3 Magnetic Circuits

3.1 Introduction

All of us are familiar with a magnet. It is a piece of solid body which possesses a property of attracting iron pieces and pieces of some other metals. This is called a **natural magnet** . While as per the discovery of Scientist **Oersted** we can have an electromagnet. Scientist Oersted stated that any current carrying conductor is always surrounded by a magnetic field. The property of such current is called **magnetic effect of an electric current**. Natural magnet or an electromagnet, both have close relation with electromotive force (e.m.f.), mechanical force experienced by conductor, electric current etc. To understand this relationship it is necessary to study the fundamental concepts of magnetic circuits. In this chapter we shall study laws of magnetism, magnetic field due to current carrying conductor, magnetomotive force, simple series and parallel magnetic circuits .

3.2 Magnet and its Properties

As stated earlier, magnet is a piece of solid body which possesses property of attracting iron and some other metal pieces.

i) When such a magnet is rolled into iron pieces it will be observed that iron pieces cling to it as shown in Fig. 3.1

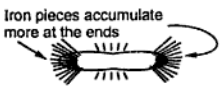


Fig. 3.1 Natural magnet

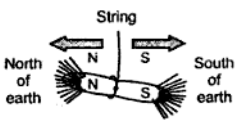


Fig. 3.2 Freely suspended

The maximum iron pieces accumulate at the two ends of the magnet while very few accumulate at the centre of the magnet.

The points at which the iron pieces accumulate maximum are called **Poles** of the magnet while imaginary line joining these poles is called **Axis** of the magnet.

ii) When such magnet is suspended freely by a piece of silk fibre, it turns and always adjusts itself in the direction of North and South of the earth.

The pole which adjusts itself in the direction of North is called North seeking or **North (N) pole**, while the pole which points in the direction of South is called South seeking or **South (S) pole**. Such freely suspended magnet is shown in the Fig. 3.2

This is the property due to which it is used in the compass needle which is used by navigators to find the directions.

iii) When a magnet is placed near an iron or steel piece, its property of attraction gets transferred to iron or steel piece. Such transfer of property of attraction is also possible by actually rubbing the pole of magnet on an iron or steel piece. Such property is called **magnetic induction.**

Magnetic Induction : The phenomenon due to which a magnet can induce magnetism in a (iron or steel) piece of magnetic material placed near it without actual physical contact is called magnetic induction.

iv) An ordinary piece of magnetic material when brought near to any pole N or S gets attracted towards the pole. But if another magnet is brought near the magnet such that two like poles ('N' and 'N' or 'S' and 'S'), it shows a repulsion in between them while if two unlike poles are brought near, it shows a force of attraction.

Key Point : *Like poles repel each other and the unlike poles attract each other. Repulsion is the sure test of magnetism as ordinary piece of magnetic material always shows attraction towards both the poles.*

Let us see the molecular theory behind this magnetism.

3.3 Molecular Theory of Magnetization

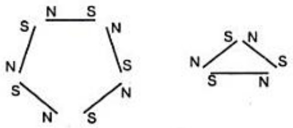


Fig. 3.3 Molecular magnets in unmagnetised material

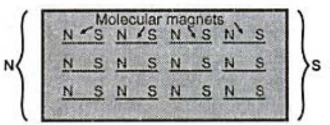


Fig. 3.4 Magnetised piece of material

Not only magnetized but materials like iron, steel are also complete magnets according to molecular theory. All materials consist of small magnets internally called **molecular magnets**. In unmagnetised materials such magnets arrange themselves in closed loops as shown in the Fig. 3.3

So at any joint, effective strength at a point is zero, due to presence of two unlike poles. Such poles cancel each other's effect. But if magnetized material is considered or unmagnetized material subjected to magnetizing force is considered, then such small molecular magnets arrange themselves in the direction of magnetizing force, as shown in the Fig. 3.4

Unlike poles of these small magnets in the middle are touching each other and hence neutralizing the effect.

But on one end 'N' poles of such magnets exist without neutralizing effect. Similarly on other end 'S' poles of such magnets exist. Thus one end behaves as 'N' pole while other as 'S' pole. So most of the iron particles get attracted towards end and not in the middle.

From this theory, we can note down the following points :

1) When magnetizing force is applied, immediately it is not possible to have alignment of all such small magnets, exactly horizontal as shown in the Fig. 3.4. There is always some limiting magnetizing force exists for which all such magnets align exactly in horizontal position .

Key Point: *Though magnetizing force is increased beyond certain value, there is no chance for further alignment of molecular magnets hence further magnetization is not possible. Such condition or phenomenon is called* ***Saturation.***

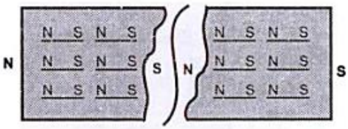


Fig. 3.5 Breaking of magnet

2) If the magnet is broken at any point, each piece behaves like an independent magnet with two poles to each, 'N' and 'S'.

3) The piece of soft iron gets magnetized more rapidly than hard steel. This is because alignment of molecular magnets in soft iron takes place quickly for less magnetizing force than in hard steel.

4) If unmagnetised piece is subjected to alternating magnetizing force i.e. changing magnetizing force, then heat is produced. This is because molecular magnets try to change themselves as per change in magnetizing force. So due to molecular friction heat is generated.

5) If a magnet is heated and allowed to cool, it demagnetizes. This is because heat sets molecular magnets into motion so that the molecules again form a closed loop, neutralizing the magnetism.

6) **Retentivity** : When a soft iron piece is magnetized by external magnetizing force due to magnetic induction, it loses its magnetism immediately if such force is removed. As against this hard steel continues to show magnetism though such force is removed. It retains magnetism for some time.

Key Point : *The power of retaining magnetism after the magnetizing force is removed is called* ***Retentivity.*** *The time for which material retains such magnetism in absence of magnetizing force depends on its retentivity.*

3.4 Laws of Magnetism

There are two fundamental laws of magnetism which are as follows :

Law 1 : *It states that 'Like magnetic poles repel and unlike poles attract each other'*

This is already mentioned in the properties of magnet.

Law 2: *This law is experimentally proved by Scientist Coulomb and hence also known as Coulomb's Law.*

The force (F) exerted by one pole on the other pole is,

a) directly proportional to the product of the pole strengths,

b) inversely proportional to the square of the distance between them, and

c) nature of medium surrounding the poles.

Mathematically this law can be expressed as,

$$F \propto \frac{M_1 M_2}{d^2}$$

where M_1 and M_2 are pole strengths of the poles while d is distance between the poles.

$$\therefore \qquad \boxed{F = \frac{K M_1 M_2}{d^2}}$$

where K depends on the nature of the surroundings and called permeability.

3.5 Magnetic Field

We have seen that magnet has its influence on the surrounding medium. 'The region around a magnet within which the influence of the magnet can be experienced is called **magnetic field.** Existence of such field can be experienced with the help of compass needle, iron or pieces of metals or by bringing another magnet in vicinity of a magnet.

3.5.1 Magnetic Lines of Force

The magnetic field of magnet is represented by imaginary lines around it which are called **magnetic lines of force.** Note that these lines have no physical existence, these are purely imaginary and were introduced by **Michael Faraday** to get the visualization of distribution of such lines of force.

3.5.2 Direction of Magnetic Field

The direction of magnetic field can be obtained by conducting small experiment.

Let us place a permanent magnet on table and cover it with a sheet of cardboard. Sprinkle steel or iron fillings uniformly over the sheet. Slight tapping of cardboard causes fillings to adjust themselves in a particular pattern as shown in the Fig. 3.6

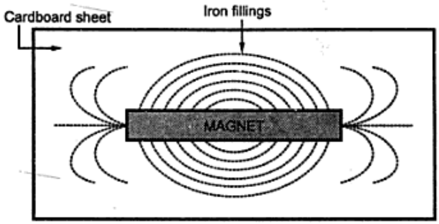


Fig. 3.6 Magnetic lines of force

The shape of this pattern projects a mental picture of the magnetic field present around a magnet.

A line of force can be defined as,

Consider the isolated N pole (we cannot separate the pole but imagine to explain line of force) and it is allowed to move freely, in a magnetic field. Then path along which it moves is called **line of force.** Its shape is as shown in the Fig. 3.6 and direction always from N-pole towards S-pole.

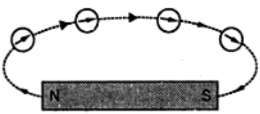


Fig. 3.7 Compass needle experiment

The direction of lines of force can be understood with the help of small compass needle. If magnet is placed with compass needles around it, then needles will take positions as shown in the Fig. 3.7. The tangent drawn at any point, of the dotted curve shown, gives direction of resultant force at that point. The N poles are all pointing along the dotted line shown,from N- pole to its S-pole.

The lines of force for a bar magnet and U-shaped magnet are shown in the Fig. 3.8.

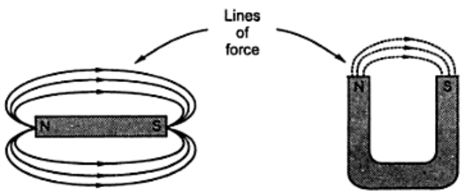


Fig. 3.8 (a) Bar magnet **Fig. 3.8 (b) U-shaped magnet**

Attraction between the unlike poles and repulsion between the like poles of two magnets can be easily understood from the direction of magnetic lines of force. This is shown in the Fig. 3.9 (a) and (b).

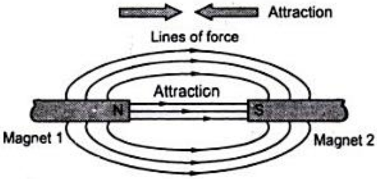


Fig. 3.9 (a) Force of attraction

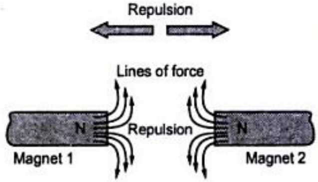


Fig. 3.9 (b) Force of repulsion

3.5.3 Properties of Lines of Force

Though the lines of force are imaginary, with the help of them various magnetic effects can be explained very conveniently. Let us see the various properties of these lines of force.

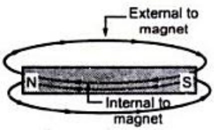


Fig. 3.10 Lines of force complete the closed path

1) Lines of force are always originating on a N-pole and terminating on a S-pole, external to the magnet.

2) Each line forms a **closed loop** as shown in the Fig. 3.10.

> **Key Point**: *This means that a line emerging from N-pole, continues upto S-pole* ***external to the magnet*** *while it is assumed to continue from S-pole to N-pole* ***internal to the magnet*** *completing a closed loop. Such lines internal to the magnet are called as* ***lines of induction.***

3) Lines of force never intersect each other.

4) The lines of force, are like stretched rubberbands and always try to contract in length.

5) The lines of force, which are parallel and travelling in the same direction repel each other.

6) Magnetic lines of force always prefer a path offering least opposition.

> **Key Point**: *The opposition by the material to the flow of lines of force is called reluctance. Air has more reluctance while magnetic materials like iron, steel etc. have low reluctance. Thus magnetic lines of force can easily pass through iron or steel but cannot pass easily through air.*

3.6 Magnetic Flux (ϕ)

The total number of lines of force existing in a particular magnetic field is called **magnetic flux.** Lines of force can be called **lines of magnetic flux.** The unit of flux is weber and flux is denoted by symbol (ϕ) . The unit weber is denoted as Wb.

$$1 \text{ weber} = 10^8 \text{ lines of force.}$$

3.7 Pole Strength

We have seen earlier that force between the poles depends on the pole strengths. As we are now familiar with flux, we can have idea of pole strength. Every pole has a capacity to radiate or accept certain number of magnetic lines of force i.e. magnetic flux which is called its **strength.** Pole strength is measurable quantity assigned to poles which depends on the force between the poles. If two poles are exerting equal force on one other, they are said to have equal pole strengths.

Unit of pole strength is **weber** as pole strength is directly related to flux i.e. lines of force

Key Point: *A unit pole may be defined as that pole which when placed from an identical pole at a distance of 1 metre in free space experiences a force of* $\frac{10^7}{16\pi^2}$ *newtons.*

So when we say Unit N-pole, it means a pole is having a pole strength of 1 weber.

3.8 Magnetic Flux Density (B)

It can be defined as 'The flux per unit area (a) in a plane at right angles to the flux is known as '**flux density**'. Mathematically,

$$B = \frac{\phi}{a} \quad \frac{Wb}{m^2} \text{ or Tesla}$$

It is shown in the Fig. 3.11.

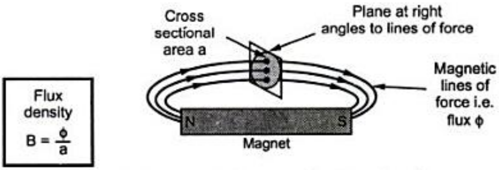


Fig. 3.11 Concept of magnetic flux density

Key Point: *The unit of flux density is* Wb/m^2*, also called* ***tesla*** *denoted as T.*

3.9 Magnetic Field Strength (H)

This gives quantitative measure of strongness or weakness of the magnetic field. Note that pole strength and magnetic field strength are different. This can be defined as 'The force experienced by a unit N-pole (i.e. N pole with 1 Wb of pole strength) when placed at any point in a magnetic field is known as **magnetic field strength** at that point.

It is denoted by H and its unit is newtons per weber i.e. (N/Wb) or amperes per metre (A/m.) or ampere turns per metre (AT/m). The mathematical expression for calculating magnetic field strength is,

$$H = \frac{\text{ampere turns}}{\text{length}}$$

$$\therefore \quad H = \frac{NI}{l} \text{ AT/m}$$

Key Point: *More the value of 'H', more stronger is the magnetic field. This is also called magnetic field intensity.*

Example 3.1 : *A pole having strength of* 0.5×10^{-3} *Wb is placed in a magnetic field at a distance of 25 cm from another pole. It is experiencing a force of 0.5 N. Assume constant of medium* $\left(\frac{1}{36\pi^2\times10^{-7}}\right)$. *Determine,*

a) magnetic field strength at the point. *b) the strength of other pole.*

c) distance at which force experienced will be doubled.

Solution : The given values are,

$$M_1 = 0.5\times10^{-3} \text{ Wb}, \qquad d = 25 \text{ cm} = 0.25 \text{ m}, \qquad F = 0.5 \text{ N}$$

$$K = \left(\frac{1}{36\pi^2\times10^{-7}}\right) = 28144.773$$

(a) Magnetic field strength,

$$H = \frac{\text{Newton}}{\text{Wb}} = \frac{\text{Force experienced}}{\text{Pole strength}} = \frac{0.5}{0.5\times10^{-3}}$$

$$\mathbf{=1000 \text{ N / Wb.}}$$

b) According to Coulomb's law,

$$F = \frac{KM_1M_2}{d^2}$$

$$\therefore \quad 0.5 = \frac{0.5\times10^{-3}\times28144.773\times M_2}{(0.25)^2}$$

$$\therefore \quad M_2 = \mathbf{2.22\times10^{-3} \text{ Wb}}$$ ... pole strength of other pole

c) $F = 1\ N$

$\therefore \quad 1 = \dfrac{28144.773 \times 0.5 \times 10^{-3} \times 2.22 \times 10^{-3}}{d^2}$

$\therefore \quad d = 0.1767\ m = \mathbf{17.67\ cm}$

At a distance of 17.67 cm from another pole, the first pole will experience a force 1 N.

> **Key Point**: *When poles are brought nearer and nearer, force experienced by them increases.*

3.10 Magnetic Effect of an Electric Current (Electromagnets)

When a coil or a conductor carries a current, it produces the magnetic flux around it. Then it starts behaving as a magnet. Such a current carrying coil or conductor is called an **electromagnet.** This is due to magnetic effect of an electric current.

If such a coil is wound around a piece of magnetic material like iron or steel and carries current then piece of material around which the coil is wound, starts behaving as a magnet, which is called an electromagnet.

The flux produced and the flux density can be controlled by controlling the magnitude the current.

The direction and shape of the magnetic field around the coil or conductor depends on the direction of current and shape of the conductor through which it is passing. The magnetic field produced can be experienced with the help of iron fillings or compass needle.

Let us study two different types of electromagnets,

1) Electromagnet due to straight current carrying conductor
2) Electromagnet due to circular current carrying coil

3.10.1 Magnetic Field due to Straight Conductor

When a straight conductor carries a current, it produces a magnetic field all along its length. The lines of force are in the form of concentric circles in the planes right angles to the conductor. This can be demonstrated by a small experiment.

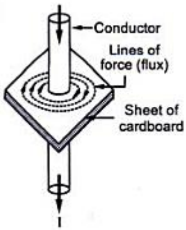


Fig. 3.12 Magnetic field due to a straight conductor

Consider a straight conductor carrying a current, passing through a sheet of cardboard as shown in the Fig. 3.12. Sprinkle iron fillings on the cardboard. Small tapping on the cardboard causes the iron filling to set themselves, in the concentric circular pattern. The direction of the magnetic flux can be determined by placing compass needle near the conductor. This direction depends on the direction of the current passing through the conductor. For the current direction shown in the Fig. 3.12 i.e. from top to bottom the direction of flux is clockwise around the conductor.

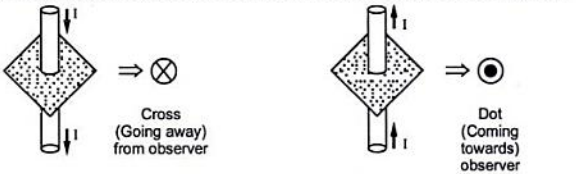
Conventionally such current carrying conductor is represented by small circle, (top view of conductor shown in the Fig. 3.12). Then current through such conductor will either come out of paper or will go into the plane of the paper.

> **Key Point** : *When current is going into the plane of the paper, i.e. away from observer, it is represented by a '**cross**', inside the circle indicating the conductors.*

The cross indicates rear view of feathered end of an arrow.

> **Key Point**: *The current flowing towards the observer i.e. coming out of the plane of the paper is represented by a '**dot**' inside the circle.*

The dot indicates front view i.e. tip of an arrow. This is shown in the Fig. 3.13.



(a) Current into the paper **(b) Current out of the paper**

Fig. 3.13 Cross and Dot convention

3.10.1.1 Rules to Determine Direction of Flux Around Conductor

1) Right Hand Thumb Rule : It states that, hold the current carrying conductor in the right hand such that the thumb pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of the magnetic field or flux around it. The Fig. 3.14 explains the rule.

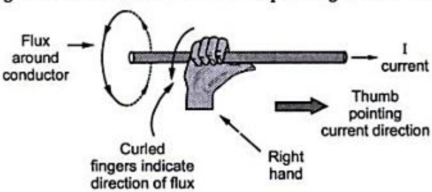


Fig. 3.14 Right hand thumb rule

Let us apply this rule to the conductor passing through card sheet considered earlier. This can be explained by the Fig. 3.15.

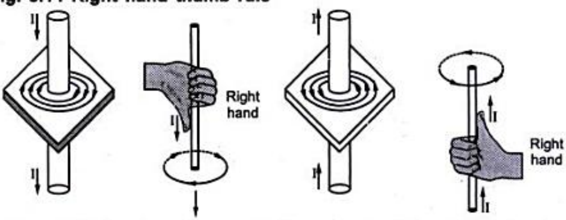


Fig. 3.15 Direction of magnetic lines by Right hand thumb rule

Conventionally it is shown as in the Fig. 3.16.

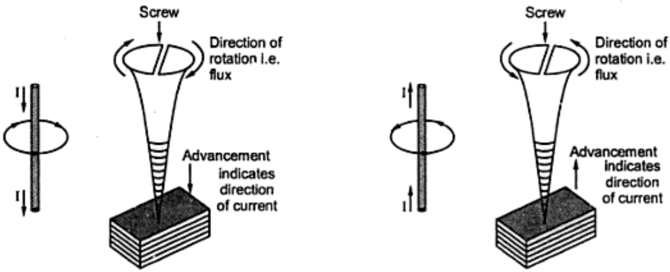
(a) Clockwise (b) Anticlockwise

Fig. 3.16 Representation of direction of flux

2) Corkscrew Rule : Imagine a right handed screw to be along the conductor carrying current with its axis parallel to the conductor and tip pointing in the direction of the current flow.

Then the direction of the magnetic field is given by the direction in which the screw must be turned so as to advance in the direction of the current.

This is shown in the Fig. 3.17.



(a) Clockwise rotation (b) Anticlockwise rotation

Fig. 3.17 Corkscrew rule

3.10.2 Magnetic Field due to Circular Conductor i.e. Solenoid

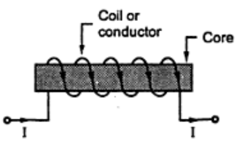


Fig. 3.18(a) Solenoid

A **solenoid** is an arrangement in which long conductor is wound with number of turns close together to form a coil. The axial length of conductor is much more than the diameter of turns. The part or element around which the conductor is wound is called as **core** of the solenoid. Core may be air or may be some magnetic material. Solenoid with a steel or iron core in shown in Fig. 3.18(a).

When such conductor is excited by the supply so that it carries a current then it produces a magnetic field which acts through the coil along its axis and also around the solenoid. Instead of using a straight core to wound the

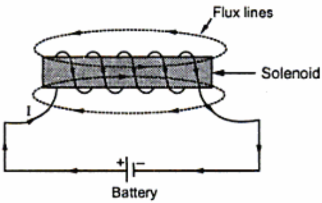


Fig. 3.18(b) Flux around a solenoid

conductor, a circular core also can be used to wound the conductor. In such case the resulting solenoid is called **Toroid.** Use of magnetic material for the core produces strong magnet. This is because current carrying conductor produces its own flux. In addition to this, the core behaves like a magnet due to magnetic induction, producing its own flux. The direction of two fluxes is same due to which resultant magnetic field becomes more strong.

The pattern of the flux around the solenoid is shown in the Fig. 3.18(b).

The rules to determine the direction of flux and poles of the magnet formed:

1) The right hand thumb rule :

Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid point to the **North pole** of the solenoid or point the direction of flux lines **inside the core.**

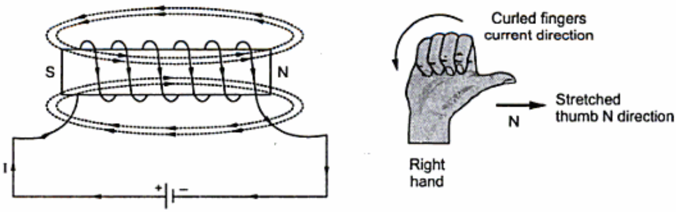


Fig. 3.19 (a) Direction of flux around a solenoid

This is shown in Fig. 3.19 (a) and (b).

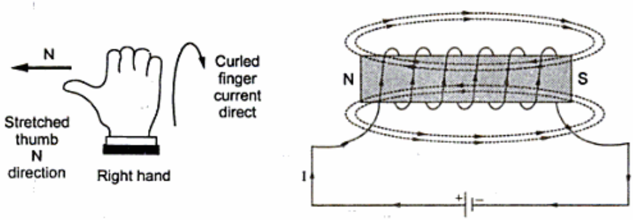


Fig. 3.19 (b) Direction of flux around a solenoid

In case of toroid, the core is circular and hence using right hand thumb rule, the direction of flux in the core, due to current carrying conductor can be determined. This is shown in the Fig. 3.20(a) and (b). In the Fig. 3.20 (a), corresponding to direction of winding, the flux set in the core is anticlockwise while in the Fig. 3.20 (b) due to direction of winding, the direction of flux set in the core is clockwise. The winding is also called **magnetising winding** or **magnetising coil** as it magnetises the core.

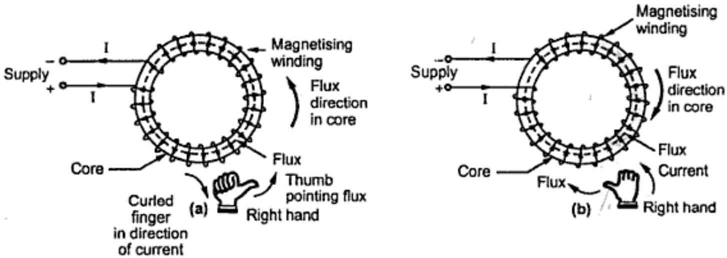


Fig. 3.20

2) Corkscrew rule : If axis of the screw is placed along the axis of the solenoid and if screw is turned in the direction of the current, then it travels towards the **N-pole** or in the direction of the magnetic field **inside** the solenoid.

3) End rule : If solenoid is observed from any one end then its polarity can be decided by noting direction of the current.

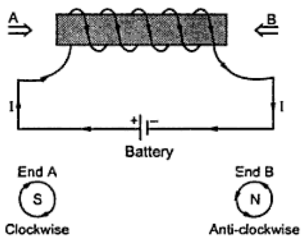


Fig. 3.21 End rule

Consider solenoid shown in the Fig. 3.21.

If it is seen from the end A, current will appear to flow in clockwise direction, so that end behaves as S-pole of the magnet. While as seen from the end B, current appears to flow in anticlockwise direction then that end which is B, behaves as N-pole of the magnet.

Generally right hand thumb rule is used to determine direction of flux and nature of the poles formed. Using such concept of an electromagnet, various magnetic circuits can be obtained.

3.11 Nature of Magnetic Field of Long Straight Conductor

We have seen that any current carrying conductor produces magnetic field around it and behaves like a permanent magnet with its field around.

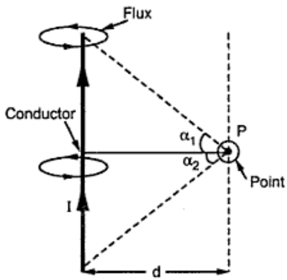


Fig. 3.22 (a) Field strength due to

Consider a conductor carrying current I amperes of length 'l' meters. Consider point P in the vicinity of such conductor. There will be influence of magnetic field on point P which can be quantified by magnetic field strength H at point P. This is definitely proportional to current I and inversely proportional to distance of point P from the conductor.

The magnitude of such magnetic field strength 'H' can be calculated by using the expression

$$\boxed{H = \frac{I}{4\pi d}(\sin \alpha_1 + \sin \alpha_2)}$$

The proof of this is out of the scope of this book.

For infinitely long conductor i.e. length 'l' is very very large then α_1 and α_2 tend to 90°.

$$H = \frac{I}{4\pi d}[\sin 90 + \sin 90] = \frac{2I}{4\pi d}$$

$$\boxed{H = \frac{I}{2\pi d} \quad A/m}$$

This unit of magnetic field strength A/m is mentioned earlier when field strength is defined.

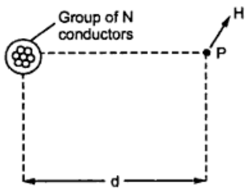


Fig. 3.22(b)

If such 'N' conductors are grouped together to form a coil or a cable then field strength due to current I passing through each conductor of the group can also be calculated by using same expression. The only change will be the field strength calculated above will get multiplied by 'N'.

So magnetic field strength at a point 'd' metres away from centre of such group of 'N' conductors each carrying current I amperes is

$$\boxed{H = \frac{NI}{2\pi d} \quad AT/m}$$

where AT/m means ampere turns per metre.

3.11.1 Magnetic Field Strength due to a Long Solenoid

Similar to the case of long straight conductor, we can decide field strength along the axis of a long solenoid. Such field strength depends on the number of turns of the conductor around the core and magnitude of current I passing through the conductor. If 'l' is the length of the solenoid in metres then H can be determined by the expression :

$$\boxed{H = \frac{NI}{l} \text{ AT/m}}$$

Key Point: *The expression is applicable for solenoids which are very very long but in practice the expression is used for all types of solenoids.*

Example 3.2 : *A current of 2 amp is flowing through each of the conductors in a coil containing 15 such conductors. If a point pole of unit strength is placed at a perpendicular distance of 10 cm from the coil, determine the field intensity at that point.*

Solution : I = 2 A, N = 15, d = 10 cm = 0.1 m.

$$H = \frac{NI}{2\pi d} = \frac{15\times 2}{2\times\pi\times 0.1} = \mathbf{47.74\ AT/m}$$

Example 3.3 : *A solenoid of 100 cm is wound on a brass tube. If the current through the coil is 0.5 A, calculate the number of turns necessary over the solenoid to produce a field strength of 500 AT/m at the center of the coil.*

Solution : The field strength on the axis of a long solenoid is given by

$$H = \frac{NI}{l} \text{ AT/m}$$

$$l = \text{Length of coil} = 100 \text{ cm} = 1 \text{ m},\ N = \text{Number of turns}$$

$$I = \text{Current} = 0.5 \text{ A}$$

$$\therefore \quad 500 = \frac{N\times 0.5}{1}$$

$$N = \mathbf{1000}$$

So 1000 turns on solenoid are necessary to produce the required field strength.

3.12 Force on a Current Carrying Conductor in a Magnetic Field

We have already mentioned that magnetic effects of electric current are very useful in analysing various practical applications like generators, motors etc. One of such important effects is force experienced by a current carrying conductor in a magnetic field.

Let a straight conductor, carrying a current is placed in a magnetic field as shown in the Fig. 3.23 (a).

The magnetic field in which it is placed has a flux pattern as shown in the Fig. 3.23 (a).

(a) Flux due to magnet **(b) Flux due to current carrying conductor**

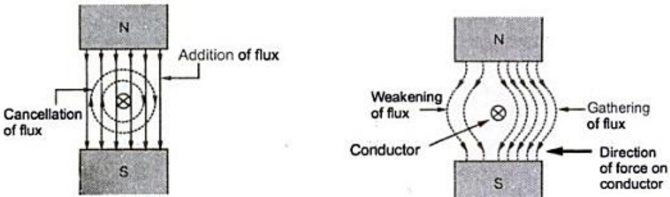
Fig. 3.23 Current carrying conductor in a magnetic field

Now current carrying conductor also produces its own magnetic field around it. Assuming current direction away from observer i.e. into the paper, the direction of its flux can be determined by right hand thumb rule. This is clockwise as shown in the Fig. 3.23 (b). [For simplicity, flux only due to current carrying conductor is shown in the Fig. 3.23 (b).]

Now there is presence of two magnetic fields namely due to permanent magnet and due to current carrying conductor. These two fluxes interact with each other. Such interaction is shown in the Fig. 3.24 (a).

This interaction as seen is in such a way that on one side of the conductor the two lines help each other, while on other side the two try to cancel each other. This means on left hand side of the conductor shown in the Fig. 3.24 the two fluxes are in the same direction and hence assisting each other . As against this, on the right hand side of the conductor the two fluxes are in opposite direction hence trying to cancel each other. Due to such interaction on one side of the conductor, there is **accumulation** of flux lines (gathering of the flux lines) while on the other side there is **weakening** of the flux lines.

The resultant flux pattern around the conductor is shown in the Fig. 3.24 (b).



(a) Presence of the two fluxes **(b) Resultant flux pattern**

Fig. 3.24 Interaction of the two flux lines

According to properties of the flux lines, these flux lines will try to shorten themselves. While doing so, flux lines which are gathered will exert force on the conductor. So conductor experiences a mechanical force from high flux lines area towards low flux lines area i.e. from left to right for a conductor shown in the Fig. 3.24.

Key Point: *Thus we can conclude that current carrying conductor placed in the magnetic field, experiences a mechanical force, due to interaction of two fluxes.*

This is the basic principle on which D.C. electric motors work and hence also called **motoring action.**

3.12.1 Fleming's Left Hand Rule

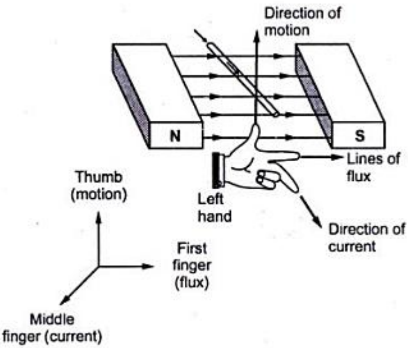


Fig. 3.25 Fleming's left hand rule

The direction of the force experienced by the current carrying conductor placed in magnetic field can be determined by a rule called **'Fleming's Left Hand Rule'**. The rule states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor'.

The rule is explained in the diagrammatic form in the Fig. 3.25.

Apply the rule to crosscheck the direction of force experienced by a single conductor, placed in the magnetic field, shown in the Fig. 3.26 (a), (b), (c) and (d).

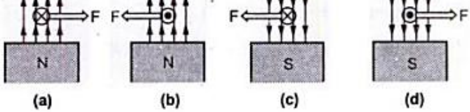


Fig. 3.26 Direction of force experienced by conductor

3.12.2 Magnitude of Force Experienced by the Conductor

The magnitude of the force experienced by the conductor depends on the following factors,

1) Flux density (B) of the magnetic field in which the conductor is placed measured in Wb/m^2 i.e. Tesla.
2) Magnitude of the current I passing through the conductor in Amperes.
3) Active length 'l' of the conductor in metres.

The **active length** of the conductor is that part of the conductor which is actually under the influence of magnetic field.

If the conductor is at right angles to the magnetic field as shown in Fig. 3.27 (a) then force F is given by,

$$F = B I l \text{ Newtons}$$

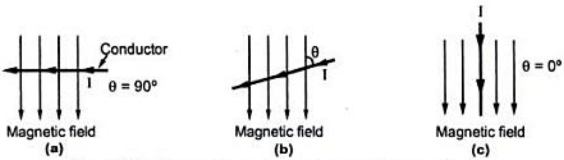


Fig. 3.27 Force on a current carrying conductor

But if the conductor is not exactly at right angles, but inclined at angle θ degrees with respect to axis of magnetic field as shown in the Fig. 3.27 (b) then force F is given by,

$$F = B I l \sin\theta \text{ Newtons}$$

As shown in the Fig. 3.27 (c), if conductor is kept along the lines of magnetic field then $\theta = 0^\circ$ and as $\sin 0^\circ = 0$, the force experienced by the conductor is also zero.

Key Point : *The direction of such force can be reversed either by changing the direction of current or by changing the direction of the flux lines in which it is kept. If both are reversed, the direction of force remains same.*

Example 3.4 : *Calculate the force experienced by the conductor of 20 cm long, carrying 50 amperes, placed at right angles to the lines of force of flux density 10×10^{-3} Wb/m².*

Solution : The force experienced is given by,

$$F = B I l \sin\theta \quad \text{where} \quad \sin(\theta) = 1 \text{ as } \theta = 90 \text{ degrees}$$

$$B = \text{Flux density} = 10\times10^{-3} \ Wb/m^2$$

$$l = \text{Active length} = 20 \text{ cm} = 0.2 \text{ m}$$

$$I = \text{current} = 50 \text{ A}$$

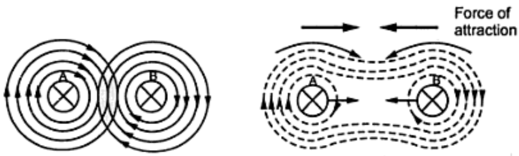
$$F = 10\times10^{-3}\times50\times0.2 = \mathbf{0.1\ N}$$

3.13 Force between Two Parallel Current Carrying Conductors

The force between two parallel current carrying conductors depends on the directions of these two currents. We have seen that whenever there is interaction of two fluxes, the force gets generated. In this case each current carrying conductor produces its own flux around it. So when such two conductors are placed nearby, due to interaction of two fluxes there exists a force between them.

3.13.1 Direction of Both the Currents Same

Consider two parallel conductors A and B which are carrying current in the same direction as shown in the Fig. 3.28 (a).



(a) Current carrying conductors (b) Flux pattern

Fig. 3.28 Force of attraction

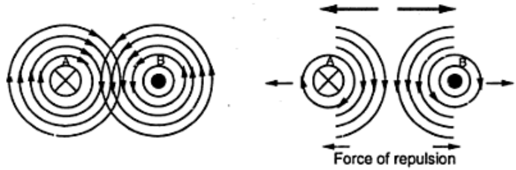
Then the direction of the two radial fields around them can be decided by right hand thumb rule. When such two conductors are placed parallel to each other and nearby the two fields interact. In space between the conductors, the two fluxes are in opposite direction and cancel each other's effect and get neutralised while in space outside the conductors two fields help or assist each other, producing high flux area around the conductors. The resultant flux pattern is shown in the Fig. 3.28 (b).

As flux lines outside try to shorten as per their property, they exert force on the conductors. Hence conductors experience a force of attraction in between them.

3.13.2 Directions of Two Currents Opposite to Each Other

Consider the conductors A and B which are carrying currents in the directions opposite to each other. Then the directions of the two radial fields can be shown in the Fig. 3.29 (a)

When such two conductors are placed nearby, parallel to each other then these two fields interact with each other. Now in space between the conductors two fluxes assist each other producing high flux zone. While surrounding the conductors, two fluxes oppose each other and cancel each other. The resultant flux pattern is shown in the Fig. 3.29 (b).



(a) Current carrying conductors (b) Flux pattern

Fig. 3.29 Force of repulsion

As flux lines within the space try to shorten as per their property and due to this they exert force on the conductors in directions opposite to each other. Due to this conductors experience a force of repulsion in between them.

3.13.3 Magnitude of Force between Two Parallel Conductors

We have seen that two current carrying conductors when placed nearby, parallel to each other experience a force. The direction of such a force depends on the directions of the flow of currents. Let us derive expression for its magnitude which requires the understanding of the permeability.

Let the two parallel conductors be 'A' and 'B' carrying currents I_1 and I_2 amperes respectively, placed in vacuum.

Now the magnetic field strength at a point 'd' metres from the axis of the current carrying conductor is given by

$$H = \frac{I}{2\pi d} \quad \text{AT/m or A/m}$$

where I is current through the conductor.

Now let distance between the centres of the conductors be 'r' metres. So magnetic field strength due to conductor A at a centre of B which is 'r' metres away is

$$H = \frac{I_1}{2\pi r}$$

Now let conductors are placed in vacuum then

$$B = \mu_0 H$$

$\therefore$ Flux density due to conductor A at center of B is

$$B = \frac{\mu_0 I_1}{2\pi r} \text{ Wb/m}^2$$

Now force experienced by conductor B is

$$F = B I l$$

when I is current through conductor B = I_2

$$\therefore \quad \boxed{F = \frac{\mu_0 I_1}{2\pi r} \cdot I_2 \cdot l \text{ Newtons.}}$$

Now $\quad \mu_0 = 4\pi\times10^{-7}$ for vacuum

$$\therefore \quad F = \frac{4\pi\times10^{-7} I_1 I_2 \times l}{2\pi r}$$

$$\therefore \quad \boxed{F = 2\times10^{-7} I_1 I_2 \frac{l}{r} \text{ Newtons}}$$

Similarly due to magnetic field of B conductor A also experiences a force of same magnitude but in opposite direction.

Example 3.5 : *Two long parallel conductors carry currents of 70 A and 120 A in opposite directions. The perpendicular distance between the conductors is 15 cm. Calculate the force per metre which one conductor will exert on the other. Also calculate the field strength at a point which is 60 mm from conductor A and 90 mm from conductor B.*

Solution : I_1 = 70 A, I_2 = 120 A, R = 15 cm = 0.15 m.

Force per metre is required i.e. l = 1m

$$\therefore \quad F = 2\times10^{-7} I_1 I_2 \frac{l}{r} = 2\times10^{-7}\times70\times120\times\frac{1}{0.15}$$

$$= \mathbf{0.0112 \ N/m}$$

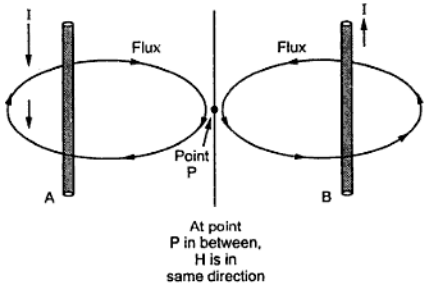


Fig. 3.30

Now magnetic field strength at a point 60 mm away from A is given by

$$H_A = \frac{I}{2\pi d} = \frac{70}{2\pi\times(60\times10^{-3})} = 185.68 \text{ A/m}$$

Magnetic field strength at a point 90 mm away from conductor B is given by

$$H_B = \frac{I}{2\pi d} = \frac{120}{2\pi\times(90\times10^{-3})} = 212.20 \text{ A/m}$$

> **Key Point:** *Since the currents in the two conductors are in opposite direction, at a point in between the two, magnetic field strengths will be in same direction, as shown.*

∴ Resultant magnetic field strength at a point 60 mm from A and 90 mm from B is

$$H = H_A + H_B = 185.68 + 212.20 = \mathbf{397.88 \text{ A/m}}$$

3.13.4 Unit of Ampere

If the current through each conductor is 1 A while the distance between their centers is 1 m and length l of each conductor which is under the influence of flux density B is 1 m then,

$$I_1 = I_2 = 1 \text{ A} \quad \text{and } l = r = 1 \text{ m}$$

$$\therefore \quad F = 2\times10^{-7}\times1\times1\times\frac{1}{1} = \mathbf{2\times10^{-7} \text{ N.}}$$

In such a case, force experienced by each conductor is 2×10^{-7} N. From this, unit of ampere can be defined.

> **Key Point:** *A current, which when flowing through each of the two long parallel straight conductors having infinite length and negligible cross-section, separated from each other by a distance of one meter in vacuum, producing a force 2×10^{-7} N per meter length in between the conductors is called **one ampere current**.*

3.14 Permeability

The flow of flux produced by the magnet not only depends on the magnetic field strength but also on one important property of the magnetic material called permeability. It is related to the medium in which magnet is placed. The force exerted by one magnetic pole on other depends on the medium in which magnets are placed.

> **Key Point:** *The **permeability** is defined as the ability or ease with which the magnetic material forces the magnetic flux through a given medium.*

For any magnetic material, there are two permeabilities,

i) Absolute permeability ii) Relative permeability.

3.14.1 Absolute Permeability (μ)

The magnetic field strength (H) decides the flux density (B) to be produced by the magnet around it, in a given medium. The ratio of magnetic flux density *B* in a particular medium (other than vacuum or air) to the magnetic field strength H producing that flux density is called **absolute permeability** of that medium.

It is denoted by μ and mathematically can be expressed as,

$$\mu = \frac{B}{H}$$

i.e. $$B = \mu H$$

The permeability is measured in units henries per metre denoted as H/m.

3.14.2 Permeability of Free Space or Vacuum (μ_0)

If the magnet is placed in a free space or vacuum or in air then the ratio of flux density B and magnetic field strength H is called **Permeability of free space** or **Vacuum** or **air**.

It is denoted as μ_0 and measured in H/m. It denotes the ease with which the magnetic flux permeates the free space or vacuum or air.

It is experimentally found that this μ_0 i.e. ratio of B and H in vacuum remains constant every where in the vacuum and its value is $4\pi\times10^{-7}$ H/m.

$$\therefore \qquad \mu_0 = \frac{B}{H} \text{ in vacuum } = 4\pi\times10^{-7} \text{ H/m}$$

Key Point *: For a magnetic material, the absolute permeability μ is not constant. This is because B and H bears a nonlinear relation in case of magnetic materials. If magnetic field strength is increased, there is change in flux density B but not exactly proportional to the increase in H.*

The ratio B to H is constant only for free space, vacuum or air which is $\mu_0 = 4\pi\times10^{-7}$ H/m.

3.14.3 Relative Permeability (μ_r)

Generally the permeability of different magnetic materials is defined relative to the permeability of free space (μ_0). The **relative permeability** is defined as the ratio of flux density produced in a medium (other than free space) to the flux density produced in free space, under the influence of same magnetic field strength and under identical conditions.

Thus if the magnetic field strength is H which is producing flux density B in the medium while flux density B_0 in free space then the relative permeability is defined as,

$$\mu_r = \frac{B}{B_0} \quad \text{where H is same.}$$

It is dimensionless and has no units.

For free space, vacuum or air, $\mu_r = 1$

According to definition of absolute permeability we can write for given H,

$$\mu = \frac{B}{H} \quad \text{in medium} \qquad ...(1)$$

$$\mu_0 = \frac{B_0}{H} \quad \text{in free space} \qquad ...(2)$$

Dividing (1) and (2) , $$\frac{\mu}{\mu_0} = \frac{B}{B_0}$$

but $$\frac{B}{B_0} = \mu_r$$

$$\therefore \quad \frac{\mu}{\mu_0} = \mu_r$$

$$\therefore \quad \boxed{\mu = \mu_0 \mu_r \quad H / m}$$

The relative permeability of metals like iron, steel varies from 100 to 100,000

> **Key Point** : *If we require maximum flux production for the lesser magnetic field strength then the value of the relative permeability of the core material should be as high as possible.*

For example if relative permeability of the iron is 1000 means it is 1000 times more magnetic than the free space or air.

3.15 Magnetomotive Force (M.M.F.or F)

The flow of electrons is current which is basically due to electromotive force (e.m.f.). Similarly the force behind the flow of flux or production of flux in a magnetic circuit is called magnetomotive force (m.m.f.) The m.m.f. determines the magnetic field strength.

It is the driving force behind the magnetic circuit. It is given by the product of the number of turns of the magnetizing coil and the current passing through it.

Mathematically it can be expressed as,

$$\boxed{\text{m.m.f.} = N\,I \quad \text{ampere turns}}$$

where N = Number of turns of magnetising coil and I = Current through coil

Its unit is ampere turns (AT) or amperes (A).

It is also defined as the work done in joules on a unit magnetic pole in taking it once round a closed magnetic circuit.

3.16 Reluctance (S)

In an electric circuit, current flow is opposed by the resistance of the material, similarly there is opposition by the material to the flow of flux which is called **reluctance**

It is defined as the resistance offered by the material to the flow of magnetic flux through it. It is denoted by 'S'. It is directly proportional to the length of the magnetic circuit while inversely proportional to the area of cross-section.

$$S \propto \frac{l}{a} \quad \text{where } 'l' \text{ in 'm' while 'a' in } 'm^2'$$

$$\therefore \quad S = \frac{Kl}{a}$$

where K = Constant of proportionality

= Reciprocal of absolute permeability of material $= \frac{1}{\mu}$

$$\therefore \quad \boxed{S = \frac{l}{\mu a} = \frac{l}{\mu_0 \mu_r a} \ \text{A / Wb}}$$

It is measured in amperes per weber (A/Wb).

The reluctance can be also expressed as the ratio of magnetomotive force to the flux produced.

$$\text{i.e.} \quad \text{Reluctance} = \frac{\text{m.m.f}}{\text{flux}}$$

$$\therefore \quad \boxed{S = \frac{NI}{\phi} \quad \text{AT / Wb or A / Wb}}$$

3.17 Permeance

The **permeance** of the magnetic circuit is defined as the reciprocal of the reluctance.

It is defined as the property of the magnetic circuit due to which it allows flow of the magnetic flux through it.

$$\therefore \quad \boxed{\text{Permeance} = \frac{1}{\text{Reluctance}}}$$

It is measured in weber per amperes (Wb/A).

3.18 Magnetic Circuits

The **magnetic circuit** can be defined as, the closed path traced by the magnetic lines of force i.e. flux. Such a magnetic circuit is associated with different magnetic quantities as m.m.f., flux reluctance, permeability etc.

Consider simple magnetic circuit shown in the Fig. 3.31 (a). This circuit consists of an iron core with cross-sectional area of 'a' m^2 with a mean length of 'l' m. (This is mean length of the magnetic path which flux is going to trace.) A coil of N turns is wound on one of the sides of the square core which is excited by a supply. This supply drives a current I through the coil. This current carrying coil produces the flux (ϕ) which completes its path through the core as shown in the Fig. 3.31 (a).

This is analogous to simple electric circuit in which a supply i.e. e.m.f. of E volts drives a current I which completes its path through a closed conductor having resistance R. This analogous electrical circuit is shown in the Fig. 3.31 (b).

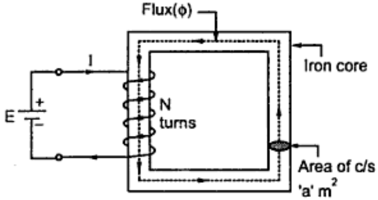


Fig. 3.31 (a) Magnetic circuit

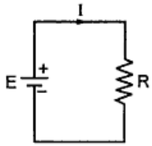


Fig. 3.31 (b) Electrical equivalent

Let us derive relationship between m.m.f., flux and reluctance.

I = Current flowing through the coil.

N = Number of turns.

ϕ = Flux in webers.

B = Flux density in the core.

μ = Absolute permeability of the magnetic material

μ_r = Relative permeability of the magnetic material

Magnetic field strength inside the solenoid is given by,

$$H = \frac{NI}{l} \quad \text{AT/m} \quad \ldots(1)$$

Now flux density is,

$$B = \mu H$$

$$B = \frac{\mu_0 \mu_r N I}{l} \quad \text{Wb/m}^2 \quad \ldots(2)$$

Now as area of cross-section is 'a' m^2, total flux in core is,

$$\phi = B a = \frac{\mu_0 \mu_r N I a}{l} \quad \text{Wb} \quad \ldots(3)$$

i.e.

$$\phi = \frac{NI}{\frac{l}{\mu_0 \mu_r a}}$$

∴

$$\boxed{\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{F}{S}}$$

where NI = Magnetomotive force m.m.f. in AT

$$S = \frac{l}{\mu_0 \mu_r a}$$

= Reluctance offered by the magnetic path.

This expression of the flux is very much similar to expression for current in electric circuit.

$$I = \frac{\text{e.m.f.}}{\text{resistance}}$$

Key Point : *So current is analogous to the flux, e.m.f. is analogous to the m.m.f. and resistance is analogous to the reluctance.*

Example 3.6 : *An iron ring of circular cross sectional area of 3.0 cm² and mean diameter of 20 cm is wound with 500 turns of wire and carries a current of 2.09 A to produce the magnetic flux of 0.5 m Wb in the ring. Determine the permeability of the material.*

(May - 2000)

Solution : The given values are :

$$a = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2, \; d = 20 \text{ cm}, \; N = 500, \; I = 2 \text{ A}, \; \phi = 0.5 \text{ m Wb}$$

Now, $l = \pi \times d = \pi \times 20 = 62.8318 \text{ cm} = 0.628318 \text{ m}$

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{0.628313}{4\pi \times 10^{-7} \times \mu_r \times 3 \times 10^{-4}} = \frac{1.6667 \times 10^9}{\mu_r} \quad \ldots (1)$$

$$f = \frac{\text{m.m.f.}}{S} = \frac{NI}{S}$$

$$\therefore \quad S = \frac{NI}{\phi} = \frac{500 \times 2}{0.5 \times 10^{-3}} = 2 \times 10^6 \text{ AT / Wb} \quad \ldots (2)$$

Equating (1) and (2),

$$\therefore \quad 2 \times 10^6 = \frac{1.6667 \times 10^9}{\mu_r}$$

$$\therefore \quad \mu_r = \mathbf{833.334}$$

3.18.1 Series Magnetic Circuits

In practice magnetic circuit may be composed of various materials of different permeabilities, of different lengths and of different cross-sectional areas. Such a circuit is called **composite** magnetic circuit.When such parts are connected one after the other the circuit is called **series magnetic circuit**.

Consider a circular ring made up of different materials of lengths l_1, l_2 and l_3 and with cross-sectional areas a_1, a_2 and a_3 with absolute permeabilities μ_1, μ_2 and μ_3 as shown in the Fig. 3.32.

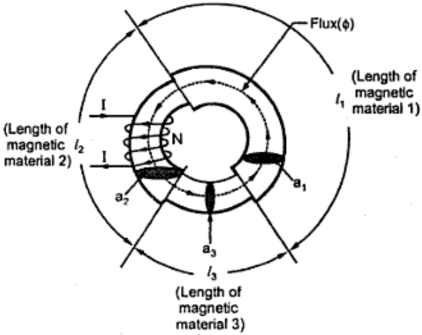


Fig. 3.32 A series magnetic circuit

Let coil wound on ring has N turns carrying a current of I amperes.

The total m.m.f. available is = NI AT

This will set the flux 'ϕ' which is same through all the three elements of the circuit.

This is similar to three resistances connected in series in electrical circuit and connected to e.m.f. carrying same current 'I' through all of them.

Its analogous electric circuit can be shown as in the Fig. 3.33.

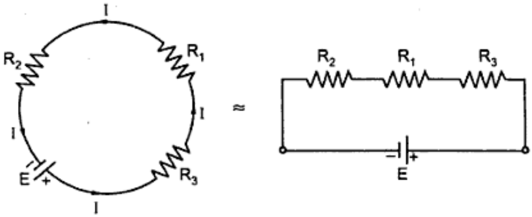


Fig. 3.33 Equivalent electrical circuit

The total resistance of the electric circuit is $R_1 + R_2 + R_3$. Similarly the total reluctance of the magnetic circuit is,

$$\text{Total } S_T = S_1 + S_2 + S_3 = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}$$

$$\therefore \quad \text{Total } \phi = \frac{\text{Total m.m.f.}}{\text{Total reluctance}} = \frac{NI}{S_T} = \frac{NI}{(S_1 + S_2 + S_3)}$$

$$\therefore \quad NI = S_T \phi = (S_1 + S_2 + S_3)\,\phi$$

$$NI = S_1 \phi + S_2 \phi + S_3 \phi$$

$$\therefore \quad (\text{m.m.f.})T = (\text{m.m.f.})_1 + (\text{m.m.f.})_2 + (\text{m.m.f.})_3$$

The total m.m.f. also can be expressed as,

$$(\text{m.m.f.})T = H_1 l_1 + H_2 l_2 + H_3 l_3$$

where $H_1 = \frac{B_1}{\mu_1}, \quad H_2 = \frac{B_2}{\mu_2}, \quad H_3 = \frac{B_3}{\mu_3}$

So for a **series magnetic** circuit we can remember,

1) The magnetic flux through all the parts is same.
2) The equivalent reluctance is sum of the reluctance of different parts.
3) The resultant m.m.f. necessary is sum of the m.m.f.s in each individual part.

3.18.2 Series Circuit with Air Gap

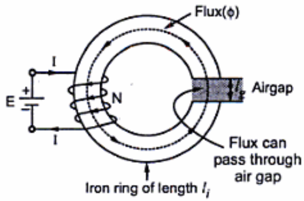


Fig. 3.34 A ring with an air gap

The series magnetic circuit can also have a short air gap.

Key Point: *This is possible because we have seen earlier that flux can pass through air also.*

Such air gap is not possible in case of electric circuit.

Consider a ring having mean length of iron part as 'l_i' as shown in the Fig. 3.34.

Total m.m.f = N I AT

Total reluctance $\boxed{S_T = S_i + S_g}$

where S_i = Reluctance of iron path

S_g = Reluctance of air gap

$$\therefore \quad S_i = \frac{l_i}{\mu a_i}$$

$$S_g = \frac{l_g}{\mu_0 a_i}$$

Key Point: *The absolute permeability of air* $\mu = \mu_0$

The cross-sectional area of air gap is assumed to be equal to area of the iron ring.

$$\therefore \quad S_T = \frac{l_i}{\mu a_i} + \frac{l_g}{\mu_0 a_i}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S_T}$$

or Total m.m.f. = m.m.f. for iron + m.m.f for air gap

$\therefore$

$$\boxed{NI = S_i\phi + S_g\phi \quad \text{AT for ring.}}$$

Example 3.7 : *An iron ring 8 cm. mean diameter is made up of round iron of diameter 1 cm and permeability of 900, has an air gap of 2mm wide. It consists of winding with 400 turns carrying a current of 3.5A. Determine,*

i) m.m.f. ii) total reluctance iii) the flux iv) flux density in ring **(May - 98, Dec - 99)**

Solution : The ring and the winding is shown in the Fig. 3.35.

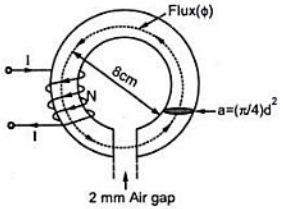


Fig. 3.35

Diameter of ring d = 8 cm,

$\therefore$ length of iron = πd − length of air gap

$$l_i = \pi\times\left(8\times10^{-2}\right)-2\times10^{-3}$$

$$= 0.2493 \text{ m.}$$

> **Key Point** : *While calculating iron length, do not forget to subtract length of air gap from total mean length.*

$$l_g = \text{Length of air gap}$$

$$= 2 \text{ mm} = 2\times10^{-3} \text{ m}$$

diameter of iron = 1 cm

$\therefore$ area of cross section $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(1\times10^{-2}\right)^2$

$$a = 7.853\times10^{-5} \quad m^2$$

Area of cross section of air gap and ring is to be assumed same.

i) Total m.m.f. produced = N I = 400 × 3.5

= **1400 AT** (ampere turns)

ii) Total reluctance $S_T = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} \qquad \text{...Given } \mu_r = 900$$

$$= \frac{0.2493}{4\pi\times10^{-7}\times900\times7.853\times10^{-5}}$$

$$= 2806947.615 \text{ AT/ Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} \qquad \text{as } \mu_r = 1 \text{ for air}$$

$$\therefore \quad S_g = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times7.853\times10^{-5}} = 20.2667\times10^{6} \quad AT / Wb$$

$$\therefore \quad S_T = 2806947.615+20.2667\times10^{6} = \mathbf{23.0737\times10^{6} \quad AT / Wb}$$

ii) $$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{N\,I}{S_T} = \frac{1400}{23.0737\times10^{6}}$$

$$= \mathbf{6.067\times10^{-5} \quad Wb}$$

iv) $$\text{Flux density} = \frac{\phi}{a} = \frac{6.067\times10^{-5}}{7.853\times10^{-5}} = \mathbf{0.7725 \quad Wb / m^2}$$

3.18.3 Parallel Magnetic Circuits

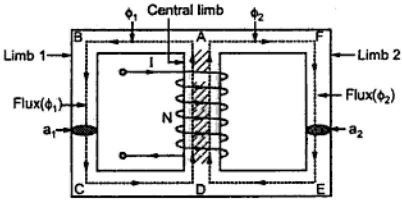
In case of electric circuits, resistances can be connected in parallel. Current through each of such resistances is different while voltage across all of them is same. Similarly different reluctances may be in parallel in case of magnetic circuits. A magnetic circuit which has more than one path for the flux is known as a **parallel magnetic circuit.**

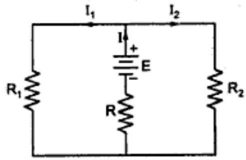
Consider a magnetic circuit shown in the Fig. 3.36 (a). At point A the total flux ϕ, divides into two parts ϕ_1 and ϕ_2.

$$\therefore \quad \boxed{\phi = \phi_1 + \phi_2}$$

The fluxes ϕ_1 and ϕ_2 have their paths completed through ABCD and AFED respectively.

This is similar to division of current in case of parallel connection of two resistances in an electric circuit. The analogous electric circuit is shown in the Fig. 3.36 (b).





(a) Magnetic circuit (b) Equivalent electrical circuit

Fig. 3.36 A parallel magnetic circuit

The mean length of path ABCD $= l_1$ m

The mean length of the path AFED $= l_2$ m

The mean length of the path AD $= l_c$ m

The reluctance of the path ABCD $= S_1$

The reluctance of path AFED $= S_2$

The reluctance of path AD $= S_c$

The total m.m.f. produced $=$ N I AT

$$\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$$

$$\therefore \quad \text{m.m.f.} = \phi \times S$$

$$\therefore \quad \text{For path ABCDA, } NI = \phi_1 S_1 + \phi S_c$$

For path AFEDA,

$$NI = \phi_2 S_2 + \phi S_c$$

where

$$S_1 = \frac{l_1}{\mu a_1}, \quad S_2 = \frac{l_2}{\mu a_2} \text{ and } S_c = \frac{l_c}{\mu a_c}$$

Generally

$$a_1 = a_2 = a_c = \text{Area of cross-section}$$

For parallel circuit,

$$\boxed{\text{Total m.m.f.} = \begin{matrix}\text{m.m.f. required} \\ \text{by central limb}\end{matrix} + \begin{matrix}\text{m.m.f. required by} \\ \text{any one of outer limbs}\end{matrix}}$$

$$NI = (NI)_{AD} + (NI)_{ABCD} \text{ or } (NI)_{AFED}$$

$$NI = \phi S_c + [\phi_1 S_1 \text{ or } \phi_2 S_2]$$

As in the electric circuit e.m.f. across parallel branches is same, in the magnetic circuit the m.m.f. across parallel branches is same.

Thus same m.m.f. produces different fluxes in the two parallel branches. For such parallel branches,

$$\boxed{\phi_1 S_1 = \phi_2 S_2}$$

Hence while calculating total m.m.f., the m.m.f. of **only one** of the two parallel branches must be considered.

3.18.4 Parallel Magnetic Circuit with Air Gap

Consider a parallel magnetic circuit with air gap in the central limb as shown in the Fig. 3.37.

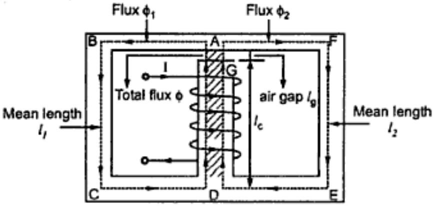


Fig. 3.37 Parallel circuit with air gap

The analysis of this circuit is exactly similar to the parallel circuit discussed above. The only change is the analysis of central limb. The central limb is series combination of iron path and air gap. The central limb is made up of,

$$\text{path GD} = \text{iron path} = l_c$$

$$\text{path GA} = \text{air gap} = l_g$$

The total flux produced is ϕ. It gets divided at A into ϕ_1 and ϕ_2.

$$\therefore \quad \phi = \phi_1 + \phi_2$$

The reluctance of central limb is now,

$$S_c = S_i + S_g = \frac{l_c}{\mu a_c} + \frac{l_g}{\mu_0 a_c}$$

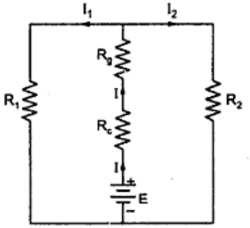


Fig. 3.38 Electrical equivalent circuit

Hence m.m.f. of central limb is now,

$$\boxed{(\text{m.m.f.})_{AD} = (\text{m.m.f.})_{GD} + (\text{m.m.f.})_{GA}}$$

Hence the total m.m.f. can be expressed as,

$$\boxed{(NI)_{total} = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD} \text{ or } (NI)_{AFED}}$$

Thus the electrical equivalent circuit for such case becomes as shown in the Fig. 3.38.

Similarly there may be air gaps in the side limbs but the method of analysis remains the same.

➡ **Example 3.8 :** *The magnetic circuit shown in Fig. 3.39 is constructed of wrought iron. The cross-section of the centre limb is 8 cm^2 and of each other limb, 5 cm^2. If the coil on centre limb is wound with 1000 turns, calculate the exciting current required to set up a flux of 1.2 mWb in the centre limb. Width of each air gap is 1 mm. Points on the B/H curve of wrought iron are as follows -* **(May - 2002)**

B (in Tesla)	1.2	1.35	1.5
H (in AT/m)	625	1100	2000

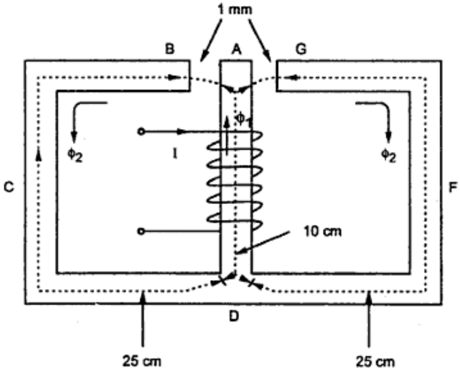


Fig. 3.39

Solution : Given; l_c = length of central limb = 10 cm = 0.1 m

a_c = 8 cm^2 = 8×10^{-4} m^2, ϕ_c = 1.2 m Wb = 1.2×10^{-3} Wb

l_i = Length of iron path of side limb = 25 cm

= 0.25 m (on each side)

l_g = Length of air gap = 1 mm = 1×10^{-3} m

a_i = 5 cm^2 = 5×10^{-4} m^2

This is the example of **parallel** magnetic circuit.

The flux in central limb 1.2 mWb gets divided into two equal paths as shown in Fig. 3.39.

$$\text{Flux in side limbs} = \frac{1.2}{2} \text{ i.e. } \therefore \phi_i = 0.6 \text{ mWb}$$

Flux density in central limb is,

$$B_c = \frac{\phi_c}{a_c} = \frac{1.2\times10^{-3}}{8\times10^{-4}} = 1.5 \text{ Tesla}$$

Flux density in air gap is,

$$B_g = \frac{\phi_i}{a_i} = \frac{0.6\times10^{-3}}{5\times10^{-4}} = 1.2 \text{ Wb/m}^2 \text{ i.e. Tesla}$$

Flux density in side limb is,

$$B_i = \frac{\phi_i}{a_i} = \frac{0.6\times10^{-3}}{5\times10^{-4}} = 1.2 \text{ Wb/m}^2 \text{ i.e. Tesla}$$

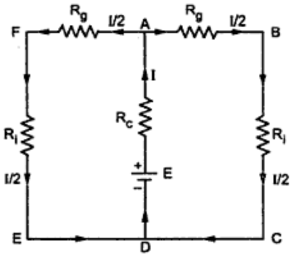


Fig. 3.40

The equivalent circuit in electrical form is shown in Fig. 3.40 (a).

Applying KVL to loop ABCD,

$$E - IR_c - \frac{I}{2} R_g - \frac{I}{2} R_i = 0$$

$$\therefore \quad E = IR_c + \frac{I}{2} R_g + \frac{I}{2} R_i$$

Similarly applying Kirchhoff's mmf law to the loop,

$$\text{mmf} = H_c\, l_c + H_g\, l_g + H_i\, l_i$$

where

$H_c\, l_c$ = m.m.f. required by central limb

$H_g\, l_g$ = m.m.f. required by air gap

$H_i\, l_i$ = m.m.f. required by iron path on any one side

I) Central limb

$$B_c = 1.5 \text{ Tesla}$$

From B-H table given, corresponding, $H_c = 2000$

$$H_c\, l_c = 2000\times0.1 = 200 \text{ AT}$$

II) Side limb

$$B_i = 1.2 \text{ Tesla}$$

From B-H table, given corresponding $H_i = 625$

$$H_i\, l_i = 625\times0.25 = 156.25 \text{ AT}$$

III) The air gap

Key Point: *For air gap, B–H table should not be referred but value of field strength H_g for air gap is to be calculated as,*

$$B_g = \mu_0 H_g$$

$$H_g = \frac{B_g}{\mu_0} = \frac{1.2}{4\pi\times10^{-7}}$$

$$H_g = 954929.65 \text{ AT}$$

$$\therefore \quad H_g l_g = 954929.65 \times (1\times 10^{-3})$$

$$\therefore \quad H_g l_g = 954.9296 \text{ AT}$$

$$\text{Total m.m.f. required} = H_c l_c + H_i l_i + H_g l_g$$

$$\therefore \quad NI = 200 + 156.25 + 954.9296$$

$$\therefore \quad NI = 1311.17 \text{ AT}$$

$$\therefore \quad \text{Current I} = \frac{1311.17}{\text{No. of turns}} = \frac{1311.17}{1000}$$

$$\therefore \quad I = \mathbf{1.31 \ A}$$

Example 3.9 : *A cast steel structure is made of a rod of square section 2.5 cm × 2.5 cm as shown in the Fig. 3.41. What is the current that should be passed in a 500 turn coil on the left limb so that a flux of 2.5 mWb is made to pass in the right limb. Assume permeability as 750 and neglect leakage.*

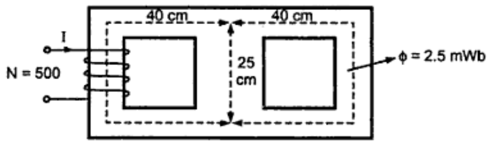


Fig. 3.41

Solution : This is parallel magnetic circuit. Its electrical equivalent is shown in the Fig. 3.41 (a).

The total flux produced gets distributed into two parts having reluctances S_1 and S_2.

S_1 = Reluctance of centre limb

S_2 = Reluctance of right side

$$S_1 = \frac{l_1}{\mu_0 \mu_r a_1} = \frac{25\times 10^{-2}}{4\pi\times 10^{-7}\times 750\times 2.5\times 2.5\times 10^{-4}}$$

$$= 424.413\times 10^3 \text{ AT/Wb}$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r a_1} = \frac{40\times 10^{-2}}{4\pi\times 10^{-7}\times 750\times 2.5\times 2.5\times 10^{-4}}$$

$$= 679.061\times 10^3 \text{ AT/Wb}$$

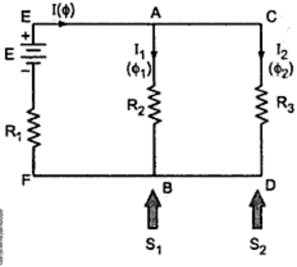


Fig. 3.41 (a)

Key Point : *For parallel branches, m.m.f. remains same.*

For branch AB and CD, m.m.f. is same.

$\therefore$ m.m.f. $= \phi_1 S_1 = \phi_2 S_2$

And $\phi_2 = 2.5$ mWb ... given

$$\therefore \quad \phi_1 = \frac{\phi_2 S_2}{S_1} = \frac{2.5\times 10^{-3}\times 679.061\times 10^3}{424.413\times 10^3} = 4 \text{ mWb}$$

$$\therefore \quad \phi = \phi_1 + \phi_2 = 2.5 + 4 = 6.5 \text{ mWb}$$

Total m.m.f. required is sum of the m.m.f. required for AEFB and that for either central or side limb.

$$S_{AEFB} = S_2 = 679.061\times 10^3 \text{ AT/Wb}$$

$$\therefore \text{ m.m.f. for AEFB} = S_{AEFB}\times\phi = 679.061\times 10^3\times 6.5\times 10^{-3}$$

$$= 4413.8965 \text{ AT}$$

$$\therefore \text{ Total m.m.f.} = 4413.8965 + \phi_1 S_1$$

$$= 4413.8965 + 4\times 10^{-3}\times 424.413\times 10^3 = 6111.548 \text{ AT}$$

But NI = total m.m.f.

$$\therefore \quad I = \frac{6111.548}{500} = 12.223 \text{ A}$$

3.19 Kirchhoff's Laws for Magnetic Circuit

Similar to the electrical circuit Kirchhoff's Laws can be used to analyse complex magnetic circuit. The laws can be stated as below :

3.19.1 Kirchhoff's Flux Law

The total magnetic flux arriving at any junction in a magnetic circuit is equal to the total magnetic flux leaving that junction.

At a junction, $\boxed{\sum \phi = 0}$

The law infact is used earlier to analyse parallel magnetic circuit at a junction A shown in the Fig.3.36 (a), where

$$\phi = \phi_1 + \phi_2$$

3.19.2 Kirchhoff's M.M.F. Law

The resultant m.m.f. around a closed magnetic circuit is equal to the algebraic sum of the products of the flux and the reluctance of each part of the closed circuit i.e. for a closed magnetic circuit.

$$\boxed{\sum \text{m.m.f.} = \sum \phi S}$$

As $\phi \times S = \text{flux} \times \text{reluctance} = \text{m.m.f.}$

M.M.F. also can be calculated as $H \times l$ where H is field strength and 'l' is mean length

$\therefore \quad \text{m.m.f.} = Hl$

Alternatively the same law can be stated as :

The resultant m.m.f. around any closed loop of a magnetic circuit is equal to the algebraic sum of the products of the magnetic field strength and the length of each part of the circuit i.e. for a closed magnetic circuit

$$\boxed{\sum \text{m.m.f.} = \sum H.l}$$

3.20 Comparison of Magnetic and Electric Circuits

Similarities between electric and magnetic circuits are listed below :

Sr. No.	Electric Circuit	Magnetic Circuit
1.	Path traced by the current is called electric circuit.	Path traced by the magnetic flux is defined as magnetic circuit.
2.	E.M.F. is the driving force in electric circuit, the unit is **volts.**	M.M.F. is the driving force in the magnetic circuit , the unit of which is **ampere turns.**
3.	There is current I in the electric circuit measured in amperes.	There is flux ϕ in the magnetic circuit measured in webers.
4.	The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
5.	Resistance oppose the flow of the current. Unit is ohm.	Reluctance is opposed by magnetic path to the flux. Unit is ampere turn/weber.

6.	$R = \rho \frac{l}{a}$. Directly proportional to l. Inversely proportional to 'a'. Depends on nature of material.	$S = \frac{l}{\mu_0 \mu_r a}$. Directly proportional to l. Inversely proportional to $\mu = \mu_0 \mu_r$. Inversely proportional to area 'a'.
7.	The current $I = \frac{\text{e.m.f.}}{\text{resistance}}$	The flux $\phi = \frac{\text{m.m.f.}}{\text{reluctance}}$
8.	The current density $\delta = \frac{I}{a}$ A/m^2	The flux density $B = \frac{\phi}{a}$ Wb/m^2
9.	Conductivity is reciprocal of the resistivity. Conductance $= \frac{1}{R}$	Permeance is reciprocal of the reluctance. Permeance $= \frac{1}{S}$
10.	Kirchhoff's current and voltage law is applicable to the electric circuit.	Kirchhoff's m.m.f. law and flux law is applicable to the magnetic circuit.

There are few dissimilarities between the two which are listed below :

Sr. No.	Electric Circuit	Magnetic Circuit
1.	In the electric circuit the current actually flows i.e. there is movement of electrons.	Due to m.m.f. flux gets established and does not flow in the sense in which current flows.
2.	There are many materials which can be used as insulators i.e. air, P.V.C., synthetic resin etc, from which current can not pass.	There is no magnetic insulator as flux can pass through all the materials, even through the air as well.
3.	Energy must be supplied to the electric circuit to maintain the flow of current.	Energy is required to create the magnetic flux, but is not required to maintain it.
4.	The resistance and the conductivity are independent of current density (δ) under constant temperature. But may change due to the temperature.	The reluctance, permeance and permeability are dependent on the flux density.
5.	Electric lines of flux are not closed. They start from positive charge and end on negative charge.	Magnetic lines of flux are closed lines. They flow from N pole to S pole externally while S pole to N pole internally.
6.	There is continuous consumption of electrical energy.	Energy is required to create the magnetic flux and not to maintain it.

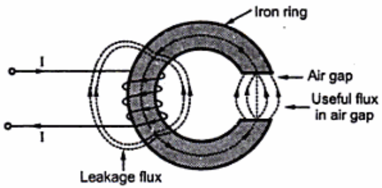
3.21 Magnetic Leakage and Fringing

Most of the applications which are using magnetic effects of an electric current, are using flux in air gap for their operation. Such devices are generators, motors, measuring instruments like ammeter, voltmeter etc. Such devices consist of magnetic circuit with an air gap and flux in air gap is used to produce the required effect.

Such flux which is available in air gap and is utilised to produce the desired effect is called **useful flux** denoted by ϕ_u .

It is expected that whatever is the flux produced by the magnetizing coil, it should complete its path through the iron and air gap. So all the flux will be available in air gap. In actual practice it is not possible to have entire flux available in air gap. This is because, we have already seen that there is no perfect insulator for the flux. So part of the flux completes its path through the air or medium in which coil and magnetic circuit is placed.

Key Point: *Such flux which leaks and completes its path through surrounding air or medium instead of the desired path is called the leakage flux.*



The Fig. 3.42 shows the useful and leakage flux.

Fig. 3.42 Leakage and useful flux

3.21.1 Leakage Coefficient or Hopkinson's Coefficient

The ratio of the total flux (ϕ_T) to the useful flux (ϕ_u) is defined as the **leakage coefficient of Hopkinson's coefficient** or **leakage factor** of that magnetic circuit.

It is denoted by λ .

$$\therefore \qquad \boxed{\lambda = \frac{\text{total flux}}{\text{useful flux}} = \frac{\phi_T}{\phi_u}}$$

The value of 'λ' is always greater than 1 as ϕ_T is always more than ϕ_u. It generally varies between 1.1 and 1.25. Ideally its value should be 1.

3.21.2 Magnetic Fringing

When flux enters into the air gap, it passes through the air gap in terms of parallel flux lines. There exists a force of repulsion between the magnetic lines of force which are parallel and having same direction. Due to this repulsive force there is tendency of the magnetic flux to bulge out (spread out) at the edge of the air gap. This tendency of flux to bulge out at the edges of the air gap is called **magnetic fringing**.

It has following two effects :

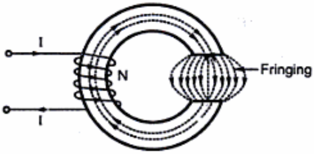


Fig. 3.43 Magnetic fringing

1) It increases the effective cross-sectional area of the air gap.

2) It reduces the flux density in the air gap.

So leakage, fringing and reluctance, in practice should be as small as possible.

Key Point : *This is possible by choosing good magnetic material and making the air gap as narrow as possible.*

Example 3.10 : *A cast iron ring of 40 cm mean length and circular cross section of 5 cm diameter is wound with a coil. The coil carries a current of 3 A and produces a flux of 3 mWb in the air gap. The length of the air gap is 2 mm. The relative permeability of the cast iron is 800. The leakage coefficient is 1.2. Calculate number of turns of the coil.*

Solution : Given, $l_t = 40 \text{ cm} = 0.4 \text{ m}$, $l_g = 2\times10^{-3}$ m

$$l_i = l_t - l_{gap} = 0.4 - 2\times10^{-3} = 0.398 \text{ m}$$

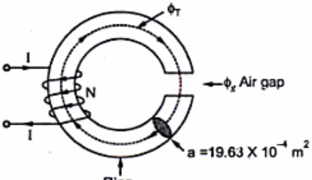


Fig. 3.44

$I = 3$ A, $\phi_g = 2\times10^{-3}$ Wb, $\mu_r = 800$, $\lambda = 1.2$

$$\lambda = \frac{\phi_T}{\phi_g} \quad \text{... Leakage coefficient}$$

$$\therefore \quad 1.2 = \frac{\phi_T}{2\times10^{-3}}$$

$$\therefore \quad \phi_T = 2.4\times10^{-3} \text{ Wb}$$

Now reluctance of iron path $S_i = \dfrac{l_i}{\mu_0\mu_r a}$

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times5^2 = 19.6349 \text{ cm}^2 = 19.634\times10^{-4}\text{m}^2$$

$$\therefore \quad S_i = \frac{0.398}{4\pi\times10^{-7}\times800\times19.63\times10^{-4}} = 201629.16 \text{ AT/Wb}$$

$$\phi_T = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S_i}$$

$$\therefore \quad 2.4\times10^{-3} = \frac{NI}{20162916}$$

$$\therefore \quad \text{NI for iron path} = 483.909 \text{ AT}$$

Reluctance of air gap $S_g = \frac{l_g}{\mu_0 a}$

$\therefore \quad S_g = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times19.634\times10^{-4}}$

$= 810608.86$ AT/Wb

Now $\phi_g = \frac{\text{m.m.f.}}{S_g}$

$\therefore \quad 2\times10^{-3} = \frac{N.I}{810608.86}$

$\therefore$ NI for air gap $= 1621.2177$ AT

$\therefore$ Total m.m.f. required $= (NI)_{iron} + (NI)_{air\ gap}$ NI = 483.909 + 1621.2177

$\therefore \quad NI = 2105.1267$ i.e. $N\times3 = 2105.1267$

$\therefore \quad N = 701.7 \approx$ **702 turns.**

Examples with Solutions

Example 3.11 : *An iron ring has circular cross-section 4 cm in radius and the average circumference of 100 cm. The ring is uniformly wound with a coil of 700 turns. Calculate,*

i) Current required to produce a flux of 2 mWb in the ring, if relative permeability of the iron is 900.

ii) If a saw cut of 1mm wide is made in the ring, calculate the current which will give same flux as in part (i). Neglect leakage and fringing. **(Dec. - 2001, May - 2003, May-2006)**

Solution : Given, $l = 100$ cm $= 1$ m, $N = 700$, $\phi = 2$ mWb $= 2\times10^{-3}$ Wb, $\mu_r = 900$

i) Radius r = 4 cm

$\therefore \quad a = \pi r^2 = \pi\times(4)^2 = 50.2654\ cm^2 = 50.2654\times10^{-4}\ m^2$

$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S}$

and $S = \frac{l}{\mu_0\mu_r a}$

$\therefore \quad \phi = \frac{NI\mu_0\mu_r a}{l}$

$\therefore \quad 2\times10^{-3} = \frac{700\times I\times4\pi\times10^{-7}\times900\times50.2654\times10^{-4}}{1}$

$\therefore \quad I = 0.5025 \text{ A}$

ii) Air gap of 1 mm is cut in the ring. So length of iron = 100 cm – 1 mm

$$l_i = 99.9 \text{ cm}$$

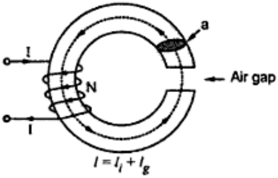


Fig. 3.45

Length of air gap $l_g = 1$ mm

$= 1 \times 10^{-3}$ m.

$\therefore$ Reluctance $S = S_i + S_g$

$$= \frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a}$$

as for air gap $\mu_r = 1$

$$S = \frac{99.9 \times 10^{-2}}{4\pi \times 10^{-7} \times 900 \times 50.26 \times 10^{-4}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 50.26 \times 10^{-4}}$$

$$\therefore \quad S = 175748.1 + 158331.62 = 334079.72 \text{ AT/Wb}$$

$$\therefore \quad \phi = \frac{NI}{S} \text{ i.e. } 2 \times 10^{-3} = \frac{700 \text{ I}}{334079.72}$$

$$\therefore \quad I = 0.9545 \text{ A}$$

Example 3.12 : *An iron ring has a mean diameter of 20 cm and a uniform circular cross section of 2.5232 cm diameter with a small brass piece fitted of 1 mm length. Three coils are wound on the ring as shown in the Fig. 3.46 and carry identical d.c. current of 2 A. If the relative permeability of iron is 800, estimate :- i) the magnetic flux produced in air-gap, ii) self-inductance of the arrangement. iii) net m.m.f. in the ring.* **(May - 2001)**

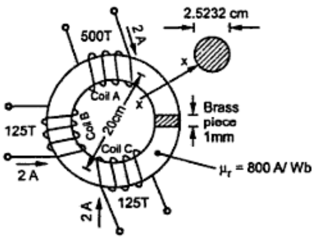


Fig. 3.46

Solution : From the various values given,

$$a = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(2.5232)^2 = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

$$l_T = \pi \times d_{mean} = \pi \times 20 = 62.8318 \text{ cm} = 0.6283 \text{ m}$$

$$\therefore \quad l_i = l_T - l_g = 0.6283 - 1 \times 10^{-3} = 0.6273 \text{ m}$$

and $$l_g = 1 \text{ mm} = 1 \times 10^{-3} \text{m}$$

$$\mu_r = 800 \quad \text{and} \quad I = 2 \text{ A}$$

$$S_T = S_i + S_g = \frac{l_i}{\mu_o \mu_r a} + \frac{l_g}{\mu_o a}$$

$$= \frac{0.6273}{4\pi \times 10^{-7} \times 800 \times 5 \times 10^{-4}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}}$$

$$= 1247973.698 + 1591549.431 = 2839523.129 \text{ AT/Wb}$$

$$\text{Net m.m.f.} = N_1 I_1 + N_2 I_2 + N_3 I_3 \quad \text{... all produce flux in same direction}$$

$$= 2\,[500 + 125 + 125] = \mathbf{1500\ AT} \quad \text{...}\ I_1 = I_2 = I_3 = 2\text{ A}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{1500}{2839523.129} = \mathbf{5.282 \times 10^{-4}\ Wb}$$

$$L = \frac{N^2}{S} = \frac{(500+125+125)^2}{2839523.129} = \mathbf{0.198\ H}$$

Example 3.13 : *A magnetic circuit is excited by three coils as shown in the Fig. 3.47. Calculate the flux produced in the air gap. The material used for core is iron having relative permeability of 800. The length of the magnetic circuit is 100 cm with an air gap of 2 mm in it. The core has uniform cross-section of 6 cm².* **(Dec. - 2002)**

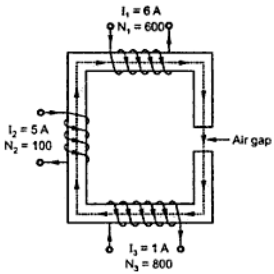


Fig. 3.47

Solution : Given, $N_1 = 600$, $I_1 = 6$ A, $N_2 = 100$, $I_2 = 5$ A

$$N_3 = 800, \quad I_3 = 1 \text{ A}, \qquad l_T = 100 \text{ cm} = 1 \text{ m}$$

$$l_i = l_T - l_g = 1\text{ m} - 2\times10^{-3} = 0.998\text{ m}$$

$$l_g = 2\times10^{-3}\text{ m}, \qquad \mu_r = 800,\ a = 6\text{ cm}^2 = 6\times10^{-4}\text{ m}^2$$

Now total reluctance $S = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.998}{4\pi\times10^{-7}\times800\times6\times10^{-4}} = 1654548.263\text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times6\times10^{-4}} = 2652582.385\text{ AT/Wb}$$

$$\therefore \quad S = 4307130.648\text{ AT/Wb.}$$

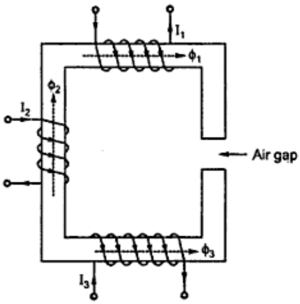


Fig. 3.47 (a)

Let us find the direction of flux due to various coils using **right hand thumb rule.**

As shown in the Fig. 3.47(a) m.m.f of coil (1) and (2) are in same direction while m.m.f. of coil (3) is in opposite direction.

$$\therefore \quad \text{Net m.m.f.} = (N_1 I_1) + (N_2 I_2) - (N_3 I_3)$$

$$= (600\times6) + (100\times5) - (1\times800)$$

$$NI = 3300\text{ AT}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S} = \frac{3300}{4307130.648}$$

$\therefore$ Flux in air gap ϕ = **0.7661 mWb**

Example 3.14 : *A magnetic circuit consists of two materials as shown in Fig. 3.48. The core has uniform cross-section of 6 cm². The core carries a winding with 900 turns. The current in the coil is 3 A. Calculate the flux produced in the air gap if the length of the air gap is 1mm. Relative permeability of material A is 1000 and that for B is 1500. The length of the magnetic circuit for A is 80 cm and for B it is 50 cm.*

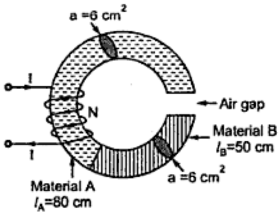


Fig. 3.48

Solution : $l_A = 80$ cm $= 0.8$ m, $l_B = 50$ cm $= 0.5$ m

$$a = 6 \text{ cm}^2 = 6\times10^{-4} \text{ m}^2, \quad l_g = 1 \text{ mm} = 1\times10^{-3} \text{ m}$$

$$N = 900, \quad I = 3 \text{ A}, \quad (\mu_r)_A = 1000, \quad (\mu_r)_B = 1500$$

The total reluctance $S = S_A + S_B + S_g$

$$S_A = \frac{l_A}{\mu_0(\mu_r)_A a} = \frac{0.8}{4\pi\times10^{-7}\times1000\times6\times10^{-4}} = 1061033 \text{ AT/Wb}$$

$$S_B = \frac{l_B}{\mu_0(\mu_r)_B a} = \frac{0.5}{4\pi\times10^{-7}\times1500\times6\times10^{-4}} = 442097 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{1\times10^{-3}}{4\pi\times10^{-7}\times6\times10^{-4}} = 1326291.2 \text{ AT/Wb}$$

$\therefore$ $S = 2894221.2$ AT/Wb

$\therefore$ The flux $\phi = \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{NI}{S}$

$$\therefore \quad \phi = \frac{3\times900}{2829421.2} = 9.542\times10^{-4} \text{ Wb} = \mathbf{0.9542 \text{ mWb}}$$

Example 3.15 : *A ring of cast steel has an external diameter of 24 cm and internal diameter of 18 cm. The area of cross-section is 3 cm × 3 cm. Inside and across the ring an ordinary steel bar 18 cm × 3 cm × 0.4 cm is fitted with negligible air gap. Calculate the m.m.f. required to produce a flux density of 1 Wb/m² in the other half ABD. Neglect leakage. The B-H curves are given below in table form. Refer the Fig. 3.49.*

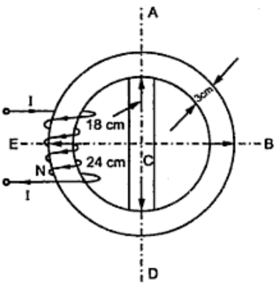


Fig. 3.49

For cast steel ring

B Wb/m²	1	1.193	1.4	1.6
H AT/m	900	800	1750	2000

For steel strip

B Wb/m²	1.4	1.45	1.5	1.6
H AT/m	1200	1650	1700	2000

Solution : Mean diameter of ring $= \dfrac{18+24}{2} = 21$ cm

$\therefore$ Mean circumference $= \pi D = \pi \times 21 = 65.9732$ cm

$= 0.659734$ m.

Length of path AED = Length of path ABD

$= \dfrac{0.659734}{2} = 0.32986$ m

For required flux density of 1 Wb/m^2 in path ABD, referring to corresponding B-H curve, value of H is 900 AT/m.

$\therefore$ m.m.f. for path ABD $= H \times l = 900 \times 0.32986$

$= 296.874$ AT

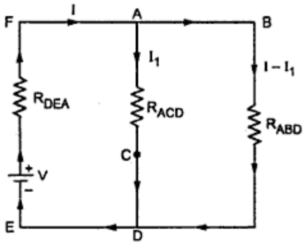


Fig. 3.49 (a)

Now the circuit is **parallel magnetic circuit,** the corresponding equivalent electric circuit is shown in the Fig. 3.49(a)

The m.m.f. across path ACD is same as across path ABD as both paths are parallel to each other, across the path DEA which is supplying m.m.f.

$\therefore$ m.m.f. across ACD = 296.874 AT

$\therefore$ H for path ACD $= \dfrac{\text{m.m.f}}{l_{ACD}}$

$= \dfrac{296.874}{0.18} = 1650$

The corresponding B = 1.45 from B-H curve given for the **steel strip,**

$\therefore$ Flux through path ACD $= B \times a$

Cross sectional area of steel $= 3 \times 0.4 = 1.2\ cm^2 = 1.2 \times 10^{-4}\ m^2$

$\phi_{ACD} = 1.45 \times 1.2 \times 10^{-4} = 1.74 \times 10^{-4}$ Wb

Similarly flux through path ABD $= B \times a$

Cross sectional area of ABC $= 3 \times 3 = 9\ cm^2 = 9 \times 10^{-4}\ m^2$

$\phi_{ABD} = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4}$ Wb

$\therefore$ Total flux supplied by path AED $= \phi_{ACD} + \phi_{ABD}$

$\phi_{AED} = 1.74 \times 10^{-4} + 9 \times 10^{-4} = 1.074 \times 10^{-3}$ Wb.

∴ Flux density for path AED, $B = \frac{\phi_{AED}}{a}$

∴ $B = \frac{1.074\times10^{-3}}{9\times10^{-4}} = 1.1933\ Wb/m^2$

The corresponding 'H' value required, from the table given is 1200.

∴ m.m.f. for path AED = $H \times l = 1200 \times 0.32986 = 395.832$ AT

∴ Total m.m.f. required = m.m.f. for path AED + m.m.f for path ACD or ABD

> **Key Point:** *The m.m.f. for one path ACD or ABD is to be added, as the two paths in parallel, same m.m.f. is required for both paths.*

∴ Total m.m.f = 395.832 + 296.874 = **692.706 AT**

Example 3.16 : *A soft iron ring of 20 cm mean diameter and circular cross-section of 4 cm diameter is wound with a magnetising coil. A current of 5 A flowing in the coil produces flux of 2.5 mWb in the air gap which is 2.2 mm wide. Taking relative permeability to be 1000 at this flux density and allowing for a leakage coefficient of 1.2, find the number of the turns on the coil.* **(Dec. - 97)**

Solution : $d_{mean} = 20$ cm, d = 4 cm, I = 5 A, $\phi_g = 2.5$ mWb, $l_g = 2.2$ mm, $\lambda = 1.2$

∴ mean length $l = \pi\times d_{mean} = \pi\times 20\times10^{-2} = 0.6283$ m

Cross section diameter = 4 cm

∴ $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(4)^2 = 12.566\ cm^2 = 12.566\times10^{-4}\ m^2$

l_g = length of air gap = 2.2 mm = 2.2×10^{-3} m

∴ l_i = length of iron path = $l - l_g = 0.6261$ m

Now $\lambda = \frac{\text{total flux}}{\text{air gap flux}} = \frac{\phi}{\phi_g}$ i.e. $1.2 = \frac{\phi}{2.5\times10^{-3}}$

∴ $\phi = 3\times10^{-3}$ Wb.

The total reluctance of the magnetic circuit,

$$S = S_i + S_g$$

Now $S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.6261}{4\pi\times10^{-7}\times1000\times12.566\times10^{-4}} = 396494.15$ AT/Wb

While $S_g = \frac{l_g}{\mu_0 a} = \frac{2.2\times10^{-3}}{4\pi\times10^{-7}\times12.566\times10^{-4}} = 1393207.4$ AT/Wb

Now $\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{Si+Sg}$

$$\phi_g = \frac{\text{m.m.f. for air gap}}{S_g} \text{ i.e. } 2.5\times10^{-3} = \frac{\text{m.m.f. for air gap}}{1393207.4}$$

$\therefore$ m.m.f. for air gap = 3483.01

$$\phi = \frac{\text{m.m.f. for iron}}{S_i} \text{ i.e. } 3\times10^{-3} = \frac{\text{m.m.f. for iron}}{396494.15}$$

$\therefore$ m.m.f. for iron = 1189.4825

Hence the total m.m.f. can be obtained as :

Total = m.m.f. for air gap + m.m.f. for iron

$\therefore$ m.m.f. = 3483.01 + 1189.48 = **4672.501 AT/Wb**

Now m.m.f = N × I i.e. 4672.501 = N × 5

$$\therefore \quad N = \frac{4672.501}{5} = \mathbf{934.5}$$

Hence the number of turns on the coil required is approximately **935.**

Example 3.17 : *An iron ring, cross sectional area of 5 cm² and mean length of 100 cm, has an air gap of 2 mm cut in it. Three separate coils having 100, 200 and 300 turns are wound on the ring and carry currents of 1 A, 2.5 A and 3 A respectively such that they produce additive fluxes in the ring. Relative permeability of the ring material is 1000. Calculate the flux in the air gap.* **(Dec. - 98)**

Solution : $a = 5\ \text{cm}^2 = 5\times10^{-4}\ \text{m}^2$, $l_T = 100\ \text{cm} = 1\ \text{m}$, $l_g = 2\ \text{mm} = 2\times10^{-3}\ \text{m}$

$$l_i = l_T - l_g = 1 - 2\times10^{-3} = 0.998\ \text{m}$$

Total reluctance $S = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.998}{4\pi\times10^{-7}\times1000\times5\times10^{-4}} = 1588366.332\ \text{AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times5\times10^{-4}} = 3183098.862\ \text{AT/Wb}$$

$$\therefore \quad S = 1588366.332 + 3183098.862 = 4771465.194\ \text{AT/Wb}$$

$$\text{Net m.m.f.} = N_1I_1 + N_2I_2 + N_3I_3 = 100\times1 + 200\times2.5 + 300\times3 = 1500\ \text{AT}$$

All m.m.f.s help each other as they produce **additive fluxes** in the ring.

$$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{1500}{4771465.194} = 0.0003143 \text{ Wb}$$

$$= \mathbf{0.3143 \text{ mWb}}$$

Example 3.18 : *The Fig. 3.50 shows a magnetic circuit with two similar branches and an exciting coil of 1500 turns on central limb. The flux density in the air gap is 1 Wb/m² and leakage coefficient 1.2. Determine exciting current through the coil. Assume relative permeability of the iron constant equal to 600.*

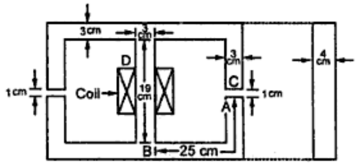


Fig. 3.50

Solution : Flux density in air gap $B_g = 1 \text{ Wb/m}^2$

$$a = 3 \times 4 = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad \phi_g = B_g \times a = 1 \times 12 \times 10^{-4} = 12 \times 10^{-4} \text{ Wb.}$$

$$\lambda = \frac{\phi_T}{\phi_g} = \frac{\phi_T}{12 \times 10^{-4}} = 1.2$$

$$\phi_T = \phi_{side} = 1.2 \times 12 \times 10^{-4}$$

$$\phi_{side} = 1.44 \times 10^{-3} \text{ Wb}$$

As the circuit is parallel magnetic circuit,

$$\phi_{coil} = \phi_{side1} + \phi_{side\ 2} = 2 \times 1.44 \times 10^{-3}$$

$$= 2.88 \times 10^{-3} \text{ (as sides are similar)}$$

Section I] Central limb

$$l_c = 19 \text{ cm} = 0.19 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_c = \frac{l_c}{\mu_0 \mu_r a} = \frac{0.19}{4\pi \times 10^{-7} \times 600 \times 12 \times 10^{-4}}$$

$$= 209996.11 \text{ AT/Wb}$$

Section II] One side limb

$$l_i = 25 + 25 = 50 \text{ cm} = 0.5 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.5}{4\pi \times 10^{-7} \times 600 \times 12 \times 10^{-4}}$$

$$= 552621.33 \text{ AT/Wb}$$

Section III] Air gap

$$l_g = 1 \text{ cm} = 0.01 \text{ m}$$

$$a = 12 \times 10^{-4} \text{ m}^2$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{0.01}{4\pi \times 10^{-7} \times 12 \times 10^{-4}}$$

$$= 6631456 \text{ AT/Wb}$$

$\therefore$ m.m.f. for central limb $= \phi_{coil} \times S_c = 2.88 \times 10^{-3} \times 209996.11$

$= 604.78$ AT

$\therefore$ m.m.f. for one side limb $= \phi_{side} \times S_i = 1.44 \times 10^{-3} \times 552621.33$

$= 795.774$ AT

$\therefore$ m.m.f. for air gap $= \phi_g \times S_g = 12 \times 10^{-4} \times 6631456$

$= 7957.7472$ AT

$\therefore$ Total m.m.f. = m.m.f. for central limb + m.m.f. for one side + m.m.f. for air gap

$$NI = 604.78 + 795.774 + 7957.7452$$

$$\therefore \quad 1500\ I = 9358.3107$$

$$\therefore \quad I = \mathbf{6.2388\ A}$$

Example 3.19 : *A magnetic circuit has the mean length of flux path of 20 cm, and cross sectional area of $1cm^2$. Relative permeability of its material is 2400. Find the m.m.f. required to produce a flux density of 2tesla in it. If an airgap of 1mm is introduced in it, find the m.m.f. required for the air gap as a fraction of the total m.m.f. to maintain the same flux density.* **(Dec. - 2003)**

Solution : $l_i = 20$ cm, $a = 1 \text{ cm}^2$, $\mu_r = 2400$, $B = 2$ T

$$S = \frac{l_i}{\mu_0 \mu_r a} = \frac{20 \times 10^{-2}}{4\pi \times 10^{-7} \times 2400 \times 1 \times 10^{-4}}$$

$$= 663.145\times10^3 \text{ AT/Wb}$$

$$\phi = B\times a = 2\times1\times10^{-4} \text{ Wb}$$

Now $\phi = \dfrac{\text{m.m.f}}{S}$

$\therefore$ $\text{m.m.f.} = \phi\times S = 2\times10^{-4}\times663.145\times10^3 = \mathbf{132.6291 \text{ AT}}$

Now l_g = 1 mm is introduced in it.

$\therefore$ l_i = 20 cm – 1 mm = 0.199 m

$\therefore$ $S_i = \dfrac{l_i}{\mu_0\mu_r a} = 659.829\times10^3 \text{ AT/Wb}$

and $S_g = \dfrac{l_g}{\mu_0 a} = 7.9577\times10^6 \text{ AT/Wb}$ ... $\mu_r = 1$ for air

$\phi = B\times a = 2\times10^{-4}$ Wb ... same as B is same

$\therefore$ $(\text{m.m.f})_{\text{iron path}} = S_i\times\phi = 131.9658$ AT

and $(\text{m.m.f})_{\text{air gap}} = S_g\times\phi = 1591.5494$ AT

$\therefore$ Total m.m.f = **1723.5152 AT**

$\therefore$ $(\text{m.m.f})_{\text{air gap}}$ = **0.9234** times total m.m.f

Example 3.20 : *A coil is wound uniformly with 300 turns over a steel of relative permiability 900, having a mean circumference of 40 mm and corss-sectional area of 50 mm². If a current of 5 A is passed through the coil, find*

i) m.m.f. ii) reluctance of the ring and iii) flux (Dec. - 2004)

Solution : Given : N = 300, $\mu_r = 900$, $l = 40 \text{ mm} = 40\times10^{-3}$ m,

$a = 50 \text{ mm}^2 = 50\times10^{-6} \text{ m}^2$, I = 5 A

i) m.m.f. = NI = 300 × 5 = **1500 AT**

ii) $$S = \frac{l}{\mu_0\mu_r a} = \frac{40\times10^{-3}}{4\pi\times10^{-7}\times900\times50\times10^{-6}} = \mathbf{70.7355\times10^3 \text{ AT/Wb}}$$

This is reluctance of the ring.

iii) $$S = \frac{\text{m.m.f.}}{\phi}$$

$\because$ $$\phi = \frac{\text{m.m.f}}{S} = \frac{1500}{70.7355\times10^3} = \mathbf{21.2057 \text{ mWb}}$$ ... Flux

Example 3.21 : *An iron ring has its mean length of flux path as 60 cm and its cross-sectional areas as 15 cm^2. Its relative permeability is 500. Find the current required to be passed, through a coil of 300 turns wound uniformly around it, to produce a flux density of 1.2 tesla. What would be the flux density with the same current, if the iron ring is replaced by air-core ?* **(May - 2005)**

Solution : Given : $l = 60$ cm $= 60 \times 10^{-2}$ m, $a = 15$ $cm^2 = 15 \times 10^{-4}$ m^2, $\mu_r = 500$

$N = 300$, $B = 1.2$ T, $I = ?$

Case 1 :

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{60\times10^{-2}}{4\pi\times10^{-7}\times500\times15\times10^{-4}} = 636.6197 \times 10^3 \text{ AT/Wb}$$

$$B = \frac{\phi}{a} \text{ i.e. } \phi = B \times a = 1.2 \times 15 \times 10^{-4} = 1.8 \times 10^{-3} \text{ Wb}$$

Now $$S = \frac{\text{m.m.f.}}{\phi} = \frac{NI}{\phi}$$

$$\therefore \quad 636.6197 \times 10^3 = \frac{300\ I}{1.8\times10^{-3}}$$

$$\therefore \quad I = \mathbf{3.8197\ A} \quad \text{... Current required}$$

Case 2 : Ring replaced by air core for which $\mu_r = 1$ hence $\mu = \mu_0$

$$\therefore \quad S = \frac{l}{\mu_0 a} = \frac{60\times10^{-2}}{4\pi\times10^{-7}\times15\times10^{-4}} = 318.3098 \times 10^6 \text{ AT/Wb}$$

I is same as calculated above i.e. I = 3.8197 A

$$\therefore \quad \text{m.m.f} = NI = 300 \times 3.8197 = 1145.91 \text{ AT}$$

$$\therefore \quad \phi = \frac{\text{m.m.f}}{S} = \frac{1145.91}{318.3098\times10^6} = 3.6 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad B = \frac{\phi}{a} = \frac{3.6\times10^{-6}}{15\times10^{-4}} = \mathbf{2.4 \times 10^{-3}\ T\ or\ Wb/m^2} \quad \text{... New flux density}$$

Example 3.22 : *A conductor of length 10 cm carrying 5 A is placed in a uniform magnetic field of flux density 1.25 tesla. Find the force acting on the conductor, if it is placed (i) along the lines of magnetic flux, (ii) perpendicular to the lines of flux, and (ii) at 30° to the flux.* **(May - 2005)**

Solution : $l = 10$ cm $= 10 \times 10^{-2}$ m, $I = 5$ A, $B = 1.25$ T

Fig. 3.51

Case 1 : Along lines of magnetic flux

θ = Angle between conductor and axis of magnetic field

$= 0°$

$\therefore$ $F = B.I.l. \sin\theta = 1.25 \times 5 \times 10 \times 10^{-2} \sin 0° = 0$ N

Case 2 : Perpendicular to lines of flux i.e. $\theta = 90°$

$$\therefore \quad F = B.I.l. \sin 90 = B.I.l. = 1.25 \times 5 \times 10 \times 10^{-2} = \mathbf{0.625\ N}$$

Case 3 : At 30° to the flux i.e. $\theta = 30°$

$$\therefore \quad F = B.I.l. \sin 30° = 1.25 \times 5 \times 10 \times 10^{-2} \times \frac{1}{2} = \mathbf{0.3125\ N.}$$

Example 3.23 : *A ring shaped core is made up of two parts of same material. Part one is a magnetic path of length 25 cm and with cross sectional area 4 cm^2, whereas part two is of length 10 cm and cross sectional area of 6 cm^2. The flux density in part two is 1.5 Tesla. If the current through the coil, wound over core, is 0.5 Amp., calculate the number of turns of coil. Assume μ_r is 1000 for material.* **[Dec.-2005]**

Solution : The arrangement is shown in the Fig. 3.52.

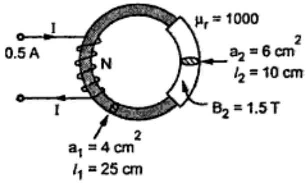


Fig. 3.52

$$B_2 = \frac{\phi}{a_2}$$

$$\therefore \quad \phi = 1.5 \times 6 \times 10^{-4}$$

$$= 9 \times 10^{-4}\ \text{Wb}$$

Key Point : *The flux ϕ is same through both the parts, as series circuit.*

$$S = S_I + S_{II} = \frac{l_1}{\mu_0 \mu_r a_1} + \frac{l_2}{\mu_0 \mu_r a_2}$$

$$= \frac{25 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 4 \times 10^{-4}} + \frac{10 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 6 \times 10^{-4}}$$

$$= 629988.3164 \ \text{AT/Wb}$$

$$\phi = \frac{\text{m.m.f}}{S} = \frac{NI}{S}$$

$$\therefore \quad 9 \times 10^{-4} = \frac{N \times 0.5}{629988.3164}$$

$$\therefore \quad N = 1133.979 \approx \mathbf{1134}$$ ... Number of turns

Example 3.24 : *The mean diameter of steel ring is 40 cm and flux density of 0.9 Wb/m^2 is produced by 3500 Aturns/meter. If the cross-section of the ring is 15 cm^2 and number of turns 440, calculate:*

i) the exciting current ii) the self inductance in henry and iii) exciting current and inductance when air gap of 1 cm is cut in the ring. **[Dec.-2006]**

Solution : d_{mean} = 40 cm, B = 0.9 Wb/m², H = 3500 AT/m, a = 15 cm², N = 440

$$l_T = \pi \times d_{mean} = \pi \times 40 = 125.6637 \text{ cm}$$

$$B = \mu_0 \mu_r H = \mu H$$

$$\therefore \quad \mu = \frac{B}{H} = \frac{0.9}{3500} = 2.5714 \times 10^{-4}$$

$$S_T = \frac{l_T}{\mu a}$$

$$\therefore \quad S_T = \frac{125.6637 \times 10^{-2}}{2.5714 \times 10^{-4} \times 15 \times 10^{-4}} = 3.2579 \times 10^6 \text{ AT/Wb}$$

$$\text{m.m.f.} = NI = H \times l_T = 3500 \times 125.6637 \times 10^{-2} = 4398.2295 \text{ AT}$$

i) $$I = \frac{4398.2295}{N} = \frac{4398.2295}{440} = 9.99 \approx \mathbf{10\ A}$$

ii) $$L = \frac{N^2}{S_T} = \frac{(440)^2}{3.2579 \times 10^6} = \mathbf{0.0594\ H}$$

iii) Now air gap l_g = 1 cm is cut.

$$\therefore \quad l_i = l_T - l_g = 124.6637 \text{ cm}$$

$$\therefore \quad S_T = S_i + S_g = \frac{l_i}{\mu a} + \frac{l_g}{\mu_0 a}$$

$$= \frac{124.6637 \times 10^{-2}}{2.5714 \times 10^{-4} \times 15 \times 10^{-4}} + \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 15 \times 10^{-4}}$$

$$= 3.232 \times 10^6 + 5.3051 \times 10^6 = 8.5371 \times 10^6 \text{ AT/Wb}$$

$$\phi = B \times a = 0.9 \times 15 \times 10^{-4} = 1.35 \times 10^{-3} \text{ Wb}$$

Now $$\phi = \frac{\text{m.m.f}}{\text{Reluctance}}$$

$$\therefore \quad 1.35 \times 10^{-3} = \frac{\text{m.m.f}}{8.5371 \times 10^6}$$

$$\therefore \quad \text{m.m.f} = NI = 11525.085$$

$$\therefore \quad I = \frac{11525.1494}{N} = \mathbf{26.1935\ A}$$

and $$L = \frac{N^2}{S_T} = \frac{(440)^2}{8.5371 \times 10^6} = \mathbf{0.0226\ H}$$

Example 3.25 : *An iron ring of mean length 50 cm has air gap of 1mm and a winding of 200 turns. If the relative permeability of iron is 300, find the flux density when a current of 1 amp flows through the coil.* **[Dec.-2007]**

Solution : The total m.m.f. is,

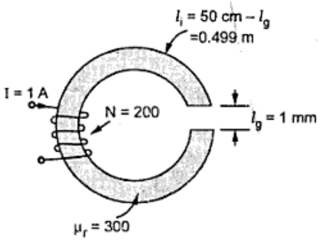


Fig. 3.53

$$\text{m.m.f.} = NI = 200 \times 1$$

$$= 200 \text{ AT}$$

The total reluctance is,

$$S_T = S_i + S_g$$

$$= \text{iron path} + \text{air gap}$$

$$= \frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a} \qquad \ldots \mu_r = 1 \text{ for air}$$

$$= \frac{1}{a}\left[\frac{0.499}{4\pi\times10^{-7}\times300} + \frac{1\times10^{-3}}{4\pi\times10^{-7}}\right]$$

$$= \frac{2119.4133}{a}$$

$$\therefore \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{S_T} = \frac{200}{\left(\frac{2119.4133}{a}\right)}$$

$$\therefore \quad \frac{\phi}{a} = \frac{200}{2119.4133} = 0.0943 \quad \text{but } \frac{\phi}{a} = B = \text{flux density}$$

$$\therefore \quad B = \mathbf{0.0943 \ Wb/m^2}$$

Example 3.26 : *A ring has diameter of 21 cm and a cross-sectional area of 10 cm². The ring is made up of semi-circular sections of cast iron and cast steel with each joint having a reluctance equal to an air gap of 0.2 mm. Find the Amp-Turns required to produce a flux of 8×10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively.* **[May-2008]**

Solution : The arrangement is shown in the Fig. 3.54.

At each joint there is an air gap. This is a series magnetic cirucit.

The total reactance is,

$$S_T = S_{iron} + S_{steel} + S_g + S_g$$

$$d = 21 \text{ cm} = \text{total diameter of ring}$$

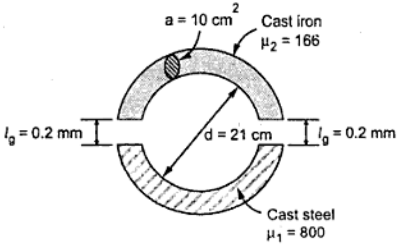


Fig. 3.54

$\therefore$ Total circumference $= \pi \times d = 65.973445$ cm $= 0.659734$ m

Total air gap length $= 2 \times l_g = 0.4$ mm $= 0.4 \times 10^{-3}$ m

$$\therefore \quad l_{iron} = \frac{\pi d - 2 l_g}{2} = 0.329667 \text{ m} = l_{steel}$$

$$\therefore \quad S_T = \frac{l_{steel}}{\mu_0 \mu_{r1} a} + \frac{l_{iron}}{\mu_0 \mu_{r2} a} + 2 \times \frac{l_g}{\mu_0 a}$$

$$= \frac{1}{4\pi \times 10^{-7} \times 10 \times 10^{-4}} \left[\frac{0.329667}{800} + \frac{0.329667}{166} + \frac{2 \times 0.2 \times 10^{-3}}{1} \right]$$

$$= 2226601.156 \text{ AT/Wb}$$

and $\phi = 8 \times 10^{-4}$ Wb $= \dfrac{\text{m.m.f.}}{S_T}$

$\therefore$ m.m.f. $= \phi \times S_T = 8 \times 10^{-4} \times 2226601.156 =$ **1781.281 AT**

Review Questions

1. *State and explain the laws of magnetism.*
2. *What is magnetic field and magnetic lines of force? State the properties of lines of force.*
3. *Define and state the units of following parameters:*

   i) magnetic flux *ii) magnetic pole strength*

   iii) magnetic flux density *iv) magnetic field strength*

   v) absolute permeability *vi) relative permeability*

   vii) m.m.f. *viii) reluctance*
4. *Derive the relation between m.m.f., reluctance and the flux.*
5. *State and explain the following rules:*

   i) Right hand thumb rule

   ii) Fleming's left hand rule

   iii) Fleming's right hand rule

   iv) Lenz's law

   v) Kirchhoff's laws for magnetic circuits
6. *Explain the proceduce to analyse following circuit, with suitable example :*

   i) Series magnetic circuit

   ii) Series magnetic circuit with air gap

   iii) Parallel magnetic circuit
7. *What is an electromagnet ? What is solenoid?*
8. *Point out the analogy between electric and magnetic circuits.*
9. *Explain the magnetic leakage and magnetic fringing.*

10. *Define leakage coefficient*

11. *Explain how current carrying conductor when placed in a magnetic field experiences a force.*

12. *A steel ring of 180 cm mean diameter has a cross sectional area of 250 mm^2. Flux developed in the ring is 500 μWb when a 4000 turns coil carries certain current. Find*

    i) m.m.f. required ii) reluctance iii) current in the coil

    Given that the relative permeability of the steel is 1100.

    (Ans. : 8181.72 AT, 1.6363×10^7, 2.045 A)

13. *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 25 A is passed through the coil, determine*

    i) m.m.f. ii) reluctance of ring and

    iii) flux **(Ans. : 7500 AT, 707355.3 AT/Wb, 0.0106 Wb)**

14. *Find the number of ampere turns required to produce a flux of 0.44 milli-weber in an iron ring of 100 cm mean circumference and 4 cm^2 in cross-section. B Vs μ_r test for the iron gives the following result :*

B in Wb/m^2	0.8	1.0	1.1	1.2	1.4
μ_r	2300	2000	1800	1600	1000

If a saw cut of 2 mm wide is made in the above ring, how many extra ampere turns are required to maintain same flux ? **(Ans. : 486.307 AT, 1744 AT)**

15. *An iron ring of 20 cm mean diameter and 10 cm^2 cross-section is magnetised by a coil of 500 turns. The current through the coil is 8 A. The relative permeability of iron is 500. Find the flux density inside the ring .* **(Ans. : 4 Wb/m^2)**

16. *An iron ring of 100 cm mean circumference is made from round iron of cross-section $10 cm^2$, it relative permeability is 800. If it is wound with 300 turns, what current is required to produce a flux of 1.1×10^{-3} Wb ?* **(Ans. 3.647 A)**

17. *A coil of 300 turns and of resistance 10 Ω is wound uniformly over a steel ring of mean circumference 30 cm, and cross-sectional area 9 cm^2. It is connected to a supply at 20 V d.c. If the relative permeability of the ring is 1500, find : (i) the magnetising force ; (ii) the reluctance ; (iii) the m.m.f. ; and (iv) the flux.*

    (Ans. : 600 AT, 176838.82 AT/Wb, 2000 AT/m, 3.3929 mWb)

18. *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900 having a mean circumference of 400 mm and cross-sectional area of 500 mm^2. If a current of 25 A is passed through the coil find*

    i) m.m.f. ii) reluctance and iii) flux **(Ans. : 7500 AT, 707355.3 AT/Wb, 10.6 mWb)**