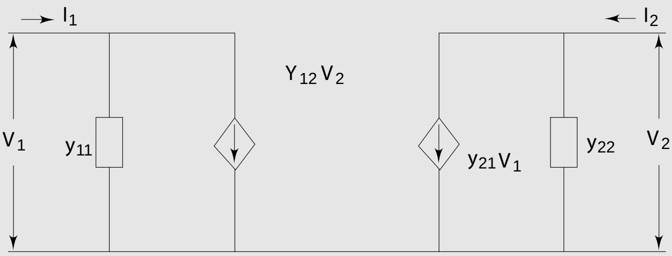

Fig. 16.99

16.9 $Z_i = 1.5\ k\Omega; Z_0 = 0.033 \times 10^{-3}\ \Omega$

16.15 $\begin{bmatrix} 5.71 & -4.29 \\ 2.14 & 2.14 \end{bmatrix}$

16.17 $\begin{bmatrix} 0.857\angle -31^\circ k\Omega & 0.17\angle 59^\circ \\ 8.58\angle -32.1^\circ & 1.89\angle 61.1^\circ m \end{bmatrix}$

16.22 $Y_{11} = -0.5 \quad Y_{12} = 0.25 \quad Y_{22} = 0.2$

16.23 $Z = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}; \quad Y = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{bmatrix}$

16.24 $Y_1 = \dfrac{2R_2 - 0.8R_3}{\Delta Z}; \quad Y_2 = \dfrac{-R_2}{\Delta Z}$

$Y_3 = \dfrac{R_1}{\Delta Z}; \quad \Delta Z = \begin{vmatrix} R_1 + R_2 & R_2 \\ R_2 - 0.2R_3 & R_2 + R_3 \end{vmatrix}$

16.25 $Y_{11} = \dfrac{2}{3}; \quad Y_{12} = \dfrac{-1}{3}; \quad Y_{21} = \dfrac{5}{3}; \quad Y_{22} = \dfrac{-1}{3}$

Objective-Type Questions

16.1 A two-port network is simply a network inside a black box, and the network has only

(a) two terminals
(b) two pairs of accessible terminals
(c) two pairs of ports

16.2 The number of possible combinations generated by four variables taken two at a time in a two-port network is

(a) four (b) two (c) six

16.3 What is the driving-point impedance at port one with port two open circuited for the network in Fig. 16.100?

(a) 4 Ω (c) 3 Ω
(b) 5 Ω

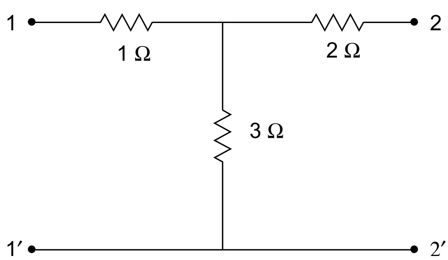

Fig. 16.100

16.4 What is the transfer impedance of the two-port network shown in Fig. 16.87?

(a) 1 Ω (b) 2 Ω (c) 3 Ω

16.5 If the two-port network in Fig. 16.87 is reciprocal or bilateral then

(a) $Z_{11} = Z_{22}$ (b) $Z_{12} = Z_{21}$ (c) $Z_{11} = Z_{12}$

16.6 What is the transfer admittance of the network shown in Fig. 16.101.

(a) –2 ℧ (c) –4 ℧
(b) –3 ℧

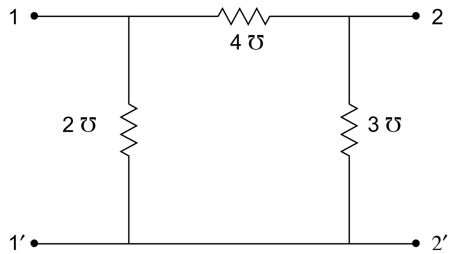

Fig. 16.101

16.7 If the two-port network in Fig. 16.88 is reciprocal then

(a) $Y_{11} = Y_{22}$ (b) $Y_{12} = Y_{22}$ (c) $Y_{12} = Y_{11}$

16.8 In describing the transmission parameters

(a) the input voltage and current are expressed in terms of output voltage and current
(b) the input voltage and output voltage are expressed in terms of output current and input current
(c) the input voltage and output current are expressed in terms of input current and output voltage

16.9 If $Z_{11} = 2\ \Omega$; $Z_{12} = 1\ \Omega$; $Z_{21} = 1\ \Omega$ and $Z_{22} = 3\ \Omega$, what is the determinant of admittance matrix.

(a) 5 (b) 1/5 (c) 1

16.10 For a two-port bilateral network, the three transmission parameters are given by $A = \frac{6}{5}$; $B = \frac{17}{5}$ and $C = \frac{1}{5}$, what is the value of D?

(a) 1 (b) $\frac{1}{5}$ (c) $\frac{7}{5}$

16.11 The impedance matrices of two, two-port networks are given by $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 15 & 5 \\ 5 & 25 \end{bmatrix}$. If the two networks are connected in series. What is the impedance matrix of the combination?

(a) $\begin{bmatrix} 3 & 5 \\ 2 & 25 \end{bmatrix}$ (b) $\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$ (c) $\begin{bmatrix} 15 & 2 \\ 5 & 3 \end{bmatrix}$

16.12 The admittance matrices of two two-port networks are given by $\begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1/2 \\ -1/2 & 5/4 \end{bmatrix}$. If the two networks are connected in parallel, what is the admittance matrix of the combination?

(a) $\begin{bmatrix} 1 & -1/2 \\ -1/2 & 5/4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ -1 & 5/2 \end{bmatrix}$ (c) $\begin{bmatrix} 3/2 & -3/4 \\ -3/4 & 15/8 \end{bmatrix}$

16.13 If the Z parameters of a two-port network are $Z_{11} = 5\ \Omega$ $Z_{22} = 7\ \Omega$; $Z_{12} = Z_{21} = 3\ \Omega$ then the A, B, C, D parameters are respectively given by

(a) $\frac{5}{3}; \frac{26}{3}; \frac{1}{3}; \frac{7}{3}$ (b) $\frac{10}{3}; \frac{52}{3}; \frac{2}{3}; \frac{14}{3}$ (c) $\frac{15}{3}; \frac{78}{3}; \frac{3}{3}; \frac{21}{3}$

16.14 For a symmetric lattice network the value of the series impedance is 3 Ω and that of the diagonal impedance is 5 Ω, then the Z parameters of the network are given by

(a) $Z_{11} = Z_{22} = 2\ \Omega$, $Z_{12} = Z_{21} = 1/2\ \Omega$ (b) $Z_{11} = Z_{22} = 4\ \Omega$, $Z_{12} = Z_{21} = 1\ \Omega$ (c) $Z_{11} = Z_{22} = 8\ \Omega$, $Z_{12} = Z_{21} = 2\ \Omega$

16.15 For a two-port network to be reciprocal.

(a) $Z_{11} = Z_{22}$
(b) $y_{21} = y_{22}$
(c) $h_{21} = -h_{12}$
(d) $AD - BC = 0$

16.16 Two-port networks are connected in cascade. The combination is to be represented as a single two port network. The parameters of the network are obtained by adding the individual

(a) Z parameter matrix
(b) h parameter matrix
(c) $A^1\ B^1\ C^1\ D^1$ matrix
(d) $ABCD$ parameter matrix

16.17 The h parameters h_{11} and h_{12} are obtained

(a) By shorting output terminals
(b) By opening input terminals
(c) By shorting input terminals
(d) By opening output terminals

16.18 Which parameters are widely used in transmission line theory

(a) Z parameters
(b) Y parameters
(c) $ABCD$ parameters
(d) h parameters

Answers to Objective-Type Questions

16.1	(b)	**16.2**	(c)	**16.3**	(a)	**16.4**	(c)	**16.5**	(b)
16.6	(c)	**16.7**	(b)	**16.8**	(a)	**16.9**	(c)	**16.10**	(c)
16.11	(b)	**16.12**	(c)	**16.13**	(a)	**16.14**	(b)	**16.15**	(c)
16.16	(a)	**16.17**	(a)	**16.18**	(c)				